3.7 Notes and Examples

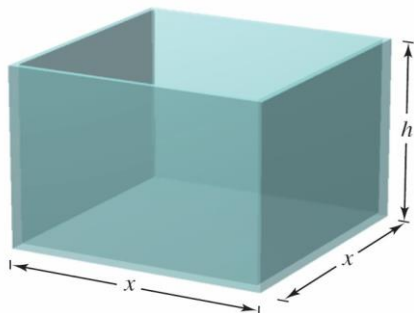Name _____

WHAT IS THE MAXIMUM SIZE YOU NEED?!

One of the most common applications of calculus involves the determining minimum or and maximum values. How often to you hear or read terms like: the greatest profit, least cost or time, the smallest size, the farthest distance, etc. Well that is what we are fixin' to do!

GUIDELINES FOR SOLVING APPLIED MINIMUM AND MAXIMUM PROBLEMS

1. Identify all *given* quantities and all quantities *to be determined*. If possible, make a sketch.

2. Write a **primary equation** for the quantity that is to be maximized or minimized. (A review of several useful formulas from geometry is presented inside the back cover.)

3. Reduce the primary equation to one having a *single independent variable*. This may involve the use of **secondary equations** relating the independent variables of the primary equation.

4. Determine the feasible domain of the primary equation. That is, determine the values for which the stated problem makes sense.

A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume?

Open box with square base:
$S = x^2 + 4xh = 108$

What are we needing to maximize?

Also given:

$$S = (area\ of\ base) + (area\ of\ four\ sides)$$
$$S = 108 = x^2 + 4xh$$

solving for h:

We are trying to maximize volume, so…
→ Write the volume as a function of one variable (x)
→ To maximize V, find the critical numbers of the volume function - take the derivative and set it equal to 0
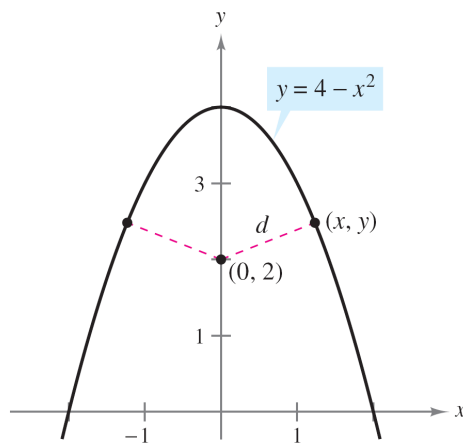→ determine the interval/domain over which there may be a solution for x

Finding the Minimum Distance

Which points on the graph $of \ y \ = \ 4 - x^2$ are closest to the point $(0, 2)$?

→ Use the distance formula:

$d = \sqrt{(x-0)^2 + (y-2)^2}$

→ because d is the smallest when the expression inside the radical is the smallest, you need to find the critical numbers of the radicand (f')

The quantity to be minimized is distance:
$d = \sqrt{(x - 0)^2 + (y - 2)^2}.$

Finding Minimum Area

A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are to be 1.5 inches, and the margins on the left and right are to be 1 inch. What should the dimensions of the page be so that the least amount of paper is used?

There are 2 things going on here:
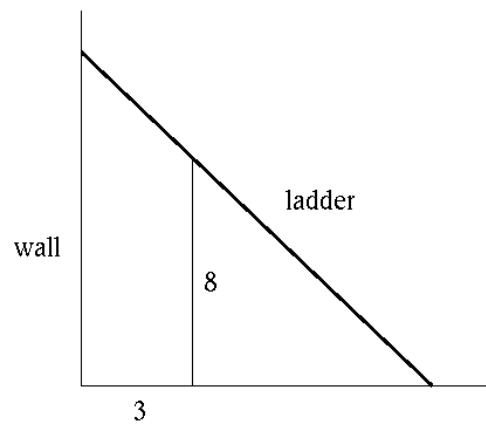→ The area to be minimized:

→ The printed area inside the margins:

- solve for y and substitute into the primary equation
- As area is positive, we need to calculate the critical numbers, differentiate with respect to x.
- domain $x > 0$

1 in.   1 in.

$1\frac{1}{2}$ in.

x

$1\frac{1}{2}$ in.

The quantity to be minimized is area:
$A = (x + 3)(y + 2).$

Find the length of the shortest ladder that will reach over an 8-ft. high fence to a large wall, which is 3 ft. behind the fence.

wall

ladder

8

3

An Endpoint Maximum

Four feet of wire is to be used to form a square and a circle. How much of the wire should be used for the circle to enclose the maximum total area?

→ Total area is given by:

$A = (area\ of\ square) + (area\ of\ circle)$

$$A = x^2 + \pi r^2$$

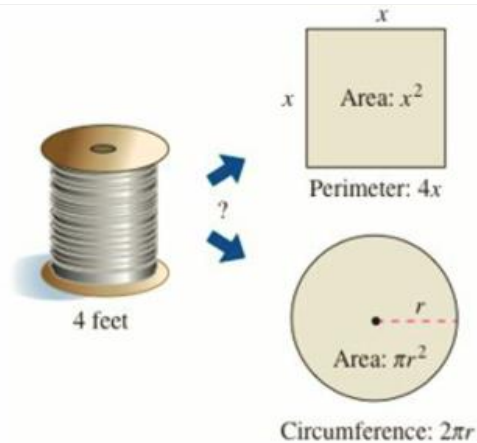→ Areas are formed by a 4 ft. length of wire, so we define the perimeter:

$4 = (P\ of\ square) + (P\ of\ circle)$

$$4 = 4x + 2\pi r$$

- Solve for r, substitute into the primary equation
- The domain of x is restricted by the fact that first, the length of the sides of the square must be greater than 0. Second, as we are limited to 4 ft. of wire and we have four sides of a square, so x can be no greater than 1.

$$D: 0 \leq x \leq 1$$

→ Differentiate A with respect to x
→ Find the critical number(s) in the domain

Area: x^2

Perimeter: $4x$

Area: πr^2

Circumference: $2\pi r$

4 feet

The quantity to be maximized is area:

$A = x^2 + \pi r^2$.