

Parabola WS

Name:

General Form of a “smiling” Parabola whose vertex is (h, k) is

$$(x - h)^2 = 4a(y - k)$$

where a is the distance between the parabola and its focus (and for that matter, its directrix too!).

A negative a means it goes down:

$$(x - h)^2 = -4a(y - k)$$

and if you want a side-ways parabola swap the x and the y :

$$(y - k)^2 = 4a(x - h) \text{ for } ($$

$$(y - k)^2 = -4a(x - h) \text{ for })$$

1. What is the equation of a parabola whose directrix is $x = -8$ and whose vertex is $(0, 0)$, and whose focus is $(8, 0)$?

2. What is the directrix, vertex and focus of $x^2 = 12y - 12$?

3. What is the equation of a parabola whose directrix is $x = -4$ and whose vertex is $(-2, 3)$, and whose focus is $(0, 3)$?

4. What is the directrix, vertex and focus of $(x + 4)^2 = 16(y + 2)$?

5. What is the directrix, vertex and focus of $x^2 - 4x = 2y$?

6. A Typical Test question is to describe a parabola given certain clues. You would then have to sketch it, find its equation, the vertex, the points that describe the latus rectum, the coordinates of the focus, and equation for its directrix, just as it is in the homework, p 741 9-50. For example:

The directrix is the line $x = 5$.

The Focus is $(1, 2)$

Answers

1. $(y - 0)^2 = 4(8)(x - 0)$, that is $y^2 = 32x$
2. $(x - 0)^2 = 4(3)(y - 1)$ so the vertex is $(0, 1)$ $a = 3$ so the focus is above at $(0, 4)$ and the directrix is below at $y = -2$
3. $(y - 3)^2 = 8(x + 2)$
4. Vertex $V(-4, -2)$ Since $a = 4$, the focus is $F(-4, 2)$ the directrix below at $y = -6$
5. The equation needs a little “tweaking” for our purposes.

$$\begin{aligned}x^2 - 4x &= 2y \\(x - 2)^2 &= 2y + 4 \\(x - 2)^2 &= 2(y + 2)\end{aligned}$$

So vertex is $V(2, -2)$ Since $a = 2$ $F(2, 0)$ and directrix is below $y = -4$

6. It goes to the left so use the form $(y - k)^2 = 4a(x - h)$

The focus is $5-1=4$ away from the directrix, so $a = 2$. We have therefore the vertex at $(3, 2)$, and $4a = 4(2) = 8$, so the formula becomes

$$\begin{aligned}(y - k)^2 &= -4a(x - h) \\(y - 2)^2 &= -8(x - 3)\end{aligned}$$

