CALCULUS AB
WORKSHEET ON PARTICLE MOTION

Work these on **notebook paper.** On problems 1 – 5, do **not** use your calculator except for parts (c), (e), and (f) on problem 1. Write your justifications in a sentence.

1. A particle moves along a horizontal line so that its position at any time is given by $s(t) = t^3 - 12t^2 + 36t,\ t \geq 0,$ where s is measured in meters and t in seconds.

(a) Find the instantaneous velocity at time t and at $t = 3$ seconds.
(b) When is the particle at rest? Moving to the right? Moving to the left? Justify your answers.
(c) (Calc) Find the displacement of the particle after the first 8 seconds.
(d) (Calc) Find the total distance traveled by the particle during the first 8 seconds.
(e) Find the acceleration of the particle at time t and at $t = 3$ seconds.
(f) (Calc) Graph the position, velocity, and acceleration functions for $0 \leq t \leq 8$.
(g) When is the particle speeding up? Slowing down? Justify your answers.

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2. The maximum acceleration attained on the interval $0 \leq t \leq 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

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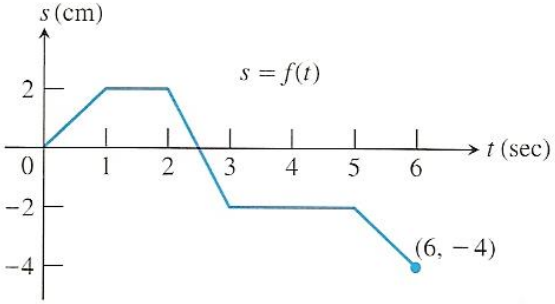
3. The figure on the right shows the position s of a particle moving along a horizontal line.

(a) When is the particle moving to the left? moving to the right? standing still? Justify your answer.
(b) For each of $v(1.5)$, $v(2.5)$, $v(4)$, and $v(5)$, find the value or explain why it does not exist.
(c) Graph the particle's velocity.
(d) Graph the particle's speed.



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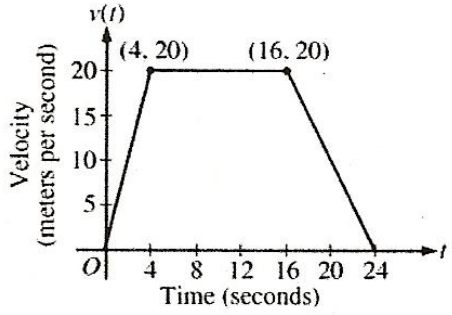
4. (2005) A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph on the right.



(a) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.
(b) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.
(c) Find the average rate of change of v over the interval $8 \leq t \leq 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

5. (Modification of 2009 Form B, Problem 6)

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	–10	–8	–4	7

The velocity of a particle moving along the *x*-axis is modeled by a differentiable function *v*, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above.

(a) Use data from the table to estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

(c) Based on the values in the table, what is the smallest number of instances at which the velocity $v(t)$ could equal $-9\,\text{m/sec}$ on the interval $0 < t < 40$? Justify your answer.

On problems 6 – 10, you must use your calculator.

6. The position of a particle at time t seconds, $t \ge 0$, is given by $s(t) = t^2 - \sin t,\ 0 \le t \le 3$, where t is measured in seconds and *s* is measured in meters. Find the particle's acceleration each time the velocity is zero.

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7. A particle's velocity at time t seconds, $t \ge 0$, is given by $v(t) = \cos\left(t^2\right) + t,\ 0 \le t \le 2$, where *t* is measured in seconds and *v* is measured in meters/second. Find the velocity of the particle each time the acceleration is zero.

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8. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$.

(a) For $0 \le t \le 8$, when is particle P moving to the left?

(b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

(c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

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9. A particle moves along the *x*-axis so that its velocity at time *t*, for $0 \le t \le 5$, is given by $v(t) = \ln\left(t^2 - 3t + 3\right)$.

(a) Find the acceleration of the particle at time $t = 4$.

(b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left? Justify your answer.

(c) Find the average rate of change of $v(t)$ on $1.5 \le t \le 3.2$.

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10. A particle moves along the x-axis so that its velocity for $0 \le t \le 4$ is given by $v(t) = 45\sqrt{t}\cos\left(0.063t^2\right)$ meters per hour. Find the time interval during which the velocity of the particle is at least 60 meters per hour. Find the distance traveled by the particle during the interval when the velocity of the particle is at least 60 meters per hour.