Counting and Probability

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Universal Set

$U = \{white, black, red, yellow, green, blue\}$

$A = \{red, blue, yellow\}$

$B = \{red, white, blue\}$

$$A \cap \overline{B}$$

What is in A, but not B?

{yellow}

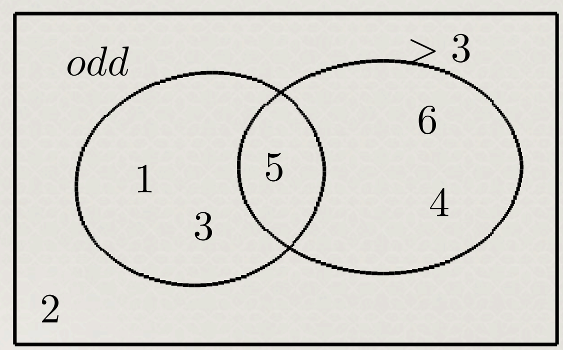
$$P(11,4) = {}_{11}P_4 = 11 * 10 * 9 * 8$$

$$= 7,920$$

$$C(8,3) = {}_8C_3 = \frac{8 * 7 * 6 * 5!}{3 * 2 * 5!}$$

$$= 8 * 6 = 56$$

If you have a fair six-sided die, what is the probability of rolling a number that is odd or greater than 3 ?



How many poker hands can be made (i.e., how many groupings of 5 cards from a deck of 52)?

$$\binom{52}{5} = {}_{52}C_5$$

$$= \frac{52!}{5!47!} = 2,598,960$$

Probability of a Royal Flush?

$$= \frac{4}{2,598,960} = \frac{1}{649,740}$$

If a fair coin is tossed 6 times, what is the probability that you get heads 3 times?

$$\frac{{}_6C_3}{2^6} = \frac{20}{64} = \frac{5}{16} \; or \; .3125$$

20 ways? The 3 heads by which toss:

123	234	345	456
124	235	346	
125	236	356	
126	245		
134	246		
135	256		
136			
145			
146			
156			

To gain access to open a door, R2D2 has to enter 3 symbols. There are 34 letters in the Aurabesh alphabet to try.

How many possible codes are there?

$$34 * 34 * 34 = 39,304$$

What is the probability that he finds the code on the second try?

$$\frac{39,303}{39,304} * \frac{1}{39,303} = \frac{1}{39,304}$$

A fair six-sided die is tossed 3 times. What is the probability that a three would be rolled at least once?

That is, Rolling a "3" 1, 2 or 3 times....seems complicated

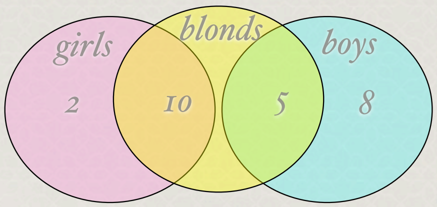
Isn't this the opposite of never tossing a 3?

$$P(1, 2, \; or \; 3) = 1 - P(never)$$

	Roll 1	Roll 2	Roll 3
ways not a 3	5	5	5
possible	6	6	6

$$1 - \frac{5^3}{6^3} = \frac{91}{216}$$

A classroom of 25 has 10 blond girls, and 13 boys. If there are 15 blonds, what is the probability that a randomly chosen student is a non-blond boy?



$$\frac{8}{25}$$

A secretary types four letters to four people and addresses the four envelopes. If she inserts the letters at random, each in a different envelope, what is the probability that exactly two letters will go into the right envelope?

Let the number be the letter, the position the envelope

For example, letter 1 in envelope 1, etc.:

1234

4!=24 permutations

1234 -4	2134 -2*	3124 -1	4123 - 0
1243 -2*	2143 -0	3142 -0	4132 - 1
1342 -1	2314 -1	3214 -2*	4213 - 1
1324 -2*	2341 -0	3241 -1	4231 - 2*
1423 -1	2413 -0	3412 -0	4312 - 0
1432 -2*	2431 -1	3421 -0	4321 - 0

6/24=.25

A 100 point exam has 11 problems. If each question is worth at least 5 points, and fractions of a point are not possible, how many ways can you assign points to the 11 problems?

*If each must have 5 points there are 100-5*11, or 45 points to "divvy up"*

WARNING! trickiness!!
we can choose to give a problem zero more points!

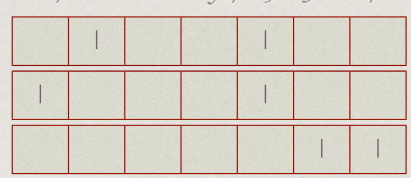
Partitioning Technique

Each point has a place, and each partition has a place

A simple example:

5 fish, divvied up among 3 dolphins, zero is a possibility for a naughty dolphin

Make 7 boxes [why 7? 5+(3-1)=7]



$1 fish \to d_1, 2 \to d_2, 2 \to d_3$

$0 fish \to d_1, 3 \to d_2, 2 \to d_3$

$5 fish \to d_1, 0 \to d_2, 0 \to d_3$

$${}_7C_2 = \frac{7!}{2!5!} = 21$$

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How many boxes? $45 + (11 - 1) = 55$

$$ {}_{55}C_{10} = \frac{55!}{10!45!} = 29,248,649,430 $$