Disk Volume

$$\text{Disk Volume}= \pi \int_a^b [R(x)]^2 \, dx \text{ or } \pi \int_c^d [R(y)]^2 \, dy$$

Washer Volume

$$\text{Washer Volume}= \pi \int_a^b [R(x)]^2 - [r(x)^2] \, dx \text{ or } \pi \int_c^d [R(y)]^2 - [r(y)^2] \, dy$$

Volume of Solids with a Known Cross Section

$$\text{Volume of Solids with a Known Cross Sections} = \int_a^b A(x) \, dx \text{ or } \int_c^d A(y) \, dy$$

Where the most common cross sections are:

Square	Rectangle	Semicircle	Triangle	Equilateral $\triangle$	Iso.Rt. $\triangle$
$A = x^2$	$A = bh$	$A = \frac{1}{2}\pi r^2$	$A = \frac{1}{2}bh$	$A = \frac{\sqrt{3}}{4}s^2$	$A = \frac{(leg)^2}{2}$ leg in region $A = \frac{(hyp)^2}{4}$ hyp in region

1. To see an interactive 3-D rendering of these, go to `http://saxocellphone.github.io/Three/` and press the Show Examples box on the right. Find the volume of each:

   (a) Rotate $y = x$ and $y = x^2$ bounded by $x = 1$ and $x = 5$ around $y = 0$ (The x-axis)

   (b) Rotate $y = 3$ and $y = -x^2$ bounded by $x = -3$ and $x = 3$ around $y = 4$ (a horizontal line)

(c) Rotate $x = y$ and $x = \sqrt{y}$ bounded by $y = 1$ and $y = 5$ around $x = 1$ (a vertical line)

(d) Rotate $y = \sin x$ bounded by $x = 0$ and $x = \pi$ around $y = 0$ (The x-axis)

2. To see an interactive 3-D rendering, go to `https://www.geogebra.org/m/XFgMaKTy`
   (a) Find the volume of the solid whose base is the enclosed area between $y = \sqrt{x}$ and $y = x^2$, whose cross section ($\perp$ to the x-axis) is a square $\left(A = s^2\right)$

(b) Find the volume of the solid whose base is the enclosed area between $y = \sqrt{x}$ and $y = x^2$, whose cross section ($\perp$ to the x-axis) is a Equilateral Triangle $\left(A = \frac{\sqrt{3}}{4}s^2\right)$

(c) Find the volume of the solid whose base is the enclosed area between $y = \sqrt{x}$ and $y = x^2$, whose cross section ($\perp$ to the x-axis) is a Semi-circle $\left(A = \frac{1}{2}\pi r^2\right)$

Answers: