Name: ________________________________

Chapter Four: Integration

4.1 Antiderivatives and Indefinite Integration

Definition of Antiderivative – A function F is an antiderivative of *f* on an interval *I* if $F'(x) = f(x)$ for all *x* in *I*.

Representation of Antiderivatives – If *F* is an antiderivative of *f* on an interval *I*, then *G* is an antiderivative of *f* on the interval *I* if and only if *G* is of the form $G(x) = F(x) + C$, for all *x* in *I* where *C* is a constant.

Examples: Find an antiderivative and then find the general antiderivative.

1. $y = 3$
2. $f(x) = 2x$
3. $f(x) = 5x^4$

Notation: If we take the differential form of a derivative, $\frac{dy}{dx} = f(x)$, and rewrite it in the form $dy = f(x)dx$ we can find the antiderivative of both sides using the integration symbol $\int$. That is,

$$y = \int dy = \int f(x)dx = F(x) + C$$

Each piece of this equation has a name that I will refer to: The integrand is *f(x)*, the variable of integration is given by *dx*, the antiderivative of *f(x)* is *F(x)*, and the constant of integration is *C*. The term indefinite integral is a synonym for antiderivative.

Note: Differentiation and anti-differentiation are "inverse" operations of each other. That is, if you find the antiderivative of a function *f,* then take the derivative, you will end up back at *f*. Similarly, if you take the derivative, the antiderivative takes you back.

Some Basic Integration Rules:

$$\int 0dx = C \qquad \int kdx = kx + C \qquad \int kf(x)dx = k\int f(x)dx$$

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx \qquad \int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

*important distinction!

We can also consider all the trig derivatives and go backwards to find their integrals.

Examples: For each function, rewrite then integrate and finally simplify.

1. $\int \sqrt[3]{x}dx$

2. $\int \frac{1}{4x^2}dx$

3. $\int \frac{1}{x\sqrt{x}}dx$

4. $\int x(x^3+1)dx$

5. $\int \frac{1}{(3x)^2} dx$

6. $\int \frac{1}{x\sqrt[5]{x}} dx$

Examples: Find the indefinite integral and check the result by differentiation.

1. $\int (12-x)\,dx$

2. $\int (8x^3 - 9x^2 + 4)\,dx$

3. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx$

4. $\int \frac{x^2+2x-3}{x^4}dx$

5. $\int \left(2t^2-1\right)^2 dt$ =

6. $\int \left(t^2-\cos t\right)dt$

7. $\int (\theta^2+\sec^2\theta)d\theta$

8. $\int \sec y\left(\tan y-\sec y\right)dy$

Example: Find the equation of *y* given $\frac{dy}{dx} = 2x - 1$ that has the particular point (1, 1) as part of its solution set.

Example: Solve the differential equation.

1. $f'(x) = 6x^2, f(0) = -1$

2. $f'(p) = 10p - 12p^3, f(3) = 2$

3. $h''(x) = \sin x, h'(0) = 1, h(0) = 6$

Example: A particle, initially at rest, moves along the x-axis such that its acceleration at time $t > 0$ is given by $a(t) = \cos t$. At the time $t = 0$, its position is $x = 3$.

a) Find the velocity and position functions for the particle.
b) Find the values of t for which the particle is at rest.