*3.5 - Limits at infinity*

## Polynomial End Behavior

|  | odd degree | even degree |
|---|---|---|
| positive leading term | | |
| negative leading term | | |
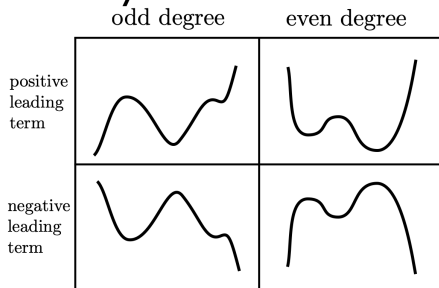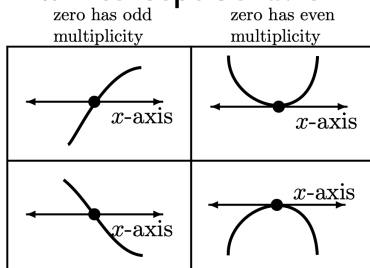
## Rational End Behavior

$$R(x) = \frac{p(x)}{q(x)} = \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0}$$

$n = $ degree of top polynomial
$m = $ degree of bottom polynomial

| case | as $x \to \pm\infty$ |
|---|---|
| $n < m$ | $f(x) \to y = 0$ |
| $n = m$ | $f(x) \to y = \frac{a_n}{b_m}$ |
| $n > m$ | $f(x) \to q(x)\,\lceil\,\overline{p(x)}$  (divide, ignore remainder) |

## $x$-intercept behavior

|  zero has odd multiplicity  |  zero has even multiplicity  |
|---|---|
| $x$-axis | $x$-axis |
| $x$-axis | $x$-axis |

### 3 zeros of Rational Functions

| $R(0)$ | $y$ intercept |
|---|---|
| All real $x$ that make top $p(x) = 0$ | $x$ intercepts |
| All real $x$ that make bottom $q(x) = 0$ | vertical asymptotes |

1. Let $f(x) = 4x^2 + 2x + 3$

   (a) $\displaystyle \lim_{x \to \infty} \frac{f(x)}{2x}$

   (b) $\displaystyle \lim_{x \to \infty} \frac{f(x)}{2x^2}$

   (c) $\displaystyle \lim_{x \to \infty} \frac{f(x)}{2x^3}$

2. Find the end behavior of the following functions:

(a) $\displaystyle\lim_{x\to\infty} \frac{\sin x}{x}$

(b) $\displaystyle\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$

(c) $\displaystyle\lim_{x\to-\infty} \left(\frac{5}{x} - \frac{x}{3}\right)$

(d) $\displaystyle\lim_{x\to\infty} \frac{|3x+2|}{x-2}$

(e) $\displaystyle\lim_{x\to-\infty} \frac{|3x+2|}{x-2}$

3. $\displaystyle\lim_{x\to\infty} x \tan \frac{1}{x}$

(a) Let $x = \dfrac{1}{t}$ and restate the limit as $t \to 0^+$:

(b) Make it all about sine and cosine:

(c) Recall a theorem from chapter 1: $\displaystyle\lim_{x\to0} \frac{\sin x}{x} = 1 \ldots$

3.6 Analyze and sketch by finding intercepts, extrema, points of inflection, and asymptotes

1. $y = -\dfrac{1}{3}(x^3 - 3x + 2) = \dfrac{(x-1)^2(x+2)}{-3}$

   (a) (intercepts and vertical asymptotes)

   (b) end behavior ($\lim\limits_{x\to\pm\infty}$ and inspecting degree)

   (c) (first derivative for rel. extrema)

   (d) (second derivative for POI and concavity)

2. $y = 6x^2 - 3x^4$

   (a) (intercepts and vertical asymptotes)

   (b) end behavior ($\lim\limits_{x\to\pm\infty}$ and inspecting degree)

   (c) (first derivative for rel. extrema)

   (d) (second derivative for POI and concavity)

3. $y = (x-1)^{2/3}$

    (a) (intercepts and vertical asymptotes)

    (b) end behavior ($\lim\limits_{x \to \pm\infty}$ and inspecting degree)

    (c) (first derivative for rel. extrema)

    (d) (second derivative for POI and concavity)