## WORKSHEET ON RELATED RATES

Work the following on notebook paper.

1. A particle moves along the curve $y=\sqrt{x^{2}+1}$ in such a way that $\frac{d x}{d t}=4$. Find $\frac{d y}{d t}$ when $x=3$.
2. An observer stands 25 feet from the base of a flagpole and watches a flag being lowered at a rate of $5 \mathrm{ft} / \mathrm{sec}$. Determine the rate at which the angle of elevation from the observer to the flag is changing at the instant that the flag is 25 feet above eye-level.
3. A man 6 feet tall walks at a rate of 5 feet per second away from a light that is 15 feet above the ground. When he is 10 feet from the base of the light,
(a) at what rate is the tip of his shadow moving?
(b) at what rate is the length of his shadow changing?
4. The length $L$ of a rectangle is decreasing at the rate of $2 \mathrm{~cm} / \mathrm{sec}$ while the width W is increasing at the rate of $2 \mathrm{~cm} / \mathrm{sec}$. When $\mathrm{L}=12$ and $\mathrm{W}=2$, find the rate of change of:
a) the area of the rectangle
b) the perimeter of the rectangle
c) the length of a diagonal of the rectangle.
5. At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at a rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when the pile is 15 feet high? (The volume of a cone is given by $V=\frac{1}{3} \pi r^{2} h$.)
6. A conical tank (with vertex down) is 10 feet across the top and 12 feet deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of the water when the water is 8 feet deep.
7. A paper cup, which is in the shape of a right circular cone, is 16 cm deep and has a radius of 4 cm . Water is draining out of the cup at a constant rate of $2 \frac{\mathrm{~cm}^{3}}{\mathrm{sec}}$. When the radius has a length of 3 cm , what is the rate of change of the radius? (Volume of a cone $\left.=\frac{1}{3} \pi r^{2} h\right)$
8. A snowball is in the shape of a sphere. Its volume is increasing at a constant rate of $10 \frac{\mathrm{in}^{3}}{\mathrm{~min}}$. How fast is the radius increasing when the volume of the sphere is $36 \pi \mathrm{in}^{3} ?\left(\right.$ Volume of a sphere $\left.=\frac{4}{3} \pi r^{3}\right)$

9. A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person's shadow is lengthening?
10. A cup has the shape of a right circular cone. The height of the cup is 12 cm , and the radius of the opening is 3 cm . Water is poured into the cup at a constant rate of $2 \mathrm{~cm}^{3} / \mathrm{sec}$. What is the rate at which the water level is rising when the depth of the water in the cup is 5 cm ? (The volume of a cone of height $h$ and radius $r$ is given by $V=\frac{1}{3} \pi r^{2} h$.)
11. The volume of a cylinder with radius $r$ and height $h$ is given by $V=\pi r^{2} h$. The radius and height of the cylinder are increasing at constant rates. The radius is expanding $\frac{1}{3} \frac{\mathrm{~cm}}{\mathrm{sec}}$ and the height is increasing at a rate of $\frac{1}{2} \frac{\mathrm{~cm}}{\mathrm{sec}}$. At what rate, in cubic centimeters per second, is the volume of the cylinder increasing when its height is 9 cm and the radius is 4 cm ?
12. Sand is deposited into a pile with a circular base. The volume $V$ of the pile is given by $V=\frac{r^{3}}{3}$, where $r$ is the radius of the base, in feet. The circumference of the base is increasing at a constant rate of $5 \pi$ feet per hour. When the circumference of the base is $8 \pi$ feet, what is the rate of change of the volume of the pile, in cubic feet per hour?
