In previous courses we "defined"

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$ ....plausible by numeric methods

But

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x \to 1^\infty$$ ..... so $1\cdot1\cdot1\cdot1=1$? No? No!

Proof (see Ex 5 p 366)

let $y = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

$$\ln y = \ln \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^x$$

$\boxed{\text{Alg. Trick: make into ratio}}$

$$\ln y = \lim_{x\to\infty} x \ln\left(1+\frac{1}{x}\right) \to \infty\cdot 0$$

$$\ln y = \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} \to \frac{0}{0}$$ form! L'Hôp permitted!

$$\ln y \overset{\text{(by LH)}}{=} \lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}}\cdot\frac{-1}{x^2}}{\frac{-1}{x^2}}$$ (chain rule) $\to \frac{0}{0}$

$$\ln y \overset{\text{(alg)}}{=} \lim_{x\to\infty} -x^2\cdot\frac{-1}{x^2+x}$$

$$\ln y \overset{\text{(alg)}}{=} \lim_{x\to\infty} \frac{x^2\cdot\frac{1}{x^2}}{(x^2+x)\cdot\frac{1}{x^2}}$$

$$\ln y \overset{\text{(alg)}}{=} \lim_{x\to\infty} \frac{1}{1+\frac{1}{x}} = 1$$

$$\ln y = 1$$

$$e^{\ln y} = e^1$$

$$y = e$$ QED ■