CALCULUS AB
WORKSHEET ON RIEMANN SUMS

Work the following on **notebook paper.** Use your calculator, and give decimal answers to four decimal places.

On problems 1 – 2, estimate the area bounded by the curve and the x-axis on the given interval using the indicated number of subintervals by finding:
(a) a left Riemann sum

(b) a right Riemann sum

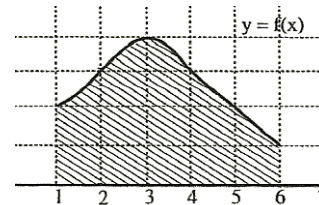1. $y = \sqrt{x}$ [0,1] $n = 4$ subintervals   2. $y = \dfrac{1}{x}$ [1, 3] $n = 4$ subintervals

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3. Estimate the area bounded by the curve and the x-axis on [1, 6] using the 5 equal subintervals by finding:

(a) a left Riemann sum

(b) a right Riemann sum

(c) a midpoint Riemann sum

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4. Oil is leaking out of a tank. The rate of flow is measured every two hours for a 12-hour period, and the data is listed in the table below.

Time (hr)	0	2	4	6	8	10	12
Rate (gal/hr)	40	38	36	30	26	18	8

(a) Draw a possible graph for the data given in the table.

(b) Estimate the number of gallons of oil that have leaked out of the tank during the 12-hour period by finding a <u>left</u> Riemann sum with <u>three</u> equal subintervals.

(c) Estimate the number of gallons of oil that have leaked out of the tank during the 12-hour period by finding a <u>right</u> Riemann sum with <u>three</u> equal subintervals.

(d) Estimate the number of gallons of oil that have leaked out of the tank during the 12-hour period by finding a <u>midpoint</u> Riemann sum with <u>three</u> equal subintervals.

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x	0	2	4	6
$f(x)$	4	k	8	12

5. The function f is continuous on the closed interval [0, 6] and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)\, dx$ found with 3 subintervals of equal length is 52. What is the value of k ?

TURN->>>

6. Oil is being pumped into a tank over a 12-hour period. The tank contains 120 gallons of oil when $t = 0$. The rate at which oil is flowing into the tank at various times is modeled by a differentiable function R for $0 \le t \le 12$, where t is measured in hours and $R(t)$ is measured in gallons per hours. Values of $R(t)$ at selected values of time t are shown in the table below.

t (hours)	0	3	5	9	12
$R(t)$ (gallons per hour)	8.9	6.8	6.4	5.9	5.7

(a) Estimate the number of gallons of oil in the tank at $t = 12$ hours by using a left Riemann sum with four subintervals and values from the table. Show the computations that lead to your answer.

(b) Estimate the number of gallons of oil in the tank at $t = 12$ hours by using a right Riemann sum with four subintervals and values from the table. Show the computations that lead to your answer.

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7. A hot cup of coffee is taken into a classroom and set on a desk to cool. When $t = 0$, the temperature of the coffee is $113°$ F. The rate at which the temperature of the coffee is dropping is modeled by a differentiable function R for $0 \le t \le 8$, where $R(t)$ is measured in degrees Fahrenheit per minute and t is measured in minutes. Values of $R(t)$ at selected values of time t are shown in the table below.

t (minutes)	0	3	5	8
$R(t)$ (° F/min.)	5.5	2.7	1.6	0.8

(a) Estimate the temperature of the coffee at $t = 8$ minutes by using a left Riemann sum with three subintervals and values from the table. Show the computations that lead to your answer.

(b) Estimate the temperature of the coffee at $t = 8$ minutes by using a right Riemann sum with three subintervals and values from the table. Show the computations that lead to your answer.

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8. (Modification of 2004 Form B AB 3/ BC 3)

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table below.

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7. 0	9.2	9.5	7. 0	4.5	2.4	2.4	4.3	7.2

Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate the distance traveled by the plane during the 40 minutes. Show the computations that lead to your answer.

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t (minutes)	0	4	7	9
$r(t)$ (gallons per minute)	9	6	4	3

9. Water is flowing into a tank at the rate $r(t)$, where $r(t)$ is measured in gallons per minute and t is measured in minutes. The tank contains 15 gallons of water at time $t = 0$. Values of $r(t)$ for selected values of t are given in the table above. Using a trapezoidal sum with the three subintervals indicated by the table, what is the approximation of the number of gallons of water in the tank at $t = 9$?