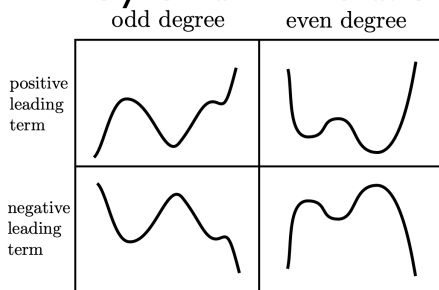
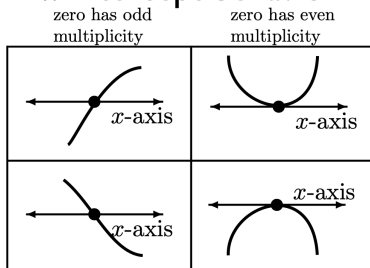


3.5 - Limits at infinity

**Polynomial End Behavior**



**x-intercept behavior**



**Rational End Behavior**

$$R(x) = \frac{p(x)}{q(x)} = \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x + b_0}$$

$n$  = degree of top polynomial  
 $m$  = degree of bottom polynomial  
 case as  $x \rightarrow \pm\infty$

$n < m$	$f(x) \rightarrow y = 0$
$n = m$	$f(x) \rightarrow y = \frac{a_n}{b_m}$
$n > m$	$f(x) \rightarrow q(x) \mid p(x)$ (divide, ignore remainder)

**3 zeros of Rational Functions**

$R(0)$	$y$ intercept
All real $x$ that make top $p(x) = 0$	$x$ intercepts
All real $x$ that make bottom $q(x) = 0$	vertical asymptotes

1. Let  $f(x) = 4x^2 + 2x + 3$

(a)  $\lim_{x \rightarrow \infty} \frac{f(x)}{2x}$

(b)  $\lim_{x \rightarrow \infty} \frac{f(x)}{2x^2}$

(c)  $\lim_{x \rightarrow \infty} \frac{f(x)}{2x^3}$

2. Find the end behavior of the following functions:

(a)  $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$

(b)  $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1}}$

(c)  $\lim_{x \rightarrow -\infty} \left( \frac{5}{x} - \frac{x}{3} \right)$

(d)  $\lim_{x \rightarrow \infty} \frac{|3x + 2|}{x - 2}$

(e)  $\lim_{x \rightarrow -\infty} \frac{|3x + 2|}{x - 2}$

3.  $\lim_{x \rightarrow \infty} x \tan \frac{1}{x}$

(a) Let  $x = \frac{1}{t}$  and restate the limit as  $t \rightarrow 0^+$ :

(b) Make it all about sine and cosine:

(c) Recall a theorem from chapter 1:  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \dots$

3.6 Analyze and sketch by finding intercepts, extrema, points of inflection, and asymptotes

1.  $y = -\frac{1}{3}(x^3 - 3x + 2) = \frac{(x-1)^2(x+2)}{-3}$

(a) (intercepts and vertical asymptotes)

(b) end behavior ( $\lim_{x \rightarrow \pm\infty}$  and inspecting degree)

(c) (first derivative for rel. extrema)

(d) (second derivative for POI and concavity)

2.  $y = 6x^2 - 3x^4$

(a) (intercepts and vertical asymptotes)

(b) end behavior ( $\lim_{x \rightarrow \pm\infty}$  and inspecting degree)

(c) (first derivative for rel. extrema)

(d) (second derivative for POI and concavity)

3.  $y = (x - 1)^{2/3}$

(a) (intercepts and vertical asymptotes)

(b) end behavior ( $\lim_{x \rightarrow \pm\infty}$  and inspecting degree)

(c) (first derivative for rel. extrema)

(d) (second derivative for POI and concavity)