

Curve Sketching

Date _____ Period _____

For each problem, find the: x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

1) $y = -\frac{x^3}{3} + x^2$

① Domain / Range
 $\mathbb{R}$

② Intercepts, asymptotes
 $(0,0)$
 $(3,0)$ $x^2(1 - \frac{x}{3}) = 0$
 $\lim_{x \rightarrow \infty} y = -\infty$, $\lim_{x \rightarrow -\infty} y = \infty$

③ CP & f'' for
 extrema, POI

$$y' = -x^2 + 2x$$

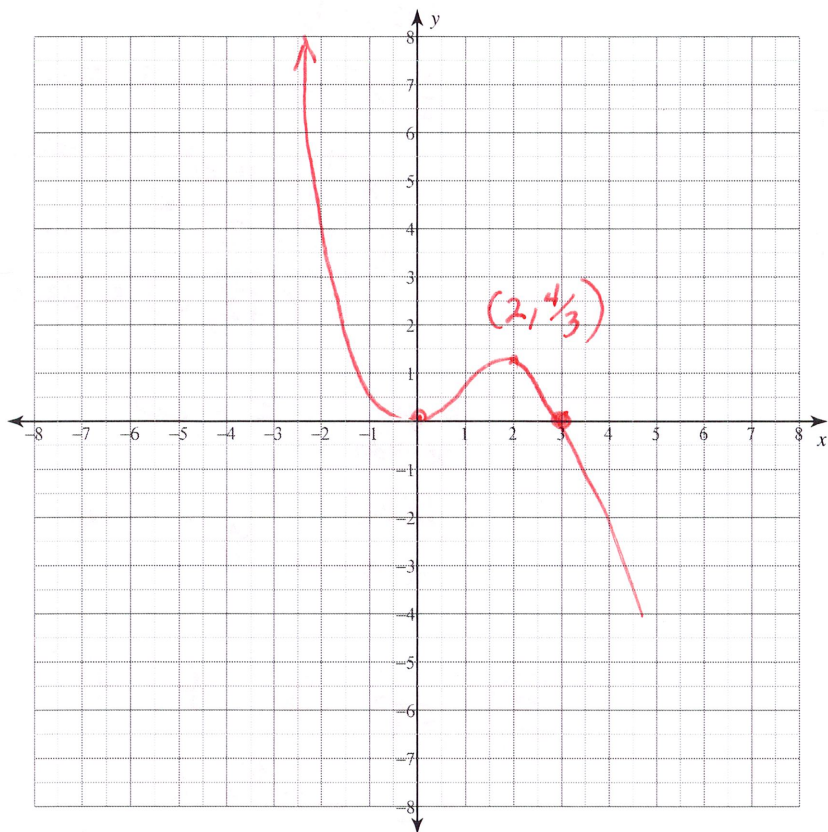
$$0 = x(2 - x)$$

y'	$\ominus$	$\oplus$	$\ominus$
	$-$	0	2
		min	max

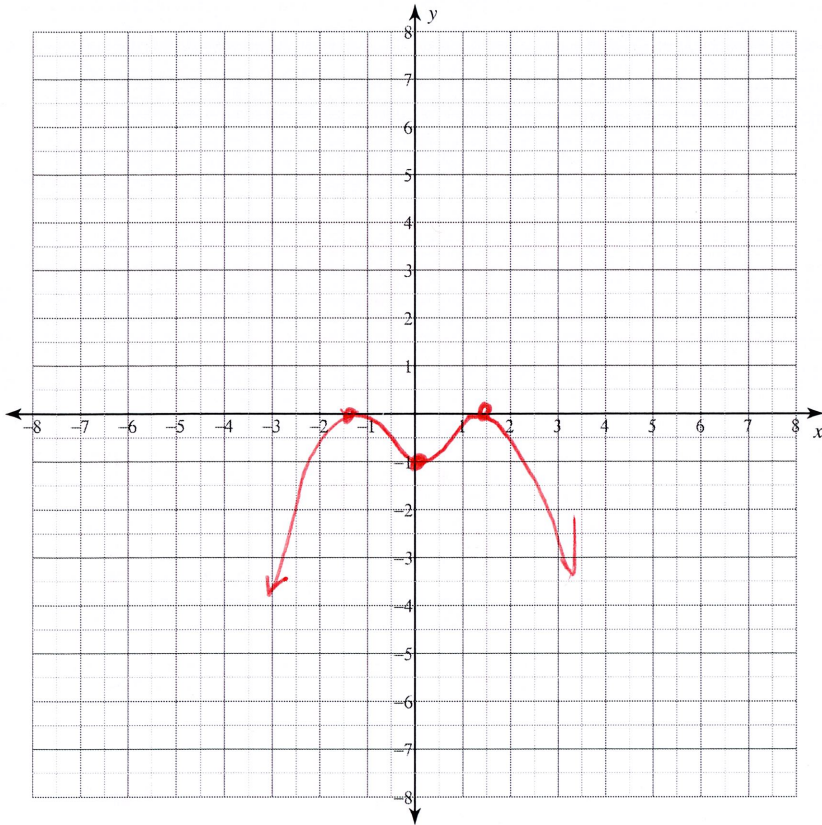
$$y'' = -2x + 2$$

$$0 = 2(1 - x)$$

y''	$\oplus$	$\ominus$
	up	down
	1	
	POI $(1, \frac{2}{3})$	



$$2) y = -\frac{x^4}{4} + x^2 - 1$$



① Domain: $\mathbb{R}$

② intercepts, asymptotes.

$(0, -1)$

