4.4a Notes and Examples

Name: Block: Seat:

Introducing 1st and 2nd Fundamental Theorem of Calc (FTC1, FTC2), Average Value (MVT for integrals)

$\boxed{\text{FTC 1}}$ **The Fundamental Theorem of Calculus**

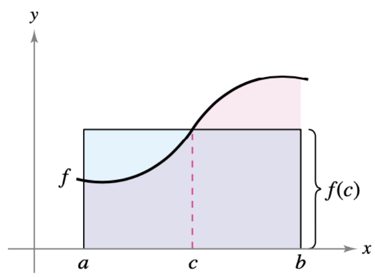
$$\int_a^b f(x)\ dx = F(x)\Big|_a^b = F(b) - F(a)$$

1. Step 1: Find the ______________________ of f, $F(x)$.
2. Step 2: Subtract $F($______________________bound) from $F($_____________________ bound).
3. We can also write
   $\displaystyle\int_a^b f'(x)\ dx =$
4. Or the "Net Change" Version: $f(a) + \displaystyle\int_a^b f'(x)\ dx =$ _____________

   or $F(a) + \displaystyle\int_a^b f(x)\ dx =$ _____________

1. $\displaystyle\int_1^2 3x^2\ dx =$

2. $\displaystyle\int_0^{\pi/2} \cos y\ dy =$



Mean Value over an Interval (MVT for Integrals)
If f is continuous on the closed interval $[a, b]$ then there exists a number c (where $a \leq c \leq b$) such that

$$\int_a^b f(x)\, dx =$$

So the area of the under the curve from a to b must match the area of a rectangle with base $(b - a)$ and height $f(c)$.

1. If we solve for $f(c)$,

$$f(c) =$$

2. $f(c)$ is called: ________________________ on ______________

3. Find the average value of $f(x) = 3x^2 - 2x$ over the interval $[0, 2]$.

4. Water is flowing into a tank over a 24-hour period. The rate at which water is flowing into the tank is modeled by the function $R(t) = \frac{1}{75}(600 + 20t - t^2)$, where $R(t)$ is measured in gallons per hour and t is measured in hours. The tank contains 150 gallons of water when $t = 0$.

   (a) Use this function to find the number of gallons of water in the tank at the end of 24 hours. (Use your TI-84's $\boxed{\text{MATH}}$ $\boxed{\text{9:fnInt}}$, but write the integral expression, the answer (exact or 3 decimal places), and units for full AP credit)

   (b) Use this function to find the average rate of water flow over the 24-hour period.

$\boxed{\textbf{FTC 2}}$ **The Second Fundamental Theorem of Calculus:**

If f is continuous on an open interval I containing a, then, for every x in the interval I:

$$\frac{d}{dx}\left[\int_a^x f(t)\ dt\right] =$$

1. The **Chain Rule Version:**

$$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\ dt\right] =$$

2. Note that in the integral $a, x, g(x)$ are the ______________ of the integral,

   and t is the __________________ of itegration.

In general, the 2nd FTC has 2 steps:

1. __________________ the variable of integration with the _____________ bound.

2. Use the chain rule: _____________ by the derivative of the _____________ bound of the integral.

5. $\displaystyle \frac{d}{dx}\int_2^x \tan(t^3)\ dt =$

6. $\displaystyle \frac{d}{dx}\int_2^{\sin x} \sqrt[3]{1+t^2}\ dt =$

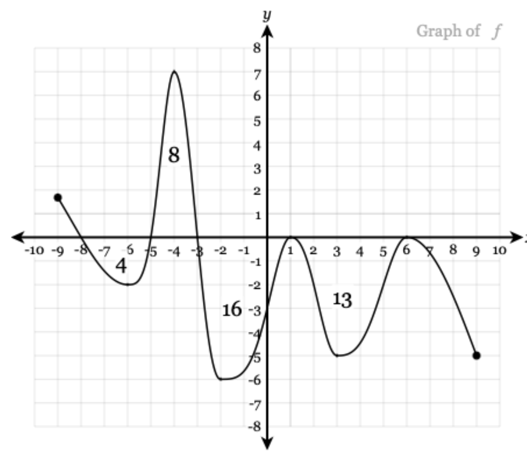
Definition of Integrally Defined Functions

Functions defined by definite integrals like:

$$g(x) = \int_a^x f(t)\ dt$$

are sometimes called ____________________ functions. Note that the x is the ______________ bound of the integral, and a is a ______________.

7. The areas of the regions bounded by the graph of the function f and the x-axis are labeled in the figure below. Let the function g be defined by the equation $g(x) = \int_1^x f(t)\ dt$



(a) $g(1) =$

(b) $g(6) =$

(c) $g(-3) =$

(d) $g(-5) =$

(e) $g(-8) =$

(f) What is the absolute maximum of g on $[-8, 6]$?

(g) What is the absolute minimum of g on $[-8, 6]$?