Integration by Parts (Ultra-Violet VooDoo)

This is way to integrate a product of unrelated functions:

$$\int f(x) \cdot g(x)\ dx$$

Think of $f(x)$ as the u function, and the $g(x)$ as dv, the derivative of the function v. Then we have:

$$\int u\ dv = uv - \int v\ du$$

To remember the right side, we recall the uv by thinking "ultraviolet" and the $v\ du$ as "Voodu."

Proof: We start with the product rule to derive $f(x) \cdot g(x)$ or $f \cdot g$:

$$(f \cdot g)' = fg' + gf'$$

Then integrate both sides:

$$\int (f \cdot g)'\ dx = \int fg'dx + \int gf'dx$$

$$f \cdot g = \int fg'dx + \int gf'dx$$

Subtract to solve for our result:

$$f \cdot g - \int gf'dx = \int fg'dx$$

or

$$\int fg'dx = f \cdot g - \int gf'dx$$

or

$$\int u\ dv = uv - \int v\ du$$

Example: $\int 3x^2 \ln x\ dx$. First let u be the $\ln x$ function, so that $\frac{1}{x}$ is du, its derivative. Next let $3x^2$ be the dv (the derivative of v) so that x^3 is the v function. We apply our rule:

$$\int 3x^2 \ln x\ dx = \ln x \cdot x^3 - \int x^3 \cdot \frac{1}{x}\ dx$$

$$x^3 \ln x - \int x^2\ dx$$

$$x^3 \ln x - \frac{x^3}{3} + C$$