$$-\frac{1}{4}x^4 + x^2 - 1 = 0$$

$$x^4 - 4x^2 + 4 = 0$$

$$(x^2 - 2)^2 = 0$$

$$(\sqrt{2}, 0) \text{ and } (-\sqrt{2}, 0)$$

$$\lim_{x \rightarrow \infty} y = -\infty$$

$$\lim_{x \rightarrow -\infty} y = -\infty$$

③ cp for f' f''

$$y' = -x^3 + 2x$$

$$0 = x(2 - x^2)$$

$$y' \quad \begin{array}{c} \oplus \quad \ominus \quad \oplus \quad \ominus \\ -\sqrt{2} \quad 0 \quad \sqrt{2} \end{array}$$

$$y'' = -3x^2 + 2$$

$$0 = -3x^2 + 2$$

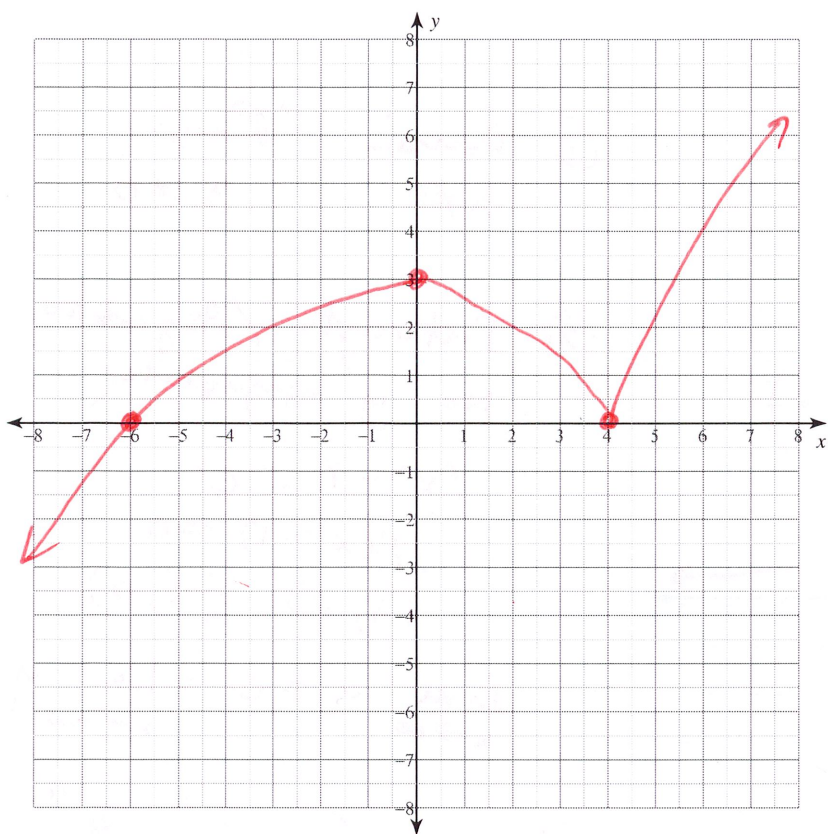
$$3x^2 = 2$$

$$x = \pm \sqrt{2/3}$$

$$y'' \quad \begin{array}{c} \ominus \quad \oplus \quad \ominus \\ -\sqrt{2/3} \quad \sqrt{2/3} \end{array}$$

↖ pos ↗

$$3) y = \frac{1}{5}(x-4)^{\frac{5}{3}} + 2(x-4)^{\frac{2}{3}}$$



① Domain / Range
 $\mathbb{R}$ $\mathbb{R}$

② intercepts, asymptotes.

