Related Rates Quiz Solutions Name: Block: Seat:

Based on HW 1-16, 2.6 (p157) 11, 13, 17

1. (6 points) (Block B common question) The radius of a sphere is increasing at a constant rate of 3 centimeters per minute. At the instant when the radius of the sphere is 444 centimeters, what is the rate of change of the volume? The volume of a sphere can be found with the equation $V = \frac{4}{3}\pi r^3$.

   (a) Find: $\left.\dfrac{dV}{dt}\right|_{r=444cm}$ (in cm^3/min)

   (b) Given: $\dfrac{dr}{dt} = 3$ cm/min

   (c) Connection: $V = \frac{4}{3}\pi r^3$

   (d) Diff: $\dfrac{dV}{dt} = 4\pi r^2 \left(\dfrac{dr}{dt}\right)$

   (e) $\dfrac{dV}{dt} = 4\pi(444)^2(3)$

   The rate of change of the volume is is **increasing** at the rate $236,532\pi$ cubic cm per minute when the radius of the sphere is 444 centimeters.

2. (6 points) (Block F common question) A cup shaped like a cone has a radius of 2 cm and measures 6 cm from top to bottom. Water leaks out of the bottom at a rate of 2 cubic cm per second. How fast is the water level dropping when the height of the water is 3 (Hint the volume of a cone is $V = \frac{1}{3}\pi r^2 \cdot h$).

   (a) Find: $\left.\dfrac{dh}{dt}\right|_{h=3cm}$ (in cm/sec)

   (b) Given: $\dfrac{dV}{dt} = -2\ \text{cm}^3/\text{sec}$

   (c) Connection: $V = \frac{1}{3}\pi r^2 \cdot h$ and because $r = 2$ when $h = 6$ similar triangles tells us $r = 1$ when $h = 3$, i.e $r = \frac{h}{3}$ so we have $V = \frac{1}{3}\pi(\frac{h}{3})^2 \cdot h$ or $V = \frac{\pi}{27}h^3$

   (d) Diff: $\dfrac{dV}{dt} = \dfrac{\pi}{9}h^2 \left(\dfrac{dh}{dt}\right)$

   (e) $-2 = \dfrac{\pi}{9}(3^2)\left(\dfrac{dh}{dt}\right)$
   $\dfrac{dh}{dt} = -\dfrac{2}{\pi}$ cm/sec

   The rate of change of height is **decreasing** at the rate $\dfrac{2}{\pi}$ cm per second when the height of the water in the cup is 3 cm.

3. (6 points) The area of a circle is decreasing at a constant rate of 60 square inches per second. At the instant when the radius of the circle is 11 inches, what is the rate of change of the radius? (Hint $A = \pi r^2$)

   (a) Find: $\left.\dfrac{dr}{dt}\right|_{r=11in}$ (in inch/sec)

   (b) Given: $\dfrac{dA}{dt} = -60$ in^2/sec

   (c) Connection: $A = \pi r^2$

   (d) Diff: $\dfrac{dA}{dt} = 2\pi r \left(\dfrac{dr}{dt}\right)$

   (e) $-60 = 2\pi(11)\left(\dfrac{dr}{dt}\right)$

   $\dfrac{dr}{dt} = -\dfrac{60}{22\pi}$ in /sec

   The rate of change of the radius is **decreasing** at the rate of $\dfrac{60}{22\pi}$ inches per second when the radius is 11 inches.

4. (6 points) The radius of a circle is increasing at a constant rate of 8 centimeters per minute. At the instant when the area of the circle is 49π square inches, what is the rate of change of the area? (Hint $A = \pi r^2$)

   (a) Find: $\left.\dfrac{dA}{dt}\right|_{A=49\pi\ cm^2}$ (in cm^2/min)

   (b) Given: $\dfrac{dr}{dt} = 8$ cm/min

   (c) Connection: $A = \pi r^2$, and $r = 7$ when the Area is 49π

   (d) Diff: $\dfrac{dA}{dt} = 2\pi r \left(\dfrac{dr}{dt}\right)$

   (e) $\dfrac{dA}{dt} = 2\pi(7)(8) = 112\pi$ cm^2/min

   The rate of change of the area of the circle is **increasing** at the rate of 112π cm per minute when the area is 49π square cm.

5. (6 points) The volume of a sphere is increasing at a constant rate of 3062 cubic feet per second. At the instant when the volume of the sphere is 653653 cubic feet, what is the rate of change of the radius? The volume of a sphere can found with the equation $V = \frac{4}{3}\pi r^3$

   (a) Find: $\left.\dfrac{dr}{dt}\right|_{V=653653\ ft^3}$ (in ft/sec)

   (b) Given: $\dfrac{dV}{dt} = 3062$ ft^3/second

   (c) Connection: $V = \frac{4}{3}\pi r^3$ and

   $r = \left(\dfrac{(3)(653653)}{4\pi}\right)^{1/3} = \left(\dfrac{196059}{4\pi}\right)^{1/3}$ when the volume is 653653 cubic feet.

   (d) Diff: $\dfrac{dV}{dt} = 4\pi r^2 \left(\dfrac{dr}{dt}\right)$

   (e) $3062 = 4\pi \left(\dfrac{196059}{4\pi}\right)^{2/3} \left(\dfrac{dr}{dt}\right)$

   $\dfrac{dr}{dt} = \dfrac{4\pi}{3062} \left(\dfrac{196059}{4\pi}\right)^{2/3}$ ft/sec

   The rate of change of the radius is **increasing** at the rate of $\dfrac{4\pi}{3062} \left(\dfrac{196059}{4\pi}\right)^{2/3}$ feet per second when the volume is 653653 cubic feet.