NAME ____________________

Antiderivatives

Definition

A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all a in I.

Essentially, the antiderivative of a function is the opposite of the derivative. If $f(x)$ is the derivative of some function, then $F(x)$ is a function that you would have taken the derivative of to get f.

The weird thing to notice is that one function has *many* antiderivatives. This may not be intuitive at first, but consider an example.

To find an antiderivative of a function $f(x) = 5x^4$, we would look for a function that we would take the derivative of to get $5x^4$.

You might see that $F(x) = x^5$ has a derivative of $F'(x) = f(x) = 5x^4$.

The thing is that we can also take the derivative of

$$F_1(x) = x^5 + 1 \text{ and get } F_1'(x) = f(x) = 5x^4$$

OR

$$F_{-5}(x) = x^5 - 5 \text{ and get } F_{-5}'(x) = f(x) = 5x^4$$

OR

$$F_{18\pi}(x) = x^5 + 18\pi \text{ and get } F_{18\pi}'(x) = f(x) = 5x^4$$

OR

you get the picture...

Theorem

If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

Antidifferentiation Formulas

Use our definition of antiderivative to find each of the following antiderivatives.

Function	General Antiderivative	Function	General Antiderivative
$bf(x)$	$bF(x)$	$\sin x$	
$f(x) + g(x)$	$F(x) + G(x)$	$\cos x$	
x^n		$\sec^2 x$	
$1/x$		$\sec x \tan x$	
e^x		$\frac{1}{\sqrt{1-x^2}}$	
		$\frac{1}{1+x^2}$	

Examples

Find the most general antiderivative of the function.
Note: You may check your answer by differentiation.

1. $f(x) = -3 + \frac{1}{4}x^2 - \frac{4}{5}x^3$

2. $g(x) = \frac{2}{3}x^{2/3}$

3. $h(x) = 2\sqrt{x} - 6\cos x$

4. $f(x) = \frac{x^5 - x^3 + 2x}{x^4}$

5. $f''(x) = 2 + \cos x$

Find f.

1. $f'(x) = \sqrt{x}(6 + 5x)$ where $f(1) = 10$

2. $f'(x) = 2x - 3/x^4$, $x > 0$, $f(1) = 3$

3. $f'(x) = \frac{4}{\sqrt{1-x^2}}$, $f(1/2) = 1$

4. $f''(x) = 8x^3 + 5$, $f(1) = 0$, $f'(1) = 8$

Recall the theorem on page 1...

THEOREM

If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

To prove this theorem, let F and G be any two antiderivatives of f on I and let $H = G - F$.

(I) If x_1 and x_2 are any two numbers in I with $x_1 < x_2$, apply the Mean Value Theorem on the interval $[x_1, x_2]$ to show that $H(x_1) = H(x_2)$.

Why does this show that H is a constant function?

(II) Deduce the theorem from the result of part (I).