(0, 3)

$$0 = \frac{1}{5}(x-4)^{\frac{5}{3}} + 2(x-4)^{\frac{2}{3}}$$

$$0 = (x-4)^{\frac{2}{3}} \left(\frac{x-4}{5} + 2 \right)$$

(4, 0)

$$\frac{x-4}{5} + 2 = 0$$

(-6, 0)

$$x-4 = -10$$

$$x = -6$$

$$\lim_{y \rightarrow \infty} y = -\infty$$

$$\lim_{y \rightarrow -\infty} y = \infty$$

③ c.p. for f' , f''
extrema, POI

$$y' = \frac{1}{3}(x-4)^{\frac{2}{3}} + \frac{4}{3}(x-4)^{-\frac{1}{3}}$$

$$0 = \frac{1}{3}(x-4)^{-\frac{1}{3}} [(x-4) + 4]$$

$$0 = \left(\frac{1}{3\sqrt[3]{x-4}} \right) (x)$$

$$y' \quad \oplus \quad | \quad \ominus \quad | \quad \oplus$$

$$y'' = \frac{2}{9}(x-4)^{-\frac{1}{3}} - \frac{4}{9}(x-4)^{-\frac{4}{3}}$$

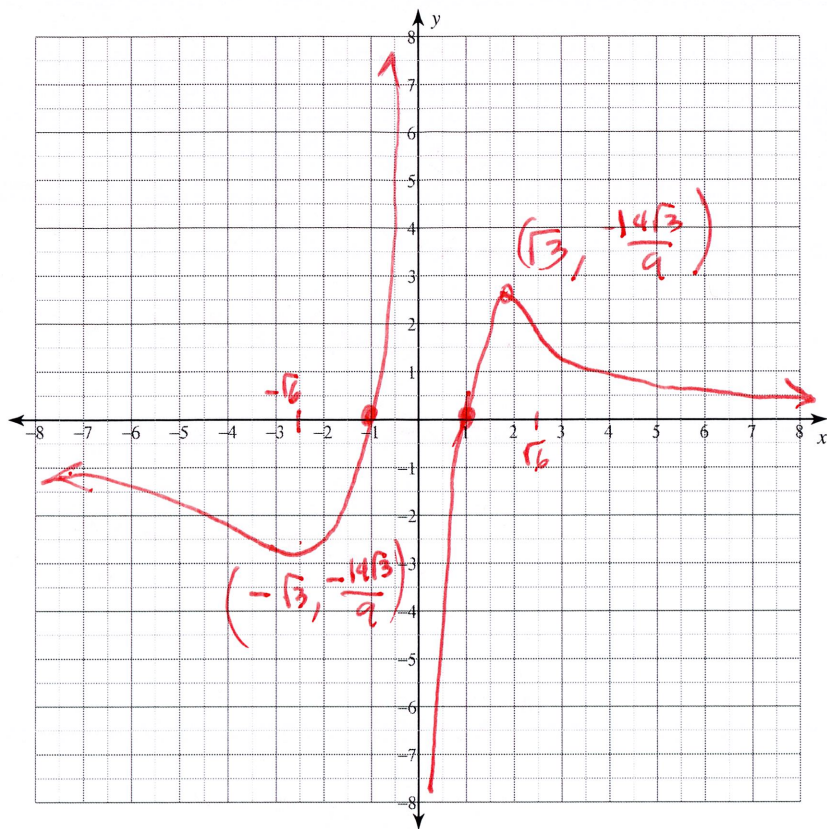
$$0 = \frac{2}{9}(x-4)^{-\frac{4}{3}} [(x-4) - 2]$$

$$0 = \left(\frac{2}{9(x-4)^{\frac{4}{3}}} \right) (x-6)$$

$$y'' \quad \ominus \quad | \quad \ominus \quad | \quad \oplus$$

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4) $y = \frac{7x^2 - 7}{x^3}$



