# Intro to Antiderivatives

## Section 4.1

Variable of integration

Constant of integration

$$y = \int f(x)\, dx = F(x) + C.$$

Integrand

An antiderivative of $f(x)$

$$\int F'(x)\, dx = F(x) + C.$$

Integration is the “inverse” of differentiation.

For example......

$$\int 3x^2\ dx = x^3 + C$$

because the derivative of $x^3 = 3x^2$

the derivative of $x^3 + 100 = 3x^2$

the derivative of $x^3 + \pi^{17} = 3x^2$

the derivative of $x^3 + C = 3x^2$ for ANY constant $C$

5. $\int \left(-\frac{6}{x^4}\right) dx = \frac{2}{x^3} + C$

add 1 to the exponent,
divide by the new exponent

$$\int -6x^{-4}\ dx \ = -6\int x^{-4}\ dx \ = -6\left(\frac{x^{-3}}{-3}\right) + C$$

$$\text{Notice } \frac{d}{dx}\left(\frac{2}{x^3}\right) = \frac{d}{dx}(2x^{-3}) = -3 * 2x^{-4} = -\frac{6}{x^4}$$

7. $\frac{dy}{dt} = 9t^2$

(You can confirm
by taking the derivative
of this)

| | Original Integral | Rewrite | Integrate | Simplify |
|---|---|---|---|---|
| a. | $\int \frac{1}{x^3}\,dx$ | $\int x^{-3}\,dx$ | $\frac{x^{-2}}{-2} + C$ | $-\frac{1}{2x^2} + C$ |
| b. | $\int \sqrt{x}\,dx$ | $\int x^{1/2}\,dx$ | $\frac{x^{3/2}}{3/2} + C$ | $\frac{2}{3}x^{3/2} + C$ |
| c. | $\int 2\sin x\,dx$ | $2\int \sin x\,dx$ | $2(-\cos x) + C$ | $-2\cos x + C$ |

add 1 to the exponent,
divide by the new exponent

**Original Integral** **Rewrite** **Integrate** **Simplify**

**11.** $\int \sqrt[3]{x}\, dx$

$$\int x^{1/3}\, dx = \frac{x^{4/3}}{\frac{4}{3}} + C = \frac{3}{4}x^{4/3} + C$$

(You can confirm
by taking the derivative
of this)

**12.** $\int \frac{1}{4x^2}\, dx$

$$\frac{1}{4}\int x^{-2}\, dx = \frac{1}{4}\left(\frac{x^{-1}}{-1}\right) + C = -\frac{1}{4x} + C$$

**14.** $\int \frac{1}{(3x)^2}\, dx$