Calc BC 5.1b Notes and Examples   Name: Block: Seat:

Intro to Euler's Method (5.1b)

1. Given the differential equation $\frac{dy}{dx} = x - 2$ and $y(0) = 5$:

   (a) Find an approximation for $y(0.8)$ using the tangent line at $(0, 5)$.

   $m = y'(0) = \left.\frac{dy}{dx}\right|_{(x,y)} =$ ______________

   The tangent line equation:

   Your estimate of $y(0.8) =$____________________________

   (b) The idea of Euler's Method is to improve this estimate by taking more than one step. Find an approximation for $y(0.8)$ by using Euler's (say "oilers") method with two equal steps.

   Let $\Delta x = \frac{0.8 - 0.0}{2} =$ ______________

   What about Δy? Let slope $m = \frac{\Delta y}{\Delta x}$. Now solve for Δy.

   $\Delta y =$ ______________

   Now for any point (x, y) of the curve, won't the slope m at (x, y) be $\frac{dy}{dx}$? So we can write

   $$\Delta y = \Delta x * \left.\frac{dy}{dx}\right|_{(x,y)}$$

   $y(0) =$ _______

   $y(0.4) \approx y(0) + \Delta y = y(0) + \Delta x * y'(0) =$ _______ + (______________) $*$ (______________) = ______________

   $y(0.8) \approx y(0.4) + \Delta y = y(0.4) + \Delta x * y'(0.4) =$ _______ + (______________) $*$ (______________) = ______________

   (c) Check your work by solving the differential equation.

   (d) Check your work with Demos: `https://www.desmos.com/calculator/p7vd3cdmei`. Change $g(x, y) = x - 2$ and on new lines add your equation, and the points $(0.8, f(0.8))$ and $(0.8, e)$ where f is your solution to the differential equation, and e is your estimate using Euler's method with two steps.

   (e) Use GeoGebra `https://www.geogebra.org/m/mPwa7SKk` to improve your estimate:

   With 4 Steps: ______________ With 8 Steps: ______________

   (f) Go to `https://mathorama.com/ti/` and use `EULERM` .

2. Given the differential equation $\frac{dy}{dx} = 2x + y$ Find the follwing

   (a) If $y(0) = 0$ estimate $y(3)$ using Euler's Method with 3 steps.

   (b) If $y(0) = 0$ estimate $y(3)$ using Euler's Method with 6 steps.

   (c) If $y(1) = 1$ estimate $y(0)$ using a step size (Δx) of 0.5.

3. (Inspired by 2013 BC 5) Consider the differential equation $\frac{dy}{dx} = y^2(2x+2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$

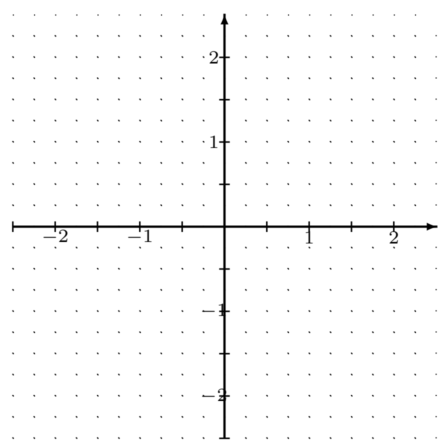
   (a) Find $\lim_{x \to 0} \frac{f(x)+1}{\sin x}$

   (b) Using the tangent line from $(0, -1)$, estimate $f\left(\frac{1}{2}\right)$.

   (c) Use $\left.\frac{d^2y}{dx^2}\right|_{(0,1)}$ to justify why you think the estimate in part (b) is an underestimate or an overestimate.

   (d) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

(e) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.



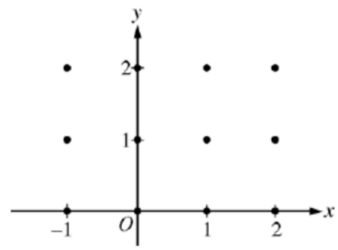
.

(f) To check your work, graph f, the tangent line at $(0, -1)$, and the point $\left(\frac{1}{2},\ f\left(\frac{1}{2}\right)\right)$. Use your calculator and/or `https://www.desmos.com/calculator/p7vd3cdmei`

4. Let $y = f(x)$ be the solutions to the differential equation $\frac{dy}{dx} = 2y - x$ with the initial condition $f(1) = 2$. What is the approximation for $f(0)$ obtained using Euler's method with two steps of equal length starting at $x = 1$?
   (a) $-\frac{5}{4}$
   (b) -1
   (c) $\frac{1}{4}$
   (d) $\frac{1}{2}$
   (e) $\frac{27}{4}$

5. Let $y = f(x)$ be the solutions to the differential equation $\frac{dy}{dx} = x - y - 1$ with the initial condition $f(1) = -2$. What is the approximation for $f(1.4)$ if Euler's method is used, starting at $x = 1$ with two steps of equal size?
   (a) -2
   (b) -1.24
   (c) -1.2
   (d) -0.64
   (e) 0.2

4. (2005 BC 4) Consider the differential equation $\frac{dy}{dx} = 2x - y$.



(a) On the axes provided, sketch a slope field for the given differential at the twelve points indicated, and sketch the solution curve that passes through the point (0,1).

(b) The solution curve that passes through the point (0,1) has a local minimum at $x = \ln(3/2)$. What is the y-value of this local minimum?

(c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$. Show the work that leads to your answer.

(d) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.