① Domain / Range
 $x \neq 0$ $y \neq 0$

② Intercepts / Asymptotes.
 No y asymptote

$$7x^2 - 7 = 0$$

$$7(x^2 - 1) = 0$$

$$(1, 0) \quad (-1, 0)$$

$$\lim_{x \rightarrow \infty} y = 0 = \lim_{x \rightarrow -\infty} y$$

③ $c.p.$ for y' , y''

$$y' = \frac{x^3(14x) - 3x^2(7x^2 - 7)}{x^6}$$

$$0 = 14x^4 - 21x^4 + 21x^2$$

$$0 = 7x^2(3 - x^2)$$

$$y' \quad \frac{-}{-} \quad \frac{+}{+} \quad \frac{+}{+} \quad \frac{-}{-}$$

$$y' = \frac{7(x^2 - 3)}{x^4}$$

$$y'' = \frac{x^4(14x) - (7x^2 - 21)(4x^3)}{x^8}$$

$$y'' = \frac{14x^3(x^2 - 6)}{x^8}$$

$$y'' = \frac{14(x^2 - 6)}{x^5}$$

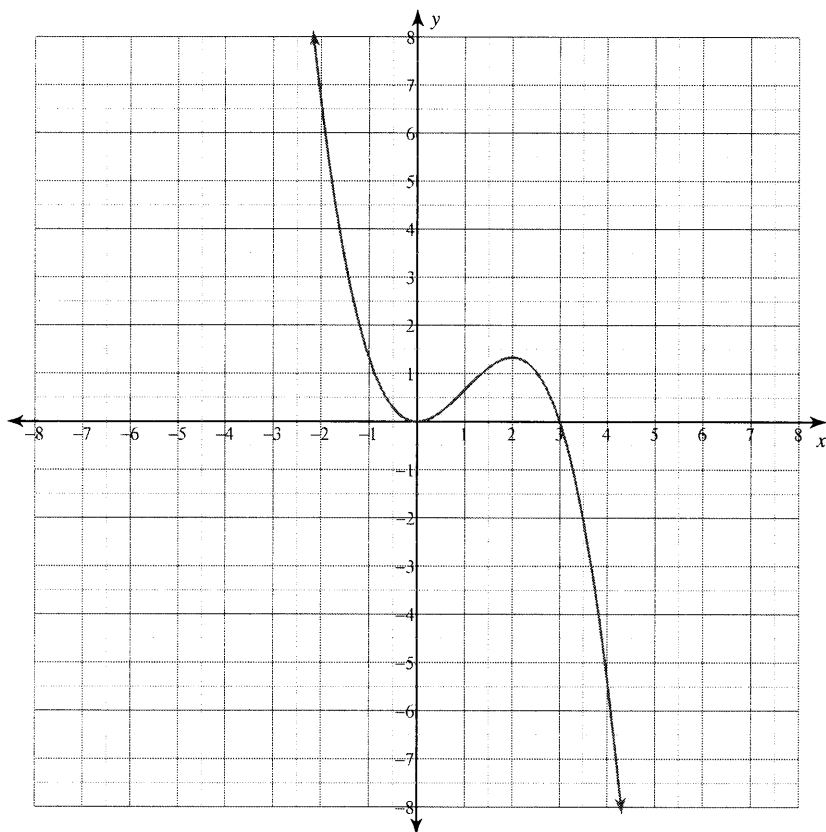
$$y'' \quad \frac{-}{-} \quad \frac{+}{+} \quad \frac{-}{-} \quad \frac{+}{+}$$

