Calc AB Ch 3 Test Practice

Name: Block: Seat:

Chapter 3 Skills Check List:

1 ...... Absolute (p166) and Relative Extrema (p167)

2 ...... Critical Numbers (168)

3 ...... Extreme Value Theorem (EVT) (p166)

4 ...... Candidates Test (p 169)

5 ...... Rolle's Theorem (p 174)

6 ...... Mean Value Theorem (MVT) for rates (p 176)

7 ...... Increasing and Decreasing (181)

8 ...... 1^{st} Derivative Test for Relative Extrema (p 183)

9 ...... Concavity (p. 192-193)

10 ...... Points of Inflection (POI) (p.193-194)

11 ...... 2^{nd} Derivative Test for Relative Extrema (p 195)

12 ...... Limits at $\pm\infty$ (end behavior) (p 199-200, 205)

13 ...... Horizontal Asymptote (p. 200)

14 ...... Curve Sketching (Section 3.6)

15 ...... Optimization Word Problems (Section 3.7)

Chapter 3 Notes and Examples

1 ...... 3.1 EVT and Candidates Test

2 ...... 3.2 Rolle's Thm and MVT

3 ...... 3.3 Increasing/Decreasing and First Derivative Test

4 ...... 3.4 Concavity and Second Derivative Test

5 ...... 3.5-3.6 Limits at Infinity and Curve Sketching and Analysis

6 ...... 3.7 Optimization

Delta Math Check List:

1 ...... Lab 2-1: Extrema (3.1)

2 ...... Lab 2-2: MVT (3.2)

3 ...... Lab 2-3: 1st Der Test (3.3)

4 ...... Lab 2-4: 2nd Der Test (3.4)

5 ...... Lab 2-5: Function Analysis

6 ...... Lab 2-6: f, f', f'' and Particle Motion

Khan Academy Check List:

Applying Derivatives to Analyze Functions Unit 5:

1 ...... Quiz 1: Using the MVT

2 ...... Quiz 2: Using the EVT / Candidate's Test / 1st Derivative Test

3 ...... Quiz 3: Concavity / 2nd Derivative Test

4 ...... Quiz 4: Connecting f, f', f''

5 ...... Quiz 5: Optimization Problems

6 ...... Unit 5 Test (18 problems)

1. Use the Candidates Test to identify the absolute extrema of $f(x) = x^3 - 6x^2 + 9x + 5$ on the interval $0 \le x \le 4$.

2. Use the Candidates Test to identify the absolute extrema of $f(x) = x + \dfrac{7}{x}$ on the interval $1 \le x \le 3$.

3. Let f be a twice differentiable function. If $f'(7) = 0$ and $f''(7) > 0$ what conclusion can be made and why?

4. (Calculator NOT Active) Find at least one c such that the Mean Value Theorem applies to the function $f(x) = x^3$ on the interval $[0, 1]$ and write the equation(s) of the tangent line(s) to the curve at $x = c$.

5. (Calculator Active) Find all c on $[0, 2]$ such that the Mean Value Theorem applies to the function $f(x) = x^2 + 3x - 4\sin(2x + 3)$ on the interval $[0, 2]$.

6. Find absolute extrema for the function $f(x) = x^3 - 3x + 2$ on the interval $[-3, 2]$. Justify your conclusion.

7. Let $f(x) = x^3 - 3x + 2$.
   (a) Find f' and f'',

   (b) Find the critical points of f' and f''.

   (c) Draw sign lines for f' and f''

   (d) On what intervals are f increasing and decreasing? Justify.

   (e) On what intervals are f concave up or down? Justify.

   (f) Find all relative extrema and justify your conclusions with the 1^{st} Derivative Test

   (g) Find all relative extrema and justify your conclusions with the 2^{nd} Derivative Test

   (h) If any exist, find any points of inflection (POI). Justify.

