

**Calculator skills to know:**

- 1..... How to make sure you are in RADIAN mode.
- 2..... How to graph two functions to find their intersections.
- 3..... How to graph a functions to find its zeros, end behavior, and asymptotes.
- 4..... How to Use the **TBL SET** and **TABLE** feature to quickly evaluate a function for values around a point
- 5..... How to Use the **WINDOW** and **ZOOM** feature to quickly **TRACE** a function for values around a point
- 6..... How to type functions that have absolute value, or greatest integer functions (**MATH** **NUM** 1:abs, 5:int)

**Precalculus skills to know:**

- 1..... How to find the equation of a line from 2 points or with slope and one point
- 2..... Know how to the equation of vertical and horizontal lines.
- 3..... Graph a piece-wise function
- 4..... Understand function notation (like  $f(x) = 5$ )
- 5..... Understand interval notation (like  $(-\infty, 4]$ )
- 6..... Finding the domain of functions (check for division by zero or the square root of a negative)
- 7..... How to factor  $(a^2 - b^2)$ ,  $(a^3 \pm b^3)$ , and expand  $(a \pm b)^2$
- 8..... How to factor polynomials & rational functions to discover the zeros, asymptotes, and holes (discontinuities)

**Calculus skills to know:**

- 1..... How to find a one or two sided limit from a graph.
- 2..... How to find a one or two sided limit from a table.
- 3..... Understand what a limit is, and whether they exist, or are infinite, or do not exist.
- 4..... How to find a one or two sided limit from a formula, using clever algebra tricks, the properties of limits, and a few theorems (like Thm 1.1-1.6, 1.9, 1.15).
- 5..... How to read and write a limit in proper mathematical notation.
- 6..... Knows what Continuity is, and how to justify whether a function is continuous or not ( at a point, an interval, or everywhere ) from a Thm (1.11, 1.12) or the definition (p 74, 77)
- 7..... How to find a value to make a piece-wise function continuous
- 8..... How to use the IVT (Thm 1.13) to justify a zero (or any other value) on a closed interval

## “No Calculator” Practice

Be neat.

For full credit show all work in an orderly way, as if to express your reasoning to another person.

2. (10 points) Evaluate the following limits. Show the work that leads to your answer.

1. (6 points) Given  $f(x) = \begin{cases} -2x^2 + 4x + 1, & x \geq 1 \\ 4 - x, & x < 1 \end{cases}$

Find:

(show the piece of the function you are using)

(a)

$$\lim_{x \rightarrow 1^+} f(x)$$

(b)

$$\lim_{x \rightarrow 4} \frac{(x-4)^2}{x^2 - 16}$$

$$\lim_{x \rightarrow \pi} \frac{\sec x}{x}$$

(c)

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\csc x}{x}$$

(b)

$$\lim_{x \rightarrow 1^-} f(x)$$

(d)

$$\lim_{x \rightarrow 0} \frac{3x}{\sin 3x}$$

(c)

$$\lim_{x \rightarrow 1} f(x)$$

(e)

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 + x + 1}$$

3. (12 points) Evaluate the following limits.  
Show the work that leads to your answer.

(a)

$$\lim_{x \rightarrow 0^-} \frac{\cos x}{x}$$

(b)

$$\lim_{x \rightarrow -2} \frac{x^2 - x - 6}{x + 2}$$

(c)

$$\lim_{x \rightarrow 0} \frac{\sqrt{1 + 2x} - 1}{x}$$

(d)

$$\lim_{x \rightarrow 0} \frac{\tan^2 x}{x}$$

4. (3 points) Given  $f(x) = \begin{cases} 10 - 2x - x^2, & x \leq 1 \\ 3 + 4e^{x-1}, & x > 1 \end{cases}$

(a)  $\lim_{x \rightarrow 1^-} f(x) =$

(b)  $\lim_{x \rightarrow 1^+} f(x) =$

(c)  $\lim_{x \rightarrow 1} f(x) =$

(d) Is  $f$  continuous at  $x = 1$ ? Why or why not?

