4.4 Notes and Examples

Name: Block: Seat:

1st Fundamental Theorem of Calc (FTC1), Average Value (MVT for integrals), 2nd Fundamental Theorem of Calculus (FTC2)

1. FTC 1 **The Fundamental Theorem of Calculus**
   If a function f is continuous on the closed interval $[a, b]$, and F is an antiderivative of f, that is $F'(x) = f(x)$ on $[a, b]$, then

   $$\int_a^b f(x)\ dx =$$

   (a) Step 1: Find ______________________

   (b) Step 2: Subtract the result $(F(x))$ at the ______________________ from ______________________.

   (c) we can also write $\int_a^b f'(x)\ dx =$

   (d) Or the "Net Change" Version: $f(a) + \int_a^b f'(x)\ dx =$ ____________

2. (a) $\int_1^2 3x^2\ dx =$

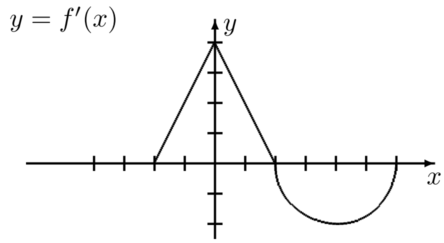
   (b) $\int_\pi^0 \sin x\ dx =$

   (c) $\int_0^{\pi/4} \sec^2 x\ dx =$

   (d) $\int_0^3 |2x - 4|\ dx =$

3. What is the area of the region bounded by $y = x^3 + 6x$, $x = 2$, and $y = 0$?

4. The graph of f' consists of two line segments and a semicircle as shown below:



Given $f(-2) = 5$ find the following:

(a) $f(0) =$

(b) $f(2) =$

(c) $f(6) =$

5. (Like 4.1, let's have a "preview" of 4.5) Recall there is no product rule for integrals, but sometimes we get the product of 2 functions due to the chain rule:

$$\int_0^{\pi/6} \sin^3 x \cos x \; dx =$$

6. MVT for Integrals. If f is continuous on the closed interval $[a, b]$ then there exists a number c (where $a \leq c \leq b$) such that

$$\int_a^b f(x)\ dx =$$

So the area of the under the curve from a to b must match the area of a rectangle with base $(b - a)$ and height $f(c)$.

   (a) If we solve for $f(c)$,

   $$f(c) =$$

   (b) $f(c)$ is called:

7. Find the average value of $f(x) = 3x^2 - 2x$ over the interval $[0, 2]$.

8. In section 3.2 we had an average RATE OF CHANGE over an interval, here in 4.4 we define an average VALUE over an interval. Try to remember the difference.

   (a) RATE OF CHANGE (AROC) of f over $[a, b]$=

   (b) AVERAGE VALUE of f on $[a, b]$=

9. Both use the word Average. To be careful to distinguish. For example, the average of a student's scores is an average value, whereas the average number of points a student's test score has increased between each test is an example of an average rate of change. Try these to see if you can tell which is which.

   (a) Find the average velocity of a particle over the interval $4 \leq t \leq 10$ if the particle's position is given by $s(t) = 2t^2 - 2t$.

   (b) Find the average velocity of a particle on the interval $4 \leq t \leq 10$ if the particle's velocity is given by $v(t) = 4t - 2$.

10. A study suggests that between the hours of 1:00 PM and 4:00 PM on a normal weekday, the speed of the traffic on a certain freeway exit is modeled by the formula $s(t) = 2t^3 - 21t^2 + 60t + 20$ where the speed is measured in kilometers per hour and t is the number of hours past noon. Compute the average speed of the traffic between the hours of 1:00 PM and 4:00 PM. (Use your calculator, and give your answer correct to three decimal places.)

11. Water is flowing into a tank over a 24-hour period. The rate at which water is flowing into the tank is modeled by the function $R(t) = \frac{1}{75}(600 + 20t - t^2)$, where $R(t)$ is measured in gallons per hour and t is measured in hours. The tank contains 150 gallons of water when $t = 0$.

    (a) Use this function to find the number of gallons of water in the tank at the end of 24 hours. (Use your TI-84's MATH 9:fnInt, but write the integral expression, the answer (exact or 3 decimal places), and units for full AP credit)

    (b) Use this function to find the average rate of water flow over the 24-hour period.

12. FTC 2 The Second Fundamental Theorem of Calculus: If f is continuous on an open interval I containing a, then, for every x in the interval I:

$$\frac{d}{dx}\left[\int_a^x f(t)\ dt\right] =$$

(a) The **Chain Rule Version:**

$$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\ dt\right] =$$

(b) Note that in the integral $a, x, g(x)$ are the _______________, and t is the variable of _____________

13. We use this evaluate the following:

(a) $\dfrac{d}{dx}\displaystyle\int_3^x \sqrt{1+t^2}\ dt =$

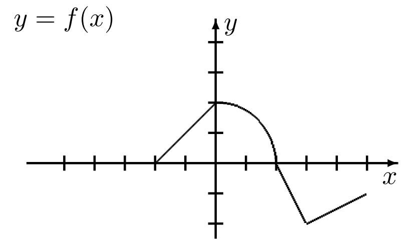
(b) $\dfrac{d}{dx}\displaystyle\int_2^x \tan(t^3)\ dt =$

(c) $\dfrac{d}{dx}\displaystyle\int_{-1}^{x^3} \frac{1}{1+t}\ dt =$

(d) $\dfrac{d}{dx}\displaystyle\int_2^{\sin x} \sqrt[3]{1+t^2}\ dt =$

(e) $\dfrac{d}{dy}\displaystyle\int_\pi^{3y} 14x^2\ dx =$

(f) In general, the 2nd FTC has 2 steps:



14. The graph of f consists of a quarter circle and line segments. Let g be the function given by

$$g(x) = \int_0^x f(t)\, dt.$$

(a) Find $g(0)$

(b) Find $g(-2)$

(c) Find $g(2)$

(d) Find $g(5)$

(e) Find all values of x on the open interval $(-2, 5)$ at which g has a relative maximum. Justify your answer.

(f) Find the absolute minimum of g on $[-2, 5]$, and the value of x at which it occurs. Justify your answer.

(g) Find the x-coordinate of each point of inflection of the graph of g on $(-2, 5)$. Justify your answer.