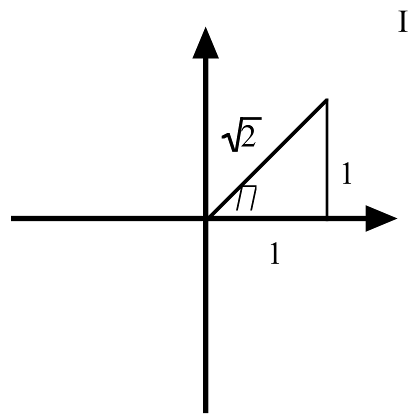
Reference Triagle Worksheet Name:

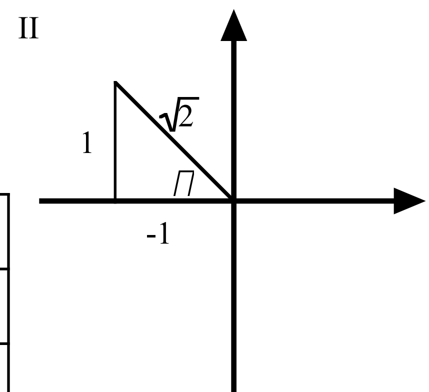
Find the exact trig ratios using the special reference triangle in each quadrant. Would you use a floor jack to lift weights? Don't use your calculator for this. This is an excercise for your brain.

θ coterminal with 45° or $\frac{\pi}{4}$



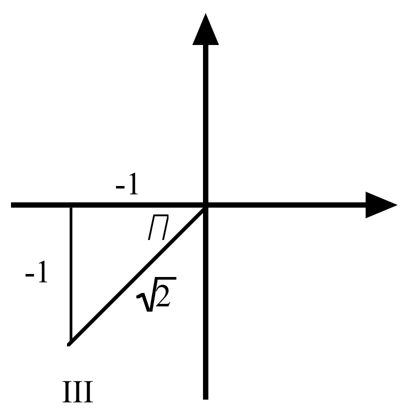
$\sin\theta =$		$\csc\theta =$	
$\cos\theta =$		$\sec\theta =$	
$\tan\theta =$		$\cot\theta =$	

θ coterminal with 135° or $\frac{3\pi}{4}$



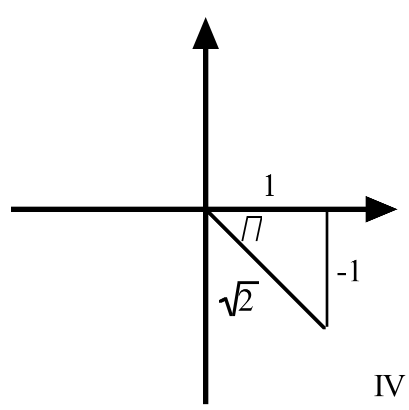
$\sin\theta =$		$\csc\theta =$	
$\cos\theta =$		$\sec\theta =$	
$\tan\theta =$		$\cot\theta =$	

θ coterminal with 225° or $\frac{5\pi}{4}$



$\sin\theta =$		$\csc\theta =$	
$\cos\theta =$		$\sec\theta =$	
$\tan\theta =$		$\cot\theta =$	

θ coterminal with 315° (-45°) or $\frac{7\pi}{4}$ $(-\frac{\pi}{4})$

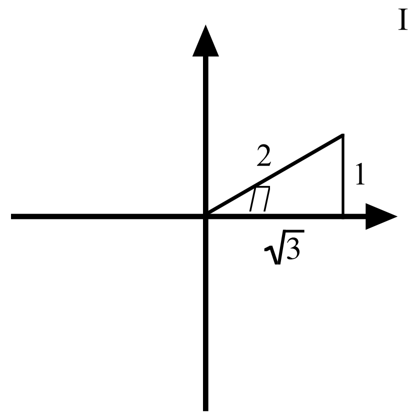


$\sin\theta =$		$\csc\theta =$	
$\cos\theta =$		$\sec\theta =$	
$\tan\theta =$		$\cot\theta =$	

Reference Triagle Worksheet Name:

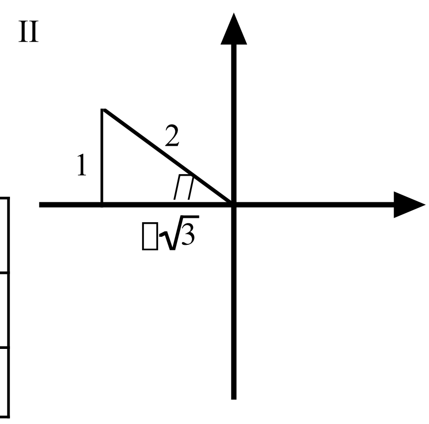
Find the exact trig ratios using the special reference triangle in each quadrant

θ coterminal with 30° or $\frac{\pi}{6}$



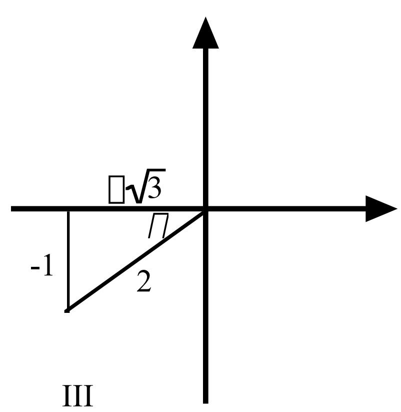
sin θ =		csc θ =	
cos θ =		sec θ =	
tan θ =		cot θ =	

