Keys to Success on AP Calculus AB Test

About the Test:

1. MC – Calculator – Usually only 6 out of 15 questions require calculators.

2. Free-Response Tips
   a. Write <u>all work in allocated space for the problem in the answer booklet.</u>
   b. Explain everything <u>*clearly*</u> using concise language and standard mathematical notation!
   c. If you are using a justification/reason/explanation from Part A or B, use an <u>arrow.</u>
   d. <u>***UNITS***</u> are important when asked for!
   e. Cross out work that you do not want to be read. <u>Do not erase!</u>
   f. A <u>justification</u> is a mathematical explanation AND/OR a written explanation.
   g. Do NOT use rounded answers in intermediate parts of a problem. Store these answers in your calculator.
   h. If you don't know something, MAKE IT UP!
   i. Even if you use your calculator, you must show your work. Do NOT use calculator jargon in your work!
   j. Be sure you have answered all parts of the question.

   ** MC – check answers backwards (plug in the answer choices)
   ** FR – they are NOT in order from easy to hard; however, MC tends to be!

3. Make sure your calculator is in RADIAN mode.

4. <u>**Always round to 3 decimal places, unless otherwise specified.**</u>

Top Student Errors

1. $f''(c)=0$ implies $(x,\ f(x))$ is a point of inflection.
2. $f'(c)=0$ implies $f(x)$ has relative extrema at $(x,\ f(x))$.
3. Average rate of change of $f(x)$ on [a, b] is $\frac{1}{b-a}\int_a^b f(x)\,dx$.
4. Volume by washers is $\pi\int_a^b (R-r)^2\,dx$
5. Separable differential equations can be solved without separating the variables.
6. Omitting the constant of integration.
7. Not showing setup work on the calculator portion.
8. Universal logarithmic antidifferentiation: $\int \frac{1}{f(x)}\,dx = \ln|f(x)|+C$
9. Forgetting to use chain rule.
10. Using calculator jargon in your work.
11. Not answering all parts of a question or the question that is asked.
12. Forgetting the units.
13. Not rounding to three decimal places.

(Mostly) Everything you should know for AP Calculus…

1. Pythagorean identities: $\cos^2 x + \sin^2 x = 1$ $\quad 1 + \tan^2 x = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$
2. Definition of absolute value: $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$
3. Definition of e: $e = \lim\limits_{n \to \infty} \left(1 + \dfrac{1}{n}\right)^n$
4. Limit Definition of the Derivative as a function: $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$
5. Limit Definition of the Derivative at a point: $f'(a) = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$
6. Limit Definition of the Derivative (Alternative Form): $f'(c) = \lim\limits_{x \to c} \dfrac{f(x) - f(c)}{x - c}$
7. Definition of continuity: f is continuous at $x = c$ if and only if

    1) $\lim\limits_{x \to c} f(x)$ exists;
    2) $f(c)$ is defined;
    3) $\lim\limits_{x \to c} f(x) = f(c)$.

8. Average rate of change of $f(x)$ on $[a, b] = \dfrac{f(b) - f(a)}{b - a}$
9. Average Value of $f(x)$ on: [a, b] is $\dfrac{1}{b-a} \int_a^b f(x)\,dx$.
10. Intermediate Value Theorem:
    If f is continuous on $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c between a and b such that $f(c) = k$.
11. Rolle's Theorem:
    If f is continuous on $[a, b]$ and differentiable on (a, b) and if $f(a) = f(b)$, then there is at least one number c on (a, b) such that $f'(c) = 0$.
12. Mean Value Theorem:
    If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c on (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.
13. Extreme Value Theorem:
    - Conditions: *f(x)* is continuous on the closed interval, [a, b]
    - Conclusion: *f(x)* has an absolute maximum and absolute minimum on [a, b]
14. Double Angle Identities:

$$\sin(2x) = 2\sin x \cos x$$

$$\cos(2x) = \begin{cases} \cos^2 x - \sin^2 x \\ 1 - 2\sin^2 x \\ 2\cos^2 x - 1 \end{cases}$$

15. Power Reducing Identities:

- $\sin^2 x = \dfrac{1-\cos 2x}{2}$
- $\cos^2 x = \dfrac{1+\cos 2x}{2}$

16. Let *f* be defined at c. If $f'(c)=0$ or if f' is undefined at c, then c is a critical number of *f*.

17. Test for Increasing and Decreasing Functions

Let *f* be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a,b).

1) If $f'(x)>0$ for all *x* in (a,b), then *f* is increasing on $[a, b]$.

2) If $f'(x)<0$ for all *x* in (a,b), then *f* is decreasing on $[a, b]$.

18. Definition of Concavity:

Let *f* be differentiable on an open interval *I*. The graph of *f* is **concave upward** on *I* if f' is increasing on the interval and **concave downward** on *I* if f' is decreasing on the interval.

19. Tests for Concavity:

Let *f* be a function whose second derivative exists on an open interval *I*.

1) If $f''(x)>0$ for all *x* in the interval *I*, then the graph of *f* is **concave upward** in *I*.

