CALCULUS AB **Name** ______________________________

4.1 – Antiderivatives and Indefinite Integrals

If you were given $f'(x)=3x^2$ and asked what function $f(x)$ had this derivative, what would you say?

$f(x)$ is called the ______________________________ of $f'(x)$.

The symbol $\int g(x)\,dx$ is the ____________________________________________________________.

The term __________________________________ is a synonym for ____________________________.

We can get formulas for antiderivatives by reversing the differentiation rules:

| Differentiation Rules | Integration Rules |
|---|---|
| $\frac{d}{dx}\left[x^n\right]=nx^{n-1}$ | $\int x^n dx=\frac{x^{n+1}}{n+1}+C,\ n\neq -1$ |
| $\frac{d}{dx}[\sin u]=\cos u\frac{du}{dx}$ | $\int \cos u\ du=\sin u+C$ |
| $\frac{d}{dx}[\cos u]=-\sin u\frac{du}{dx}$ | $\int \sin u\ du=-\cos u+C$ |
| $\frac{d}{dx}[\tan u]=\sec^2 u\frac{du}{dx}$ | $\int \sec^2 u\ du=\tan u+C$ |
| $\frac{d}{dx}[\cot u]=-\csc^2 u\frac{du}{dx}$ | $\int \csc^2 u\ du=-\cot u+C$ |
| $\frac{d}{dx}[\sec u]=\sec u\tan u\frac{du}{dx}$ | $\int \sec u\tan u\ du=\sec u+C$ |
| $\frac{d}{dx}[\csc u]=-\csc u\cot u\frac{du}{dx}$ | $\int \csc u\cot u\ du=-\csc u+C$ |
| $\frac{d}{dx}\left[e^u\right]=e^u\frac{du}{dx}$ | $\int e^u\ du=e^u+C$ |
| $\frac{d}{dx}[\ln u]=\frac{1}{u}\frac{du}{dx}$ | $\int \frac{1}{u}\ du=\ln\lvert u\rvert+C$ |

**Properties of Indefinite Integrals**

$\int \left(f(x)\pm g(x)\right)dx=\int f(x)\,dx\pm\int g(x)\,dx$

$\int k\,f(x)\,dx=k\int f(x)\,dx$, where $k$ is a constant

Note that $\int f(x)\cdot g(x)\,dx\neq\int f(x)\,dx\cdot\int g(x)\,dx$ and $\int\frac{f(x)}{g(x)}dx\neq\frac{\int f(x)\,dx}{\int g(x)\,dx}$

Ex $\int(3x^2-5x+4)\,dx=$

Ex. $\int\left(x+\frac{5}{x}\right)dx=$

There isn't a product rule or a quotient rule for antiderivatives so sometimes you must simplify first.

Ex. $\int(2x-1)(x+3)\,dx=$

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Ex. $\int\frac{x^2-2x+7}{\sqrt{x}}\,dx=$

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Ex. Solve the differential equation: $f'(x)=6x^2,\ f(1)=-3$

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Ex. Solve the differential equation: $f''(x)=\cos x,\ f'(0)=3,\ f(0)=-2$

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Ex. An object moves along a straight line so that at any time $t$ its acceleration is given by $a(t)=6t$. At time $t=0$, the object's velocity is 10 and the object's position is 7. What is the object's position at time $t=2$?

# Preview of 4.5

**4.5 Integration Using u-Substitution**

When we differentiated composite functions, we used the Chain Rule. The reverse process is called u-substitution.

Ex. $\int \left(x^2+1\right)^5 (2x)\,dx =$

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Ex. $\int x^2 \left(2x^3+5\right)^4 dx =$

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Ex. $\int x\sqrt{x^2+3}\,dx =$

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Ex. $\int \frac{x}{\sqrt{3x^2+4}}\,dx =$

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Ex. $\int \frac{x+1}{x^2+2x}\,dx =$