   (i) What is the end behavior of f (That is, find $\lim_{x \to \pm\infty} f(x)$)

8. (Calculator NOT active) Given the function $g(x) = \dfrac{4x}{x^2+1}$, its 1^{st} derivative $g'(x) = \dfrac{4(1-x^2)}{(x^2+1)^2}$ and its 2^{nd} derivative $g''(x) = \dfrac{8x(x^2-3)}{(x^2+1)^3}$

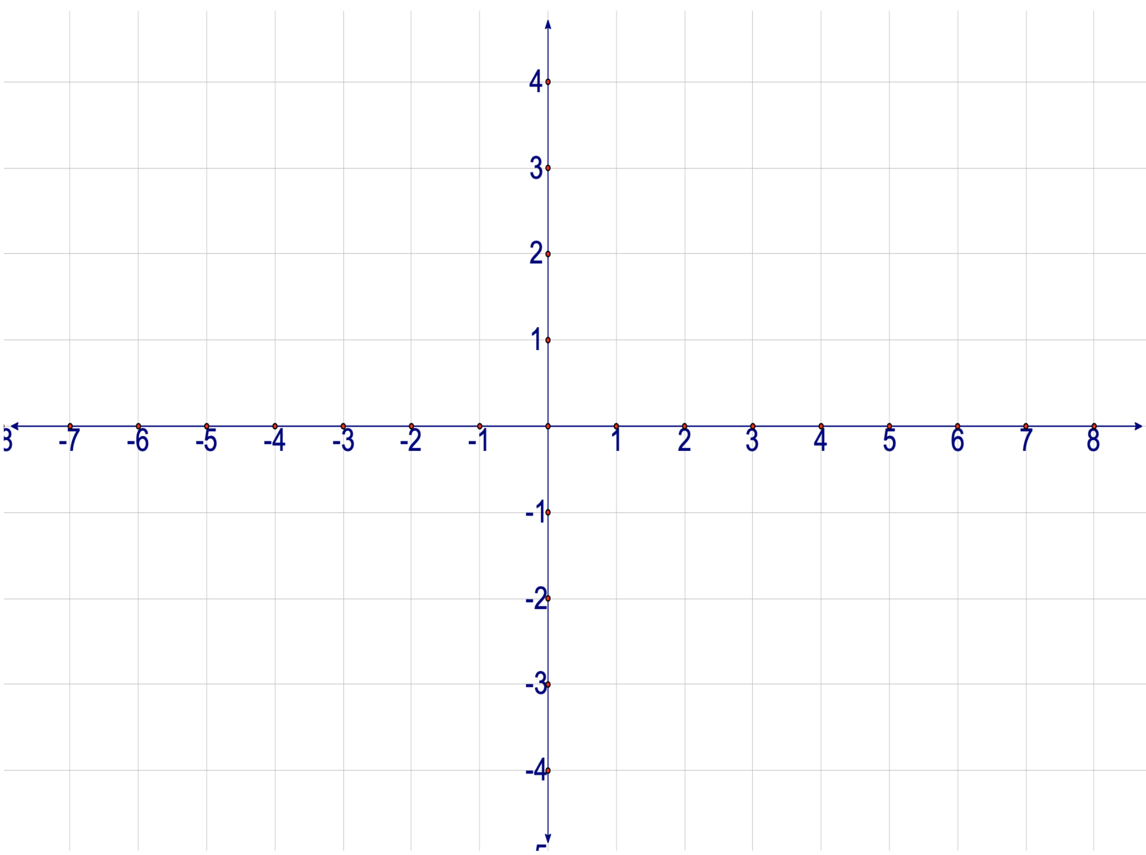
   (a) Make sign lines for g, g', and g''

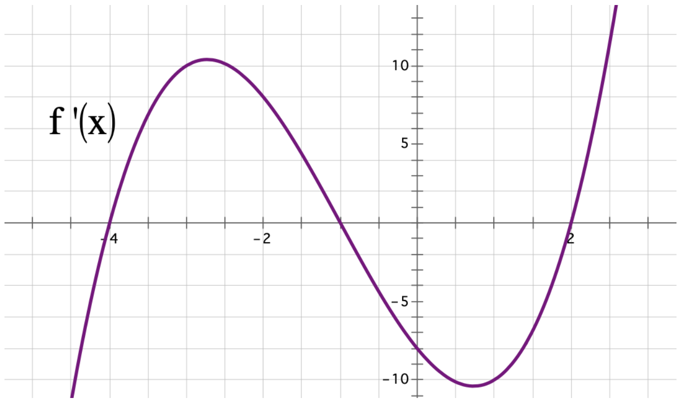
   (b) Find the (x, y) coordinates of any intercepts, and graph them.