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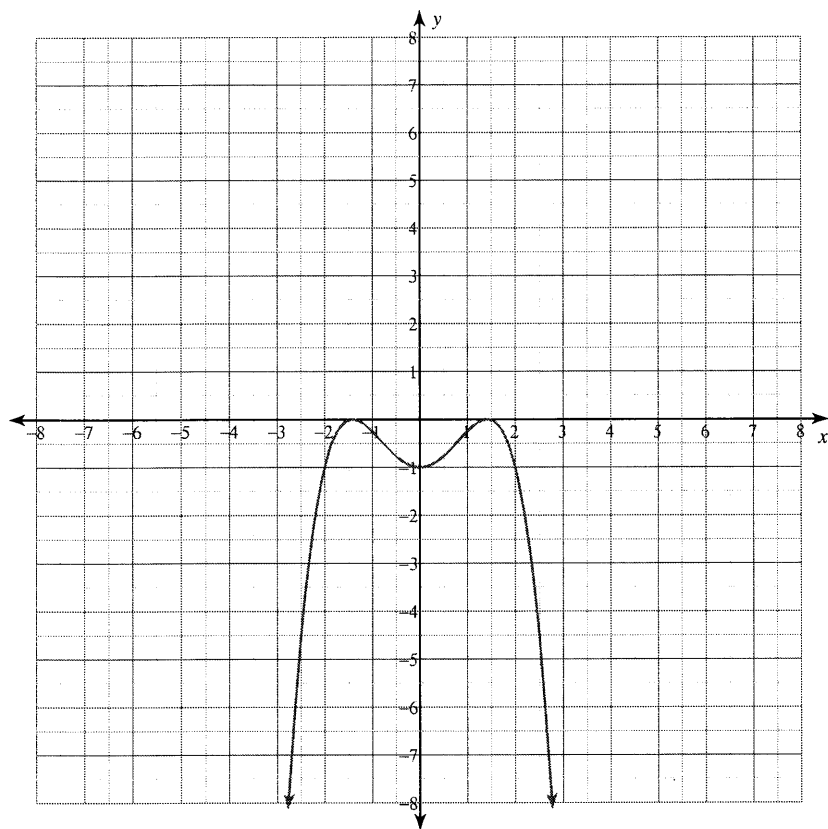
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For each problem, find the: x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

1) $y = -\frac{x^3}{3} + x^2$

x-intercepts at $x = 0, 3$ y-intercept at $y = 0$ Critical points at: $x = 0, 2$ Increasing: $(0, 2)$ Decreasing: $(-\infty, 0), (2, \infty)$ Inflection point at: $x = 1$ Concave up: $(-\infty, 1)$ Concave down: $(1, \infty)$ Relative minimum: $(0, 0)$ Relative maximum: $\left(2, \frac{4}{3}\right)$

2) $y = -\frac{x^4}{4} + x^2 - 1$



x -intercepts at $x = -\sqrt{2}, \sqrt{2}$

y -intercept at $y = -1$

Critical points at: $x = -\sqrt{2}, 0, \sqrt{2}$

Increasing: $(-\infty, -\sqrt{2}), (0, \sqrt{2})$

Decreasing: $(-\sqrt{2}, 0), (\sqrt{2}, \infty)$

Inflection points at: $x = -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}$

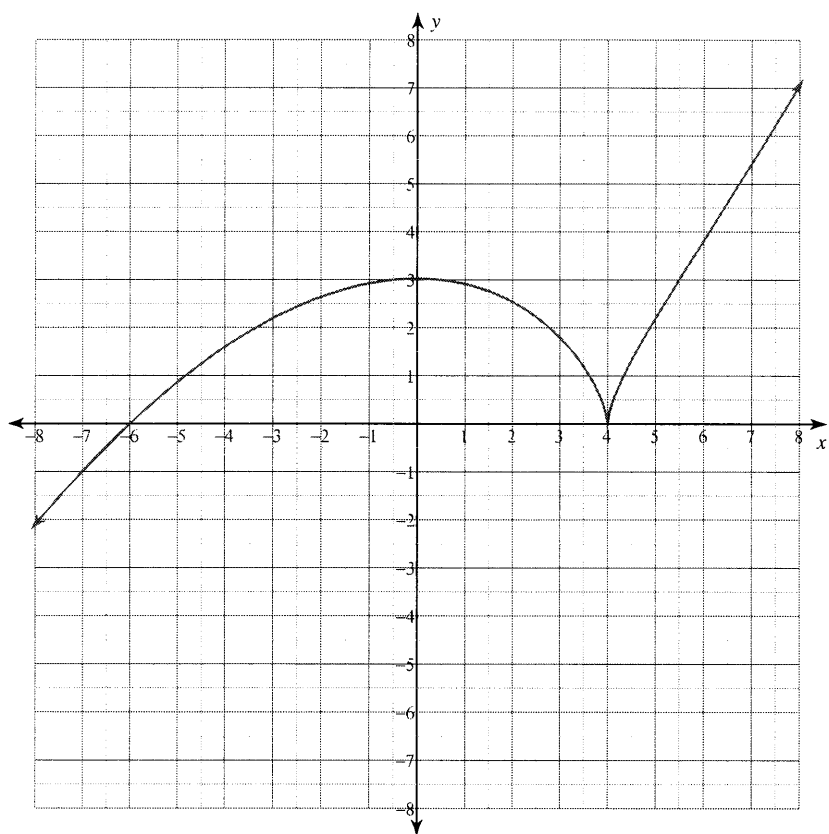
Concave up: $\left(-\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}\right)$