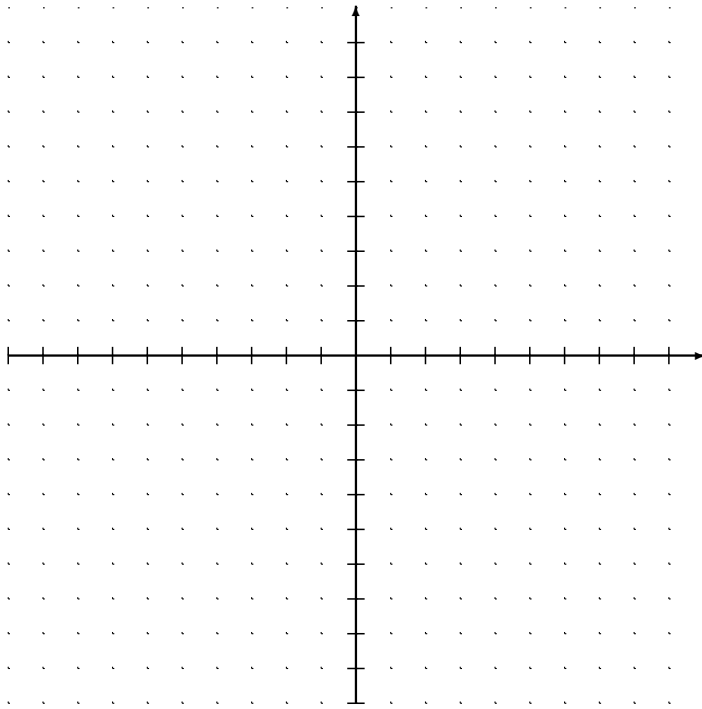
5. (2 points)  $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 + 2x - 3}$

6. (3 points)

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{2x}$$

7. (4 points) Write an equation of the linear function  $f$  where  $f(-1) = 5$  and  $f(3) = -7$

8. (6 points) Write the function  $f(x) = |x - 3| + 2x$  as a piece-wise function and sketch its graph. Be sure to include the point where the slope changes.



9. (2 points) Find the domain of  $g(x) = \sqrt{9 - 2x}$ . Show all work.

10. Let  $f(x) = x^3 - 2x^2 - 15x$

(a) (3 points) Find all the zeros of  $f(x)$

(b) (4 points) Using  $f(x)$  (defined above), let

$$h(x) = \begin{cases} \frac{f(x)}{x-5}, & x \neq 5 \\ k, & x = 5 \end{cases}$$

Find the value of  $k$  so that  $h(x)$  is continuous at  $x = 5$ . Justify your answer.

11. (3 points) Explain why the function

$$f(x) = 1 - 3x - x^3$$

has a zero on the closed interval  $[0, 1]$

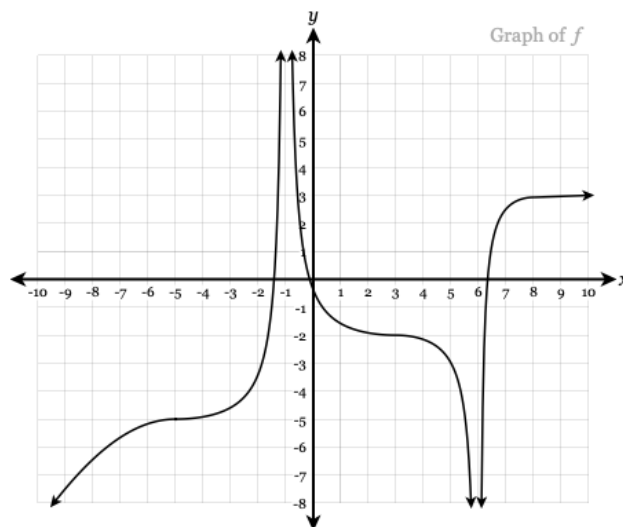
12. Let  $f(x) = \frac{x+1}{x^2-3x-4}$ .

