Circuit: Separable Differential Equations

But first…a few words:

How to solve the differential equation $\frac{dy}{dx} = ky$? This is an important concept as there are many real-world situations where the rate of change is proportional to a relationship of the original independent and dependent variables. Unfortunately, our dependent variable is making an appearance on the right side, there is no obvious independent variable and simple antidifferentiation rules won't do the trick.

Using differential notation, we can *separate the variables*. $\frac{dy}{dx} = ky \quad \rightarrow \quad dy = ky\,dx \quad \rightarrow \quad \frac{dy}{y} = k\,dx$

This allows us to get our dependent variable, y, on the left and our independent variable, x, on the right. The k is a constant and could have final position on either side.

*Note we are not multiplying by dx, though the elegance of the notation allows it to seem so.

We'll now integrate both sides. While both are indefinite, a single arbitrary constant on either side can be defined in such a way to cover both integrals.

$$\int \frac{dy}{y} = \int k\,dx$$

$$\ln|y| = kx + c$$

We'll rewrite the natural logarithm relationship using e. The absolute value is no longer necessary as this expression is always greater than zero. Continue simplifying using exponent properties for multiplication.

$$y = e^{kx+c}$$

$$y = e^{c}e^{kx}$$

$$y = Ce^{kx}$$

Since e^c is a constant, we'll replace it with a new constant, $e^c = C$.

You may recognize this form as the continuous interest formula or a form of a radioactive decay equation. This relationship is so common, that on the AP® exam you may jump from $\frac{dy}{dx} = ky$ directly to the solution $y = Ce^{kx}$. In cases where x represents time or the independent variable is t, C is the initial value of y.

If you ever need to take an even root of y to isolate it as a function, you will leave the $\pm$ in front of the radical unless you're given an initial condition that allows you to determine which path (positive or negative) that you are using.

You will use the separation of variables and your knowledge of algebra and calculus to finish the following circuit.

Circuit: Separable Differential Equations Name______________________________

Directions: Beginning in the first cell marked #1, find the requested information. To advance in the circuit, hunt for your answer and mark that cell #2. Continue working in this manner until you complete the circuit. If you do not have enough space in the cell, you may work on a separate sheet of paper and attach. You may not need to separate in every problem.

<table>
<tr>
<td>Ans: $y = 19e^{t^2}$

#1 Solve for y.
$\frac{dy}{dt} = 2t$

Find a function that might be y.</td>
<td>Ans: $y = 5e^{2t}$

______ Solve for y.
$\frac{dy}{dt} = 2$

Find a function that might be y.</td>
</tr>
<tr>
<td>Ans: $y = \frac{1}{2}t + 7$

______ Solve for y.
$\frac{dy}{dt} = \frac{1}{2}t$

Find a function that might be y.</td>
<td>Ans: $y = -\sqrt{3t^2 + 2t + 1}$

______ Solve for y.
$\frac{dy}{dt} = \frac{3t^2y + 3ty}{y}$

Find a function that might be y.</td>
</tr>
<tr>
<td>Ans: $y = 2t - 5$

______ Solve for y.
$\frac{dy}{dt} = \frac{3t}{y}$

Find a function that might be y.</td>
<td>Ans: $y = \frac{1}{3}t^3 - 4$

______ Solve for y. Sometimes, you'll need to factor to help separate.
$\frac{dy}{dt} = 2ty - 8t = 2t(y - 4)$

Find a function that might be y.</td>
</tr>
</table>

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Ans: $y = \frac{1}{4}t^2 - 2$ ______ Solve for y. $\frac{dy}{dt} = t^2$ Find a function that might be y.	Ans: $y = t^3 + \frac{3}{2}t^2 + \frac{1}{2}$ ______ Solve for y. $\frac{dy}{dt} = -\frac{3t}{y}$ Find a function that might be y.
Ans: $y = -\sqrt{9 - 3t^2}$ ______ Solve for y. $\frac{dy}{dt} = 2ty$ Find a function that might be y.	Ans: $y = 3e^{t^2} + 4$ ______ Solve for y. $\frac{dy}{dt} = 2y$ Find a function that might be y.
Ans: $y = t^2 + 8$ ______ Solve for y. $\frac{dy}{dt} = \frac{1}{2}$ Find a function that might be y.	Ans: $y = -\sqrt{3t^2 + 9}$ ______ Solve for y. $\frac{dy}{dt} = \frac{3t + 1}{y}$ Find a function that might be y.