   (c) Find any vertical or horizontal asymptotes and g's end behavior (i.e.: $\lim_{x\to\pm\infty} g(x)$) and use this to improve the graph.

   (d) Find the relative extrema and indicate them on the graph with a small horizontal bar through any extrema.

   (e) Improve the graph by noting where it is concave up or concave down, and find any points of inflection. Indicate a POI on the graph with a small perpendicular bar through the point.





9. (Calculator NOT active) Given the graph of $f'(x)$ above:
   (a) Approximate the sign lines for $f'(x)$ and $f''(x)$.

   (b) Find a relative maximum of f and use the 1^{st} Derivative Test to justify your conclusion.

   (c) Find a relative minimum of f and use the 2^{nd} Derivative Test to justify your conclusion.

   (d) On what interval(s) is f increasing? Justify.

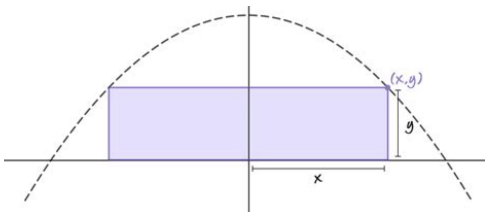
   (e) On what interval(s) is f decreasing? Justify.

   (f) On what interval(s) is f concave up? Justify.

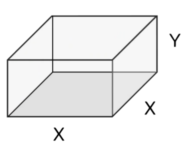
   (g) On what interval(s) is f concave down? Justify.

   (h) Find all the points of inflection of f, and justify why it is a point of inflection.

10. (Calculator NOT Active) A rectangle has its bottom edge on the x axis and its top corners are on the the graph $y = 6 - \frac{x^2}{24}$ What length and width should the rectangle have so that the rectangle has the largest area?



11. (Calculator NOT Active) Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 500 cubic feet of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?



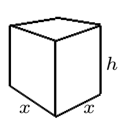
12. In 1977 P.M. Tuchinsky wrote an article called "The Human Cough". In it, the speed s of the air leaving your windpipe as you cough can be modeled by the function

$$s(x) = kx^2(R - x)$$

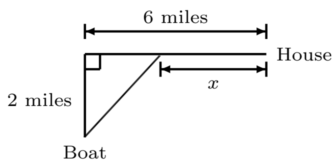
where R is radius of your windpipe (trachea) while resting (a positive constant), x is the radius of your windpipe while coughing (a positive variable), and k is positive constant.

If for a particular person $k = \frac{1}{3}$ and $R = 27$ mm, find the absolute maximum speed on the interval $0 \leq x \leq 27$. (Your answer will be in terms of mm per second)

13. A box has 2 square ends and 4 rectangular sides. The square ends are made out of plastic that costs \$5 dollars per square foot, and the cardboard sides cost \$3 dollars per square foot. Find the dimensions of the box that has a volume of 60 cubic feet, that is the cheapest to make. *Hint:* Use the candidates test, first derivative test, or second derivative test to justify your claim that it is indeed the minimum cost.



14. A person in a rowboat two miles from the nearest point on a straight shoreline wishes to reach a house six miles farther down the shore. If the person can row at a rate of 3 miles per hour and walk at a rate of 5 miles per hour, find the least amount of time required to reach the house. How far from the house should the person land the rowboat?



15. A cylindrical metal container, open at the top, is to have a capacity of 24π cubic inches. The cost of material used for the bottom of the container 15 cents per square inch, and the cost of the material used for the curved part is 5 cents per square inch. Find the dimensions that will minimize the cost of the material, and find the minimum cost.