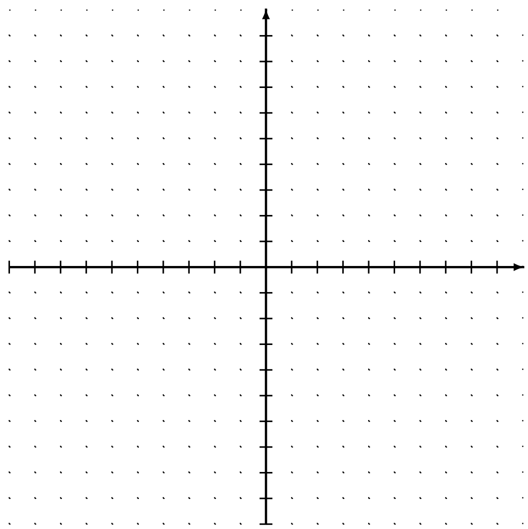
Calc AB: Indeterminate Forms and L'Hôpital

Name:

1. Sketch the graphs of
   $f(x) = x^2 - 2x$ and $g(x) = 4x - 2x^2$



   (a) Sketch the two tangent lines when $x = 2$

   (b) Write the equations (in point slope form) of the two tangent lines when $x = 2$

   (c) So what would be

   $$\lim_{x\to 2} \frac{f(x)}{g(x)} \text{ and } \lim_{x\to 2} \frac{f'(x)}{g'(x)}?$$

   (d) L'Hôpital's Rule states that if $f(x)$ and $g(x)$ are differentiable functions and $f(a) = g(a) = 0$, then

   $$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

   if the limit of the right exists. (Note this is the ratio of the derivatives, not the derivative of the quotient)

   It also works for $f(a) = g(a) = \infty$ and when $x \to \infty$

   (e) $\frac{0}{0}$ form: $\lim\limits_{x\to 0} \dfrac{e^{2x} - 1}{x}$

   (f) $\frac{\infty}{\infty}$ form: $\lim\limits_{x\to\infty} \dfrac{\ln x}{x}$

   (g) Done twice: $\lim\limits_{x\to -\infty} \dfrac{x^2}{e^{-x}}$

   (h) $0 \cdot \infty$ form: $\lim\limits_{x\to\infty} e^{-x}\sqrt{x}$

   (i) 1^∞ form (take ln to both sides) : $\lim\limits_{x\to\infty} \left(1 + \dfrac{1}{x}\right)^x$

   (j) 0^0 form (take ln to both sides) : $\lim\limits_{x\to 0^+} (\sin x)^x$

2. Find
$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$$

3. Find
$$\lim_{x\to \frac{\pi}{2}}\frac{\sec x}{1+\tan x}$$

4. Evaluate
$$\int_1^\infty (1-x)e^{-x}dx.$$

5. Find
$$\lim_{x\to\infty} x\ \sin\left(\frac{1}{x}\right) = \lim_{x\to\infty}\frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}$$

(we can move x to the denominator as $\frac{1}{x}$ to get a ratio for L'Hôpital).

6. Find
$$\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$$

7. Find

$$\lim_{x\to 0+} x^x = \lim_{x\to 0+} e^{\ln x^x} = \lim_{x\to 0+} e^{x\ln x} = e^{\lim_{x\to 0+} x\ln x}$$

or

$$\text{Let } y = \lim_{x\to 0+} x^x$$

$$\ln y = \ln\left[\lim_{x\to 0+} x\ln x\right]$$

(next move x to the denominator as $\frac{1}{x}$ to get a ratio for L'Hôpital).

8. Find

$$\lim_{x\to 0}\left(1+\frac{1}{x}\right)^x$$

$$\text{Let } y = \lim_{x\to 0}\left(1+\frac{1}{x}\right)^x$$

(take the ln of both sides and move x to the denominator as $\frac{1}{x}$ to get a ratio for L'Hôpital).

Answers

(1) $-\frac{1}{2}$ (2) $\frac{1}{2}$ (3) 1 (4) $-\frac{1}{e}$ (5) 1
(6) $\frac{1}{2}$-see p 366 ex 5 (7) 1 (8) e-see page 571 ex 5