CALCULUS AB

Notes – In and Out Problems

1. (2015 AB 1) The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

$\int_0^8 R(t)\,dt = 76.5703$

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

$R(3) - D(3) = -.3136$

The amount of water in the pipe is decreasing at $t = 3$ since $R(3) - D(3) < 0$.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

$A(t) = 30 + \int_0^t (R(x) - D(x))\,dx$

$A'(t) = R(t) - D(t) = 0$

$R(t) = D(t)$

$t = 3.2716$

The minimum amount of water is 27.9645 ft^3 at $t = 3.2716$ hours.

t	$A(t)$
0	30
3.2716	27.9645
8	48.5436

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

$A(t) = 30 + \int_0^t (R(x) - D(x))\,dx$

$50 = 30 + \int_0^w (R(x) - D(x))\,dx$

2. (2013 AB 1) On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

$P(t) = 100$

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

$G'(5) = -24.5875$

At $t = 5$ hours, the rate at which gravel arrives to the plant is decreasing at a rate of 24.5875 tons per hour per hour.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

$\int_0^8 g(t)\,dt = 825.5510$

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

$G(5) = 98.1407 < 100$

The amount of gravel is decreasing at $t = 5$ hours since $G(5) < 100$.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

$A(t) = 500 + \int_0^t (G(x) - 100)\,dx$

$A'(t) = G(t) - 100 = 0$

$G(t) = 100$

$t = 4.9234$

t	$A(t)$
0	500
4.9234	635.3761
8	525.5510

The max amount of gravel is 635.3761 tons at $t = 9234$ hours.

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

3. (2016 AB 1) Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$ where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of R(t) are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

$$R'(2) = \frac{R(3) - R(1)}{3-1} = \frac{950 - 1190}{2} \ \frac{\text{liters}}{\text{hour}^2}$$

(b) Use a left Riemann sum with four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

$$\int_0^8 r(t)\,dt \approx (1340)(1) + (1190)(2) + (950)(3) + (740)(2) = 8050$$

Overestimate b/c R(t) is decreasing and I used a left Rieman sum.

(c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

50,000

$$50{,}000 + \int_0^8 (W(t) - 8050) = 49786 \text{ liters}$$

W(t)

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

$W(t) = R(t)$

$W(t) - R(t) = 0$

$W(0) - R(0) = 660$

$W(8) - R(8) = -618.4755$

Since W & R are differentiable, they are continuous, and $-618.4755 < 0 < 660$ there must be a time t, $0 < t < 8$, such that $W(t) - R(t) = 0$, which means $W(t) = R(t)$.

4. (2018 AB 1) People enter a line for an escalator at a rate modeled by the function r given by

$$r(t)=\begin{cases} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 & \text{for } 0\le t\le 300 \\ 0 & \text{for } t>300 \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t=0$.

(a) How many people enter the line for the escalator during the time interval $0\le t\le 300$?

$\int_0^{300} r(t)\,dt = 270$ people

(b) During the time interval $0\le t\le 300$, there are always people in line for the escalator. How many people are in line at time $t=300$?

$20+270-\int_0^{300} 0.7\,dt = 80$ people

(c) For $t>300$, what is the first time t there are no people in line for the escalator?

$80-.7t=0$ $\quad 80=.7t$

$t=114.2857$

$300+114.2857$

There are 0 ppl. in line at 414.2857 seconds.

(d) For $0\le t\le 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

$A(t)=20+\int_0^{t}(R(x)-.7)\,dx$

$A'(t)=R(t)-.7=0$

$R(t)=.7$

$t=166.5747$

$t=33.0132$

The minimum number of ppl in line is 4 at $t=33.0132$ seconds.

t	$A(t)$
0	20
33.0132	3.8034
166.5747	158.0701
300	80

CALCULUS AB
FRQ PRACTICE – 2019 AB 1

Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15\sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).

(a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)? Give your answer to the nearest whole number.

$\int_0^5 E(t) = 153$ fish.

(b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)?

$\frac{1}{5-0}\int_0^5 L(t)\,dt = 6.0590$ fish

(c) At what time, t, for $0 \le t \le 8$, is the greatest number of fish in the lake? Justify your answer.

$A(t) = \int_0^t E(x) - L(x)\,dx$

$A'(t) = E(t) - L(t) = 0$

$E(t) = L(t)$

$t = 6.2035$

X	A(t)
0	0
6.2035	135.0149
8	80.9199

The greatest number of fish in the lake is 135.0149 fish at $t = 6.2035$ hours.

(d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$)? Explain your reasoning.

$E'(5) - L'(5) = -10.7227$

$-6.8017 - 3.9210$

The rate of change in the number of fish in the lake is decreasing at $t = 5$ since $E'(5) - L'(5) < 0$.