4.5 Notes and Examples

Name: Block: Seat:

Integration by Substitution or "u-sub"

Substitution in integration is how we "undo" the Chain Rule which often is needed in differentiation. In this section you will study techniques for integrating composite functions. (Undoing the Product Rule is a Chapter 8 Topic you will see next year)

Substitution with Indefinite Integrals

Let g be a function whose range is an interval I, and let f be a function that is continuous on I. If g is differentiable on its domain and F is an antiderivative of f on I, then

$$\int f(g(x))g'(x)\ dx =$$

If we let $u = g(x)$, then $\dfrac{du}{dx} =$, solving for du we get $du =$

Now we can substitute and write our first integral as

$$\int f(g(x))g'(x)\ dx = \int f(u)du =$$

1. $\displaystyle\int (x^2+1)^2(2x)\ dx$

2. $\displaystyle\int \sec^2 x(\tan x + 3)\ dx$

The Power Rule for Integration

If g is a differentiable function of x, and $n \neq -1$ then

$$\int [g(x)]^n \, g'(x) \; dx =$$

or if $u = g(x)$, then

$$\int u^n \; du =$$

3. (a) $\int (x^2 + x)(2x + 1) \; dx =$

(b) $\int \cos^2 x \sin x \; dx =$

(c) $\int \frac{1}{x} \; dx =$

4. Sometimes we need to make some adjustments:

   (a) $\int x\sqrt{x^2+5}\ dx =$

   (b) $\int \cos(3x)\ dx =$

   (c) $\int \frac{x+1}{x^2+2x}\ dx =$

   (d) $\int \sqrt{2x-1}\ dx$

   (e) $\int x\sqrt{2x-1}\ dx$

Substitution with Definite Integrals
If the function $u = g(x)$ has a continuous derivative on the closed interval $[a, b]$ and f is continuous on the range of g, then

$$\int_a^b f(g(x))g'(x)\ dx =$$

5. (a) $\displaystyle\int_{\pi/12}^{\pi/9} \sin(3x)\ dx =$

(b) $\displaystyle\int_0^2 \frac{x}{\sqrt{2x^2+1}}\ dx =$

(c) $\displaystyle\int_{-1}^0 x\sqrt{1-x^2}\ dx =$

Odd and Even Functions
Let f be integrable on $[-a, a]$.

1. If f is an **even** function,
$$\int_{-a}^a f(x)\ dx =$$
2. If f is an **odd** function,
$$\int_{-a}^a f(x)\ dx =$$

6. (a) $\displaystyle\int_{-2}^2 x(x^2+1)^3\ dx$

(b) $\displaystyle\int_{-\pi/2}^{\pi/2} \sin^2 x \cos x\ dx$