- (a) (2 points) For what value(s) of  $x$  does  $f(x)$  have a discontinuity? 13. (3 points)

- (b) (2 points) Write equation(s) of any vertical asymptotes

- (c) (4 points) At each point of discontinuity found in part (a), determine whether or not the limit exists. Justify your answer.

- (d) (2 points) A rational function  $g(x) = \frac{a}{x+b}$  is defined such that  $g(x) = f(x)$  whenever  $f(x)$  is defined. Using the function  $f(x)$  from part (a), determine the values of  $a$  and  $b$ .



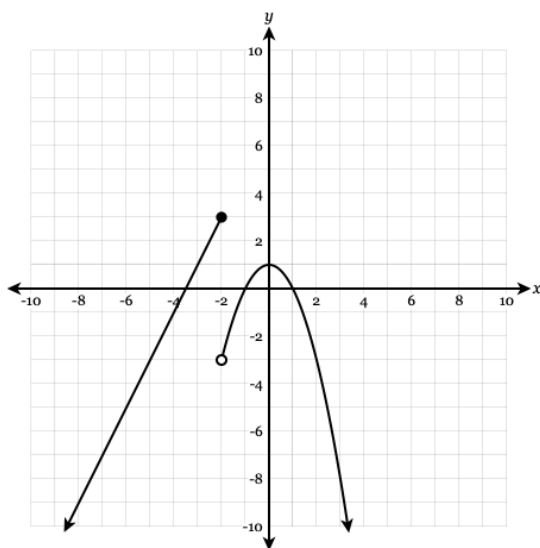
From the graph of  $f$  above, find the value.

(a) If  $\lim_{x \rightarrow a} f(x) = -\infty$ ,  $a =$

(b) If  $\lim_{x \rightarrow b} f(x) = \infty$ ,  $b =$

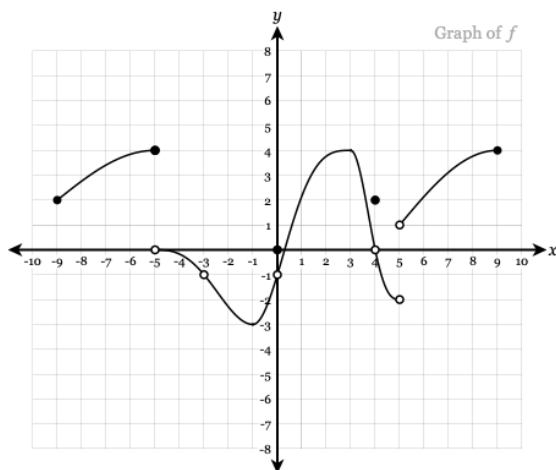
(c) If  $\lim_{x \rightarrow \infty} f(x) = c$ ,  $c =$

14. (6 points) Consider the graph of  $f$  below.



Based on this graph, for what values of  $x$  is the function  $f$  not continuous? Justify your conclusion with mathematical reasoning using the values given in the graph.

15. (5 points) Consider the graph of  $f(x)$  below. and find the missing value indicated by the question mark (?).



$$\lim_{x \rightarrow ?} f(x) = -3 \quad ? = \underline{\hspace{2cm}}$$

$$f(?) = 0 \quad ? = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow -5^-} f(x) = ? \quad ? = \underline{\hspace{2cm}}$$