θ coterminal with 150° or $\frac{5\pi}{6}$



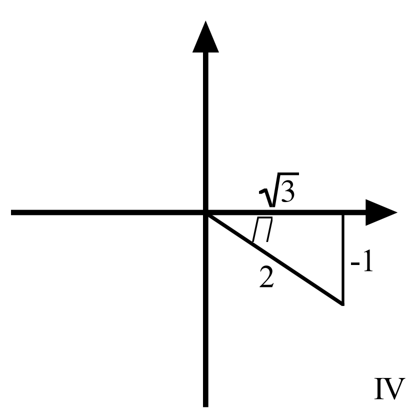
sin θ =		csc θ =	
cos θ =		sec θ =	
tan θ =		cot θ =	

θ coterminal with 210° or $\frac{7\pi}{6}$



sin θ =		csc θ =	
cos θ =		sec θ =	
tan θ =		cot θ =	

θ coterminal with 330° (-30°) or $\frac{11\pi}{6}$ ($-\frac{\pi}{6}$)

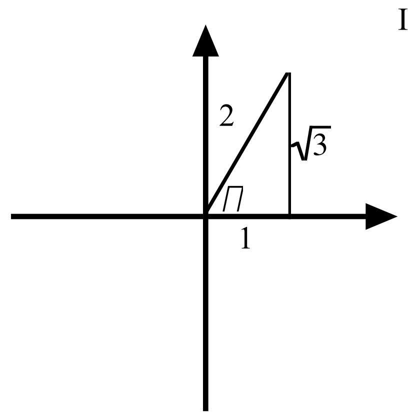


sin θ =		csc θ =	
cos θ =		sec θ =	
tan θ =		cot θ =	

Reference Triagle Worksheet Name:

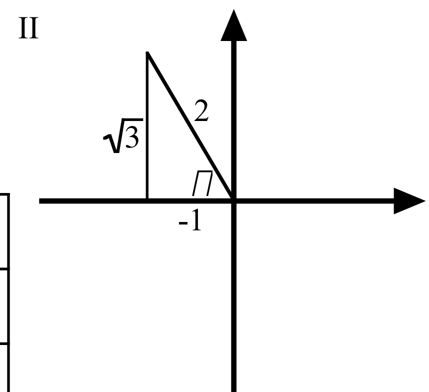
Find the exact trig ratios using the special reference triangle in each quadrant

θ coterminal with 60° or $\frac{\pi}{3}$



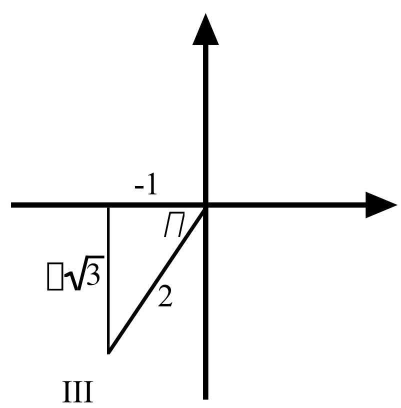
$\sin\theta =$		$\csc\theta =$	
$\cos\theta =$		$\sec\theta =$	
$\tan\theta =$		$\cot\theta =$	

θ coterminal with 120° or $\frac{2\pi}{3}$



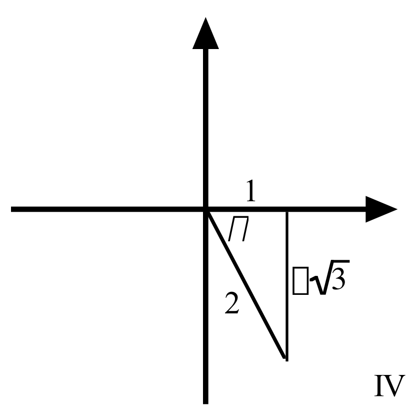
$\sin\theta =$		$\csc\theta =$	
$\cos\theta =$		$\sec\theta =$	
$\tan\theta =$		$\cot\theta =$	

θ coterminal with 240° or $\frac{4\pi}{3}$



$\sin\theta =$		$\csc\theta =$	
$\cos\theta =$		$\sec\theta =$	
$\tan\theta =$		$\cot\theta =$	

θ coterminal with 300° (-60°) or $\frac{5\pi}{3}$ $(-\frac{\pi}{3})$



$\sin\theta =$		$\csc\theta =$	
$\cos\theta =$		$\sec\theta =$	
$\tan\theta =$		$\cot\theta =$	

Now you can use the previous tables with the addition and subtraction formulas to find the exact values of other angles! For instance, since 75°=45°+30°, we can use the addition formula with the exact values:

$\sin 45° = \frac{1}{\sqrt{2}}$ $\sin 30° = \frac{1}{2}$ $\cos 45° = \frac{1}{\sqrt{2}}$ $\cos 30° = \frac{\sqrt{3}}{2}$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

For example: $\sin(75°) = \sin(45° + 30°) = \sin 45°\cos 30° + \cos 45°\sin 30°$

$$= \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\frac{1}{2} = \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}$$

Now you try:

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

1. $\sin 15° = \sin(45° - 30°) =$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

2. $\cos(75°) =$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

3. $\cos(15°) =$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

4. $\tan(75°)=$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

5. $\tan(15°)=$

6. *(Now you need to use something other than 45° and 30°)* $\sin(105°)=$

7. $\cos(120°)=$ *(Use the addition formula with 60°)*

Answers:

(1) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ or $\frac{\sqrt{6}-\sqrt{2}}{4}$ (2) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ or $\frac{\sqrt{6}-\sqrt{2}}{4}$ (the same because 15 + 75 = 90) (3) $\frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$

(4) $\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ (5) $\frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ (6) $\sin(60° + 45°) = \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$ (7) $\frac{1^2 - \sqrt{3}^2}{4} = -\frac{1}{2}$