Concave down: $\left(-\infty, -\frac{\sqrt{6}}{3}\right), \left(\frac{\sqrt{6}}{3}, \infty\right)$

Relative minimum: $(0, -1)$

Relative maxima: $(-\sqrt{2}, 0), (\sqrt{2}, 0)$

3) $y = \frac{1}{5}(x-4)^{\frac{5}{3}} + 2(x-4)^{\frac{2}{3}}$



x-intercepts at $x = -6, 4$

y-intercept at $y = \frac{12\sqrt[3]{2}}{5}$

Critical points at: $x = 0, 4$

Increasing: $(-\infty, 0), (4, \infty)$

Decreasing: $(0, 4)$

Inflection point at: $x = 6$

Concave up: $(6, \infty)$

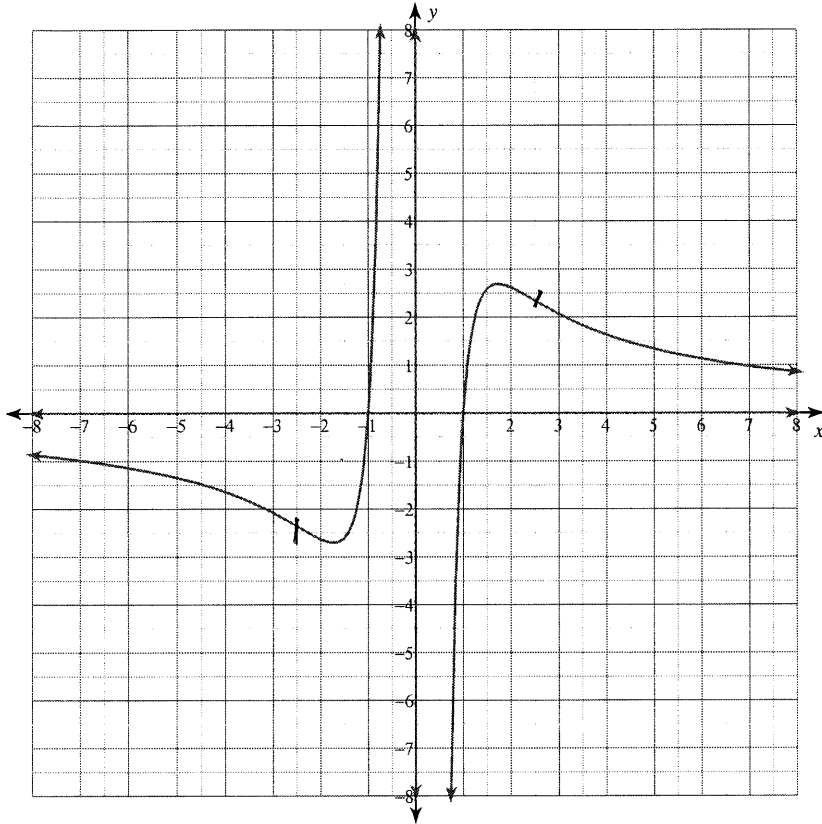
Concave down: $(-\infty, 4), (4, 6)$

Relative minimum: $(4, 0)$

Relative maximum: $\left(0, \frac{12\sqrt[3]{2}}{5}\right)$

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4) $y = \frac{7x^2 - 7}{x^3}$



x-intercepts at $x = -1, 1$

No y-intercepts.

Vertical asymptote at: $x = 0$

Horizontal asymptote at: $y = 0$

Critical points at: $x = -\sqrt{3}, \sqrt{3}$

Increasing: $(-\sqrt{3}, 0), (0, \sqrt{3})$

Decreasing: $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$

Inflection points at: $x = -\sqrt{6}, \sqrt{6}$

Concave up: $(-\sqrt{6}, 0), (\sqrt{6}, \infty)$

Concave down: $(-\infty, -\sqrt{6}), (0, \sqrt{6})$

Relative minimum: $\left(-\sqrt{3}, -\frac{14\sqrt{3}}{9}\right)$

Relative maximum: $\left(\sqrt{3}, \frac{14\sqrt{3}}{9}\right)$