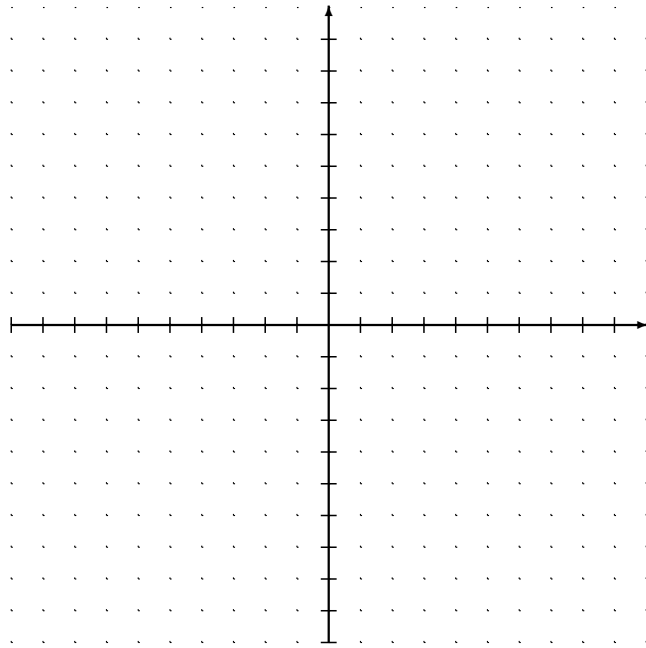
$$f(4) = ? \quad ? = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 4} f(x) = ? \quad ? = \underline{\hspace{2cm}}$$

**Part 2 “Calculators Active” Practice (Though they are not always necessary)**

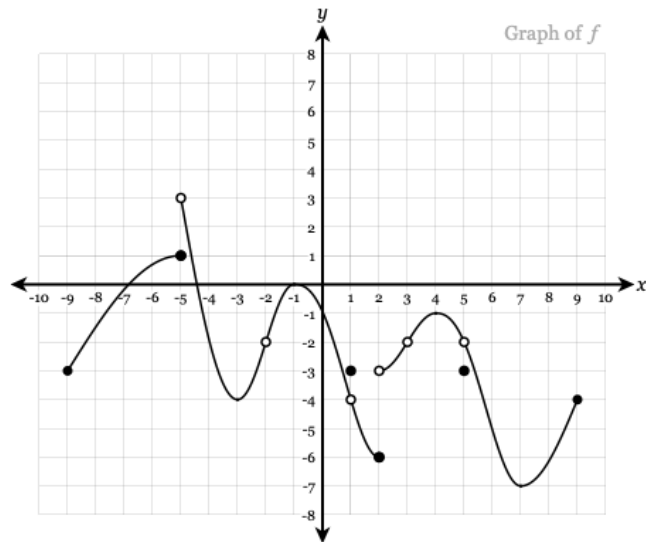
16. (5 points) Graph a function  $f$  that has the following criteria:

- (a) Domain of  $[-8, 5)$
- (b)  $\lim_{x \rightarrow 2} f(x) = -2$
- (c)  $f(-3) = 4$
- (d)  $\lim_{x \rightarrow -3^-} f(x) = 6$
- (e)  $\lim_{x \rightarrow -3^+} f(x) = -4$



17. (2 points) Using the graph of  $f$  below, choose true or false:

- (a)  $\lim_{x \rightarrow -2} f(x) = -2$
- (b)  $f$  is continuous on  $(-5, -2)$
- (c)  $f(1) = -3$
- (d)  $\lim_{x \rightarrow -5^+} f(x) = 1$
- (e)  $f$  has a removable discontinuity at  $x = 3$
- (f)  $\lim_{x \rightarrow 2^+} f(x)$  does not exist





Give final answers to three decimal places, either rounded or truncated

18. (2 points) Find the  $x$  intercept(s) of the function  $f(x) = 3x^3 - 4x^2 + 3x - 3$

19. (3 points) Find the the point(s)  $(x, y)$  of intersection of the functions  $f$  and  $g$ :

$$f(x) = \ln(x - 1)$$

$$g(x) = 2^{-x}$$

20. (2 points) Let  $f(x) = \frac{\cos x}{x - \pi}$ . Use the numerical method to evaluate

$$\lim_{x \rightarrow \pi^-} f(x)$$

$x$					
$f(x)$					

21. (2 points) Let  $f(x) = \frac{\cos x}{x - \pi}$ . Use the numerical method to evaluate

$$\lim_{x \rightarrow \pi} f(x)$$

$x$					
$f(x)$					