6.3b Notes and Examples   Name: Block: Seat:

The Logistic Model

We have seen models unrestricted growth. In exponential growth (or decay), we assume that the rate of increase (or decrease) of a population at any time t is directly proportional to the population P so that

$$\frac{dP}{dt} =$$

However, in many situations population growth levels off and approaches a limiting number L (the carrying capacity or M for Max) because of limited resources. In this situation the rate of increase

(or decrease) is directly proportional to both ______________ and ______________ This type of growth

is called _________________________

This is modeled by a differential equation which has 3 common forms, all seen in past AP exams:

1. $\frac{dP}{dt} =$
2. $\frac{dP}{dt} =$
3. $\frac{dP}{dt} =$

Examples include the spread of a rumor or disease, the population growth when there are limiting resources, and even in business. Sam Walton (CEO of Wal*Mart) used the model to stop stocking items when the growth switched to concave up to concave down!

1. The he population $P(t)$ of fish in a lake satisfies the logistic differential equation

$$\frac{dP}{dy} = 3P - \frac{P^2}{6000}$$

where t is measured in years, and $P(0) = 4000$.

(a) $\lim_{t\to\infty} P(t) =$

(b) What is the range of the solution curve?

(c) For what values of P is the solution curve increasing? Decreasing? Justify your answer.

(d) For what values of P is the solution curve concave up? Concave down? Justify your answer.

(e) Does the solution curve have an inflection point? Justify your answer.

(f) Use the information you found to sketch the graph of $P(t)$.

(g) What if the initial amount is different? What if $P(t) = 10,000$? Or if $P(t) = 20,000$? Which of your answers would change?

2. The function N satisfies the logistic differential equation $\dfrac{dN}{dt} = \dfrac{N}{10}\left(1 - \dfrac{N}{850}\right)$, where $N(0) = 105$. Which of the following statements are false?

   (a) $\lim_{t\to\infty} N(t) = 850$

   (b) $\dfrac{dN}{dt}$ has a maximum value when $N = 105$

   (c) $\dfrac{d^2N}{dt^2} = 0$ when $N = 425$.

   (d) When $N > 425$, $\dfrac{dN}{dt} > 0$ and $\dfrac{d^2N}{dt^2} < 0$

   (e) None of these

3. A population y changes at a rate modeled by the differential equation $\frac{dy}{dt} = 0.2y(1000 - y)$, where t is measured in years. What are all values of y for which the population is increasing at a decreasing rate?

4. The number of students in a cafeteria is modeled by the function P that satisfies the logistic differential equation $\frac{dP}{dt} = \frac{1}{2000}P(200 - P)$, where t is the time in seconds and $P(0) = 25$. What is the greatest rate of change, in students per second, of the number of students in the cafeteria?

5. the number of moose in a national park is modeled by the function M that satisfies the logistic differential equation $\dfrac{dM}{dt} = 0.6M\left(1 - \dfrac{M}{500}\right)$, where t is the time in years and $M(0) = 50$. What is $\lim_{t\to\infty} M(t)$?
   (a) 50
   (b) 200
   (c) 500
   (d) 1000
   (e) 2000
   (f) None of these

6. A population is modeled by the function P that satisfies the logistic differential equation $\dfrac{dP}{dt} = \dfrac{P}{5}\left(1 - \dfrac{P}{12}\right)$.
   (a) If $P(0) = 3$, what is $\lim_{t\to\infty} P(t)$?

   (b) If $P(0) = 20$, what is $\lim_{t\to\infty} P(t)$?

   (c) If $P(0) = 3$, what value of P is the population growing the fastest?