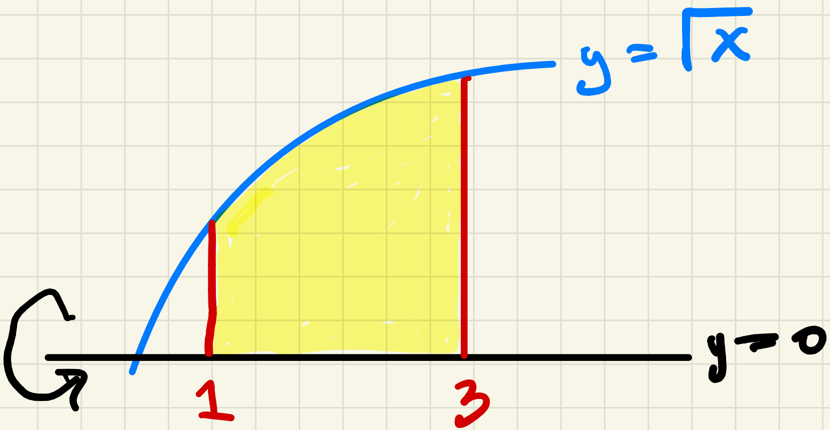
7.2 (p463) 58, 74



find the x values on $[1,3]$ that divide the solid into 3 equal volumes

① Find the entire volume

$$\pi \int_1^3 (\sqrt{x})^2\,dx = \frac{\pi x^2}{2}\Big|_1^3 = \frac{\pi}{2}(3^2 - 1^2) = 4\pi$$

② So each volume needs to be $\frac{4\pi}{3}$

$$\pi \int_1^a (\sqrt{x})^2\,dx = \frac{\pi}{2} x^2 \Big|_1^a = \frac{\pi}{2}(a^2 - 1^2)$$

$$\frac{\cancel{\pi}}{2}(a^2 - 1) = \frac{4\cancel{\pi}}{3}$$

$$a^2 = \frac{8}{3} + 1 = \frac{11}{3}$$

$$a = \sqrt{\frac{11}{3}} \quad \text{or} \quad \frac{\sqrt{33}}{3}$$

③ The middle and last section

$$\pi \int_a^b (\sqrt{x})^2\,dx = \frac{\pi}{2} x^2 \Big|_a^b = \frac{\pi}{2}\left(b^2 - \frac{11}{3}\right) = \frac{4\pi}{3}$$

$$b^2 - \frac{11}{3} = \frac{8}{3}$$

$$b^2 = \frac{8+11}{3} = \frac{19}{3}$$

$$b = \sqrt{\frac{19}{3}} \quad \text{or} \quad \frac{\sqrt{57}}{3}$$

④ Conclusion: the x values are $\frac{\sqrt{33}}{3}$ and $\frac{\sqrt{57}}{3}$

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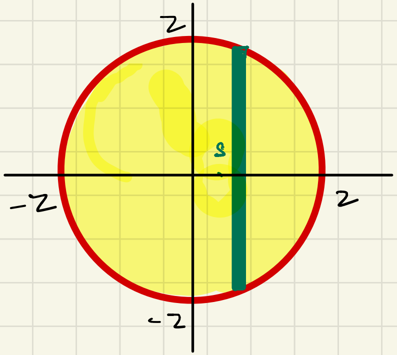


$$x^2+y^2=4$$

$$y=\pm\sqrt{4-x^2}$$

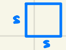
$$s = \sqrt{4-x^2} - \left(-\sqrt{4-x^2}\right)$$

$$s = 2\sqrt{4-x^2}$$

First find the cross sectional area in terms of s,

Then

$$V=\int_{-2}^{2} A(x)\,dx$$

a) Squares: 

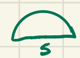
$$A=s^2=\left(2\sqrt{4-x^2}\right)^2 = 4(4-x^2)$$

$$V=\int_{-2}^{2} 4(4-x^2)\,dx = 4\left(4x-\frac{x^3}{3}\right)\Big|_{-2}^{2} = \frac{128}{3}$$

b) 

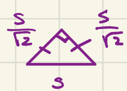
$$= \frac{s}{2}\cdot\frac{s\sqrt{3}}{2} = \frac{\sqrt{3}}{4}s^2 = \frac{\sqrt{3}}{4}\left[4(4-x^2)\right] = \sqrt{3}(4-x^2)$$

$$V=\sqrt{3}\int_{-2}^{2} 4-x^2\,dx = \sqrt{3}\left(4-\frac{x^3}{3}\right)\Big|_{-2}^{2} = \frac{32\sqrt{3}}{3}$$

c) 

$$= \frac{1}{2}\pi\left(\frac{s}{2}\right)^2 = \frac{\pi}{8}s^2 = \frac{\pi}{8}\left(4(4-x^2)\right) = \frac{\pi}{2}(4-x^2)$$

$$V = \frac{\pi}{2}\int_{-2}^{2}(4-x^2)\,dx = \frac{\pi}{2}\left(4x-\frac{x^3}{2}\right)\Big|_{-2}^{2} = \frac{32}{3}$$

d) 

$$= \frac{1}{2}\left(\frac{s}{\sqrt{2}}\right)^2 = \frac{1}{4}s^2 = \frac{4(4-x^2)}{4} = 4-x^2$$

$$V=\int_{-2}^{2} 4-x^2\,dx = 4x-\frac{x^3}{3}\Big|_{-2}^{2} = \frac{32}{3}$$