2) If $f''(x)<0$ for all *x* in the interval *I*, then the graph of *f* is **concave downward** in *I*.

20. Relative Extrema (1st Derivative Test):

Let *c* be a critical number of a function *f* that is continuous on an open interval *I* containing *c*. If *f* is differentiable on the interval, except possibly at $x=c$, then $(c,\ f(c))$ can be classified as follows:

1) If $f'(x)$ changes from negative to positive at $x=c$, then $(c,\ f(c))$ is a **relative or local minimum** of *f*.

2) If $f'(x)$ changes from positive to negative at $x=c$, then $(c,\ f(c))$ is a **relative or local maximum** of *f*.

21. Relative Extrema (2nd Derivative Test):

Let *f* be a function such that the second derivative of *f* exists on an open interval containing *c*.

1) If $f'(c)=0$ and $f''(c)>0$, then $(c,\ f(c))$ is a **relative or local minimum** of *f*.

2) If $f'(c)=0$ and $f''(c)<0$, then $(c,\ f(c))$ is a **relative or local maximum** of *f*.

22. Definition of an Inflection Point

A function *f* has an inflection point at $(c,\ f(c))$

1) if $f''(c)=0$ or $f''(c)$ does not exist and

2) if f'' changes sign from positive to negative or negative to positive at $x=c$

OR if $f'(x)$ changes from increasing to decreasing or decreasing to increasing at $x=c$.

23. First Fundamental theorem of calculus: $\int_a^b f'(x)\,dx = f(b)-f(a)$

- $\int_a^b f(x)\,dx$ is the area under the curve of f(x) on interval $a \le x \le b$.
- $\int_b^a f(x)\,dx$ is negative if the area is below the x-axis on interval $a \le x \le b$.

24. Accumulation Functions: $\int_c^{g(x)} f(t)\,dt$

- To find the derivative: $\dfrac{d}{dx}\left[\int_c^{g(x)} f(t)\,dt\right] = f(g(x))\,g'(x)$ (2ND FTC)

25. Volume by discs (horizontal axis): $V=\pi\int_a^b (r(x))^2\,dx$ for $a\le x\le b$.

26. Volume by discs (vertical axis): $V=\pi\int_c^d (r(y))^2\,dy$ for $c\le y\le d$.

27. Volume by washers (horizontal axis): $V=\pi\int_a^b \left((R(x))^2-(r(x))^2\right)dx$ for $a\le x\le b$.

28. Volume by washers (vertical axis): $V=\pi\int_c^d \left((R(y))^2-(r(y))^2\right)dy$ for $c\le y\le d$.

29. Volume by cross sections perpendicular to the x-axis with known cross-section $A(x)$: $\int_a^b A(x)\,dx$ for $a\le x\le b$.

30. Volume by cross sections perpendicular to the y-axis with known cross-section $A(y)$: $\int_c^d A(y)\,dy$ for $c\le y\le d$.

31. Position/ Velocity/Acceleration (AB):
    - Speed is increasing when: acceleration and velocity have the same signs
    - Speed is decreasing when: acceleration and velocity have opposite signs

32. Given a <u>graph</u> of *f* and $g(x)=\int_0^x f(t)\,dt$:
    - The graph *f* is the graph of g'
    - $\int_0^x f(t)\,dt$ is the AREA under the curve $f(t)$ on interval $[0, x]$.
    - To evaluate *g*(x), evaluate the integral by using geometric shapes.

33. Derivative Approximations given select values of $f(x)$ in a table.

x	**f(x)**
a	e
b	f
d	g

To approximate $f'(c)\approx\dfrac{f(d)-f(b)}{d-b}$ given $b<c<d$.

34. Tangent Line Approximations

1. Write the tangent line at the given point: $(a, f(a))$

$$y-f(a)=f'(a)(x-a)\Rightarrow y=f(a)+f'(a)(x-a)$$

2. Then plug in the point $x=c$ and solve for $y(c)$.

$$y(c)=f(a)+f'(a)(c-a)$$

35. Absolute extrema – Use candidates test, compare the y-values of the relative extrema AND the endpoints. If there is only 1 critical number, then the critical number is both a relative and absolute extremum.

36. Particle Motion - Position/ Velocity/ Acceleration
- PVAJ:
    - Position: $x(t)$
    - Velocity: $x'(t)=v(t)$
    - Acceleration: $x''(t)=v'(t)=a(t)$

- SPEED
    - Speed: $|v(t)|$
    - INCREASING – velocity and acceleration have the same signs
    - DECREASING – velocity and acceleration have opposite signs

- Initially: t = 0
- At Rest: v(t) = 0
- Particle Moving Right: v(t) > 0
- Particle Moving Left: v(t) < 0
- Total Distance on [a, b]: $\int_a^b |v(t)|\,dt$
- Average velocity on [a, b]: $\dfrac{x(b)-x(a)}{b-a}$ or $\dfrac{1}{b-a}\int_a^b v(t)\,dt$
- Instantaneous velocity at $t=a;\ v(a)=x'(a)$

