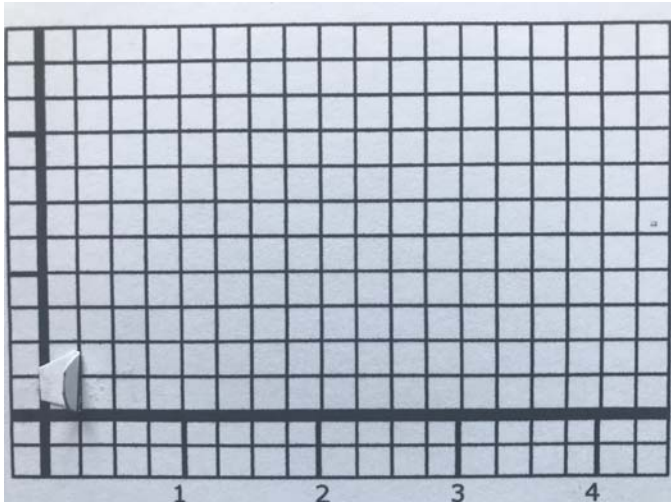
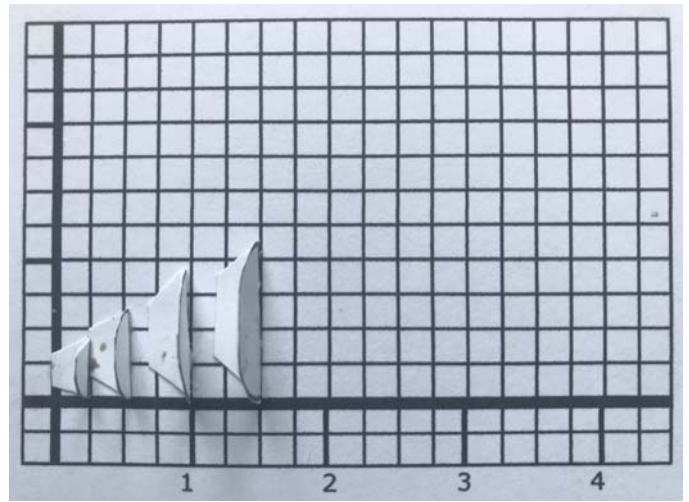


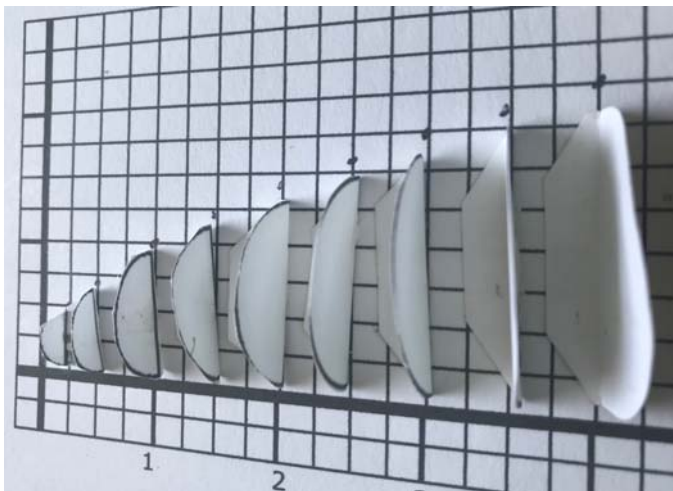
General Directions



Step 1: After students cut out their nine similar figures they should begin gluing them on to the first quadrant. This picture illustrates the gluing of the first shape at $x = 0.25$. The edge on the coordinate grid will be approximately $\sqrt{0.25} = 0.5$.



Step 2: Continue to glue additional cross sections at the appropriate x value. Each edge against the coordinate axis will be approximately the square root of that x -value.



Step 3: Continue to add additional cross section until all 9 have been glued down.

Graph and Construction

| x | $y = \sqrt{x}$ |
|------|----------------|
| 0 | 0 |
| 0.25 | 0.5 |
| 0.5 | 0.71 |
| 1.0 | 1 |
| 1.5 | 1.22 |
| 2 | 1.41 |
| 2.5 | 1.58 |
| 3 | 1.73 |
| 3.5 | 1.87 |
| 4 | 2 |

Base for region R

Finding the Volume of Your Solid

Shape of your cross section: semi-circle Area formula for your cross section: $\frac{\pi (\frac{\sqrt{x}}{2})^2$

1. As you look at the x value where each slice of your polygon touches region R, what general algebraic expression represents the length of the line where each of polygon touches region R?

2. Using your answer to question 1 what algebraic expression would represent the area of each polygon in your model? $A(x) = \frac{\pi (\sqrt{x})^2}{8}$

3. Each of your cross-sectional slices is very thin. Since the thickness of each slice is only a small change in x , what general expression represents the thickness of each polygonal slice? Δx

4. The cross sections of your figure are all similar polygons. Write a Riemann sum that will find the volume of your nine cross sections by completing this statement:

$$\text{volume} = \sum_{n=1}^9 \left(\frac{A(x)}{8} \right) (\Delta x)$$

5. If we kept increasing the number of slices until the shape became a solid, the Riemann sum can be rewritten as an integral to find the volume of the solid. Complete the general integral for the volume of your solid.

$$\text{volume} = \sum_{n=1}^{\infty} A(x) \Delta x = \sum_{n=1}^{\infty} \left(\frac{1}{8} \pi (\sqrt{x})^2 \right) (\Delta x) = \int_0^4 \left(\frac{\pi x}{8} \right) dx$$

6. Complete the integration procedure to find the volume of your solid.

$$\int_0^4 \frac{1}{8} \pi x dx = \frac{1}{8} \pi \frac{x^2}{2} \Big|_0^4 = \pi - 0 = \pi$$

Math Through Discovery LLC © 2019

Step 4: Complete the questions below the figure. Then have students collect data on the Gathering All Information page.