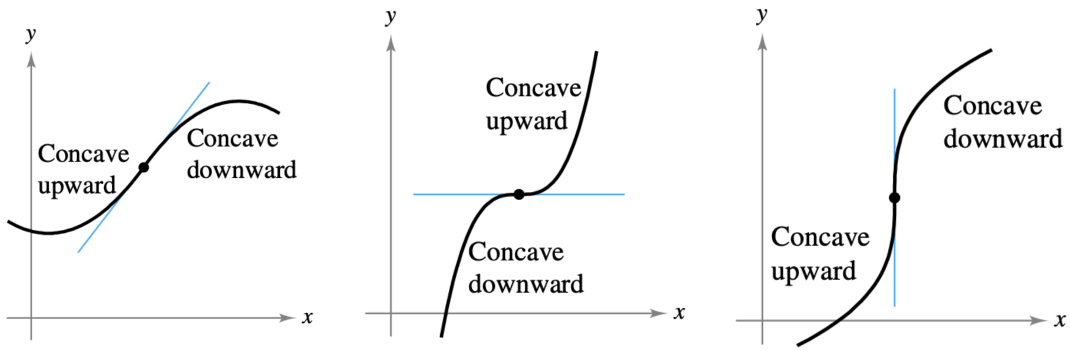
3.4 Notes and Examples

Name: Block: Seat:

Concavity & The 2nd Derivative Test



Definition of Concavity:

Let f be differentiable on an open interval (a, b).

1. The graph of f is ____________________ on (a, b) if f' is ____________________ on (a, b).

2. The graph of f is ____________________ on (a, b) if f' is ____________________ on (a, b).

Test for Concavity:

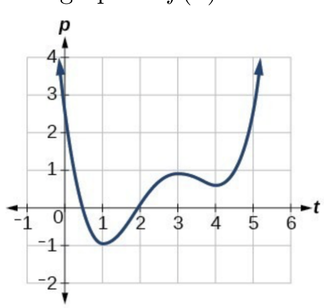
Let f be a function whose second derivative exists on an open interval (a, b).

1. If $f''(x)$ ________ 0 for all x in (a, b), then the graph of f is ____________________ on (a, b).

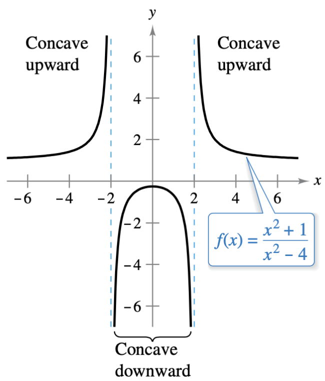
2. If $f''(x)$ ________ 0 for all x in (a, b), then the graph of f is ____________________ on (a, b).

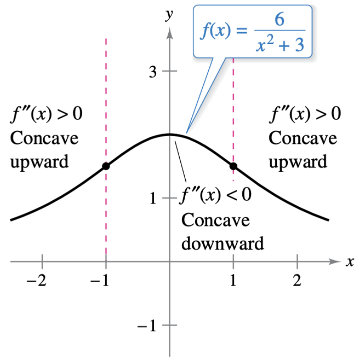
1. Determine the open intervals on which the graph of the function is concave upward or concave downward

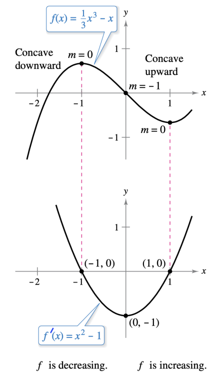
   (a) The graph of $f(x)$:



   (b) $g(x) = 3x^2 - x^3$







Definition of an Inflection Point:

A function f has an inflection point at $(c, f(c))$ IF:

1. f has a ________ line at the point $(c, f(c))$.

2. $f''(c) =$ ________ OR $f''(c)$ ____________

   AND

3. If $f''(x)$ __________________ at $x = c$.

   OR

   $f'(x)$ _____ from ____________ to ______________ or

   ____________ to ______________.

2. Find the points of inflection and discuss the concavity of the graph of the function $f(x) = -x^3 + 6x^2 - 5$
   (a) $f'(x) =$ $f''(x) =$

   (b) Sign lines for f' and f'':

   (c) f is increasing on f is concave up on

   (d) f is decreasing on f is concave down on

The 2^{nd} Derivative Test
Let f be a function such that the second derivative of f exists on an open interval containing c.

1. If $f'(c)$______________ and $f''(c)$ ______________, then $f(c)$ is a relative ______________.

2. If $f'(c)$______________ and $f''(c)$ ______________, then $f(c)$ is a relative ______________.

3. Use the Second Derivative Test, if possible, to find the relative extrema of $f(x) = -3x^5 + 5x^3$. Justify your answer.
   (a) $f'(x) =$ $f''(x) =$

   (b) $f'(x) = 0$ when $x =$: f'' at the zeros of f':

   (c) f has a is relative ______________ at $x =$ ______________ by the 2^{nd} derivative test because

   $f'($______________$) =$__________ and $f''($ ______________$)$ __________ 0

   (d) f has a is relative ______________ at $x =$ ______________ by the 2^{nd} derivative test because

   $f'($______________$) =$__________ and $f''($ ______________$)$ __________ 0

4. Let $f(x) = x^4 - 2x^2 - 1$. Find any extrema of f. Justify your conclusions.
   (a) $f'(x) =$ $f''(x) =$

   (b) $f'(x) = 0$ when $x =$: Sign line for f'' :

   (c) f has a is relative ______________ at $x =$ ______________ by the 2^{nd} derivative test because

   $f'($______________$) =$__________ and $f''($ ______________$)$ __________ 0

   (d) f has a is relative ______________ at $x =$ ______________ by the 2^{nd} derivative test because

   $f'($______________$) =$__________ and $f''($ ______________$)$ __________ 0

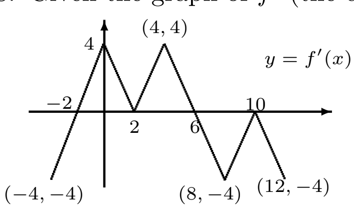
AP Style Practice

5. Suppose that the function f has a continuous second derivative for all x, and that $f(3) = -4$, $f'(3) = 1$, $f''(3) = -2$. Let g be a function whose derivative is given by $g'(x) = (x^2 - 9)(2f(x) + 5f'(x))$ for all x. Does g have a local maximum or a local minimum at $x = 3$? Justify your answer.

6. (Calculator Active) The derivative of the function f is given by $f'(x) = x\cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$? Justify your answer.

7. (Calculator Active) The derivative of the function f is given by $f'(x) = e^{\sin x} - \cos x - 1$ for $0 < x < 2\pi$. On what interval(s) is f concave down? Are there any points of inflection? Justify your answers.

8. Given the graph of f' (the derivative of f) below, do the following and justify your conclusions:



(a) Find all critical points of f

(b) Find the intervals where f is increasing

(c) Find the intervals where f is decreasing

(d) Find the local extrema of f

(e) Find the intervals where f is concave up

(f) Find the intervals where f is concave down

(g) Find all points of inflection of f.