7.2b Notes and Examples Name: Block: Seat:

Volume of Solids of Known cross-sections and Volumes of Revolutions

Disk Volume

$$\text{Disk Volume} = \pi \int_a^b [R(x)]^2 \, dx \text{ or } \pi \int_c^d [R(y)]^2 \, dy$$

Cone Example

Washer Volume

$$\text{Washer Volume} = \pi \int_a^b [R(x)]^2 - [r(x)^2] \, dx \text{ or } \pi \int_c^d [R(y)]^2 - [r(y)^2] \, dy$$

Volume of Solids with a Known Cross Section

$$\text{Volume of Solids with a Known Cross Sections} = \int_a^b A(x) \, dx \text{ or } \int_c^d A(y) \, dy$$

Where the most common cross sections are:

Square	Rectangle	Semicircle	Triangle	Equilateral $\triangle$	Iso.Rt. $\triangle$
$A = x^2$	$A = bh$	$A = \frac{1}{2}\pi r^2$	$A = \frac{1}{2}bh$	$A = \frac{\sqrt{3}}{4}s^2$	$A = \frac{(leg)^2}{2}$ leg in region $A = \frac{(hyp)^2}{4}$ hyp in region

1. To see an interactive 3-D rendering, go to `https://www.geogebra.org/m/XFgMaKTy`
   (a) Find the volume of the solid whose base is the enclosed area between $y = \sqrt{x}$ and $y = x^2$, whose cross section ($\perp$ to the x-axis) is a square $\left(A = s^2\right)$

(b) Find the volume of the solid whose base is the enclosed area between $y = \sqrt{x}$ and $y = x^2$, whose cross section ($\perp$ to the x-axis) is a Equilateral Triangle $\left(A = \frac{\sqrt{3}}{4}s^2\right)$

(c) Find the volume of the solid whose base is the enclosed area between $y = \sqrt{x}$ and $y = x^2$, whose cross section ($\perp$ to the x-axis) is a Semi-circle $\left(A = \frac{1}{2}\pi r^2\right)$

2. Class 3-18 Examples from DeltaMath

   (a) Let the region R be the area enclosed by the function $f(x) = x^2$ and $g(x) = 3x$. If the region R is the base of a solid such that each cross section perpendicular to the x-axis is an **isosceles right triangle** with a leg in the region R, find the volume of the solid. You may use a calculator and round to the nearest thousandth.

   (b) Let the region R be the area enclosed by the function $f(x) = e^x - 2$ and $g(x) = 4x - 1$. If the region R is the base of a solid such that each cross section perpendicular to the x-axis is a **square**, find the volume of the solid. You may use a calculator and round to the nearest thousandth.

(c) Let the region R be the area enclosed by the function $f(x) = \sqrt{x} - 1$ and $g(x) = \frac{1}{3}x - 1$. If the region R is the base of a solid such that each cross section perpendicular to the x-axis is a **rectangle** whose height is half the length of its base in the region R, find the volume of the solid. You may use a calculator and round to the nearest thousandth.

(d) Let the region R be the area enclosed by the function $f(x) = \ln(2x)$ and $g(x) = \frac{1}{2}x - 1$. If the region R is the base of a solid such that each cross section perpendicular to the x-axis is a **semi-circle** with diameters extending through the region R, find the volume of the solid. You may use a calculator and round to the nearest thousandth.