CALCULUS BC
WORKSHEET ON CONCAVITY AND SECOND DERIVATIVE TEST

Work the following on **<u>notebook paper</u>**. Do <u>not</u> use your calculator.

On problems 1 – 4, find the points of inflection and discuss the concavity of the graph of the function.

1. $f(x) = -x^4 + 24x^2$
2. $f(x) = \frac{1}{20}x^5 - \frac{1}{6}x^4$
3. $f(x) = x(x-4)^3$
4. $f(x) = x + 2\cos x$, $[0, 2\pi]$

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5. Given $f(x) = x^3 + 5x^2 - 8x + 7$. Use the Second Derivative Test to find whether f has a local maximum or a local minimum at $x = -4$. Justify your answer.

6. Given $f(x) = \sqrt{3}\,x - 2\sin x$. Use the Second Derivative Test to find whether f has a local maximum or a local minimum at $x = \frac{\pi}{6}$. Justify your answer.

On problems 7 – 9, find the critical points of each function, and determine whether they are relative maximums or relative minimums by using the Second Derivative Test whenever possible.

7. $f(x) = x^3 - 3x^2 + 3$
8. $f(x) = x + \frac{4}{x}$
9. $f(x) = \sin x - \cos x$, $0 \le x \le 2\pi$

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10. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.

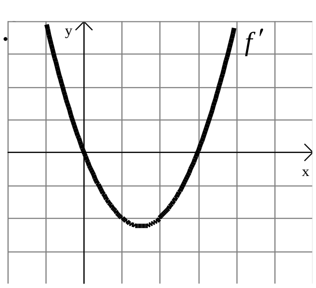
(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

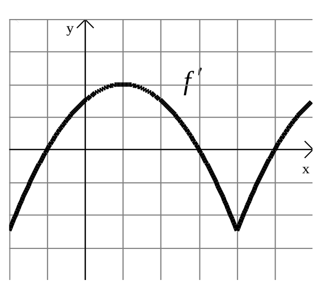
(c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at point P? Justify your answer.

On problems 11 – 12, the graph of the derivative, f', of a function f is shown.

(a) On what interval(s) is f increasing or decreasing? Justify your answer.

(b) At what value(s) of x does f have a local maximum or local minimum? Justify your answer.

11.


12.


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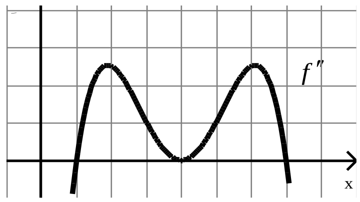
13. The graph of the second derivative, f'', of a function f is shown. State the x-coordinates of the inflection points of f. Justify your answer.



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14. For what values of a and b does the function $f(x) = x^3 + ax^2 + bx + 2$ have a local maximum when $x = -3$ and a local minimum when $x = -1$?

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15. The graph of a function f is shown on the right.
Fill in the chart with +, −, or 0.

Point	f	f'	f''
A			
B			
C