CALCULUS AB
Section 3.1 – Extrema on an Interval