37. Derivative Formulas

$$\frac{d}{dx}[c]=0 \qquad \frac{d}{dx}[x]=1 \qquad \frac{d}{dx}[cx]=c \qquad \frac{d}{dx}\left[x^n\right]=nx^{n-1}$$

$$\frac{d}{dx}\left[f(x)g(x)\right]=f(x)g'(x)+g(x)f'(x) \qquad \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{\left(g(x)\right)^2}$$

$$\frac{d}{dx}\left(f\left(g(x)\right)\right)=f'\left(g(x)\right)\cdot g'(x) \qquad \frac{d}{dx}[\ln u]=\frac{1}{u}\frac{du}{dx} \qquad \frac{d}{dx}[\log_a u]=\frac{1}{u\ln a}\frac{du}{dx}$$

$$\frac{d}{dx}[e^u]=e^u\frac{du}{dx} \qquad \frac{d}{dx}[a^u]=a^u\ln a\frac{du}{dx} \qquad \frac{d}{dx}[\sin u]=\cos u\frac{du}{dx}$$

$$\frac{d}{dx}[\cos u]=-\sin u\frac{du}{dx} \qquad \frac{d}{dx}[\tan u]=\sec^2 u\frac{du}{dx} \qquad \frac{d}{dx}[\cot u]=-\csc^2 u\frac{du}{dx}$$

$$\frac{d}{dx}[\sec u]=\sec u\tan u\frac{du}{dx} \qquad \frac{d}{dx}[\csc u]=-\csc u\cot u\frac{du}{dx} \qquad \frac{d}{dx}[\arcsin u]=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}[\arccos u]=-\frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad \frac{d}{dx}[\arctan u]=\frac{1}{1+u^2}\frac{du}{dx} \qquad \frac{d}{dx}[\operatorname{arccot} u]=-\frac{1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}[\operatorname{arc\,sec} u]=\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx} \qquad \frac{d}{dx}[\operatorname{arc\,csc} u]=-\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx} \qquad \left(f^{-1}\right)'(a)=\frac{1}{f'\left(f^{-1}(a)\right)}$$

Definition of a definite integral: $\int_a^b f(x)\,dx=\lim_{\Delta x\to 0}\sum_{k=1}^{n} f\left(x_k\right)\cdot\left(\Delta x_k\right)=\lim_{n\to\infty}\sum_{k=1}^{n} f\left(x_k\right)\cdot\left(\Delta x_k\right)$

If f is a continuous function defined on [a, b], and if [a, b] is divided into n equal subintervals of width $\Delta x=\frac{b-a}{n}$, and if $x_k=a+k\Delta x$ is the right endpoint of subinterval k, then the definite integral of f from a to b is:

$$\int_a^b f(x)\,dx=\lim_{n\to\infty}\sum_{k=1}^{n}\left[f\left(a+\frac{(b-a)k}{n}\right)\right]\left(\frac{b-a}{n}\right)$$

38. Integration Formulas

$$\int dx = x + C \qquad \int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1 \qquad \int \frac{1}{u}\,du = \ln|u| + C$$

$$\int e^u du = e^u + C \qquad \int a^u du = \frac{a^u}{\ln a} + C \qquad \int \cos u\,du = \sin u + C$$

$$\int \sin u\,du = -\cos u + C \qquad \int \sec^2 u\,du = \tan u + C \qquad \int \csc^2 u\,du = -\cot u + C$$

$$\int \sec u \tan u\,du = \sec u + C \qquad \int \csc u \cot u\,du = -\csc u + C \qquad \int \tan u\,du = -\ln|\cos u| + C$$

$$\int \cot u\,du = \ln|\sin u| + C \qquad \int \sec u\,du = \ln|\sec u + \tan u| + C \qquad \int \csc u\,du = -\ln|\csc u + \cot u| + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\frac{u}{a} + C \qquad \int \frac{du}{u^2 + a^2} = \frac{1}{a}\arctan\frac{u}{a} + C \qquad \int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$$

Interpreting the Meaning of the Derivatives in Context

Let's review the process of interpreting the meaning of the derivative in context.

Students will be required to interpret the meaning, in context, of a derivative and/or definite integral on the AP exam. When interpreting these values, it is crucial that students are **concise** and include three components in their interpretation.

For Derivatives

Note: Derivative values are INSTANTANEOUS values, meaning they occur at a precise moment. Be sure to always interpret a derivative value AT a specific time, not during or over an interval!!!

3 Components to Interpret a Derivative

1. Include units—for both the independent and dependent values
2. Include "rate" as part of your answer
3. Include context for the problem

Interpreting the Meaning of the Integral in Context

For Integrals

Note: Integrals are values that happen OVER an interval. Use phrases like "over the interval" in your interpretation and NOT phrases "at".

3 Components to Interpret an Integral

1. Include units—for both the time interval and the calculated value
2. Use phrases like "total" or "net" as part of your answer
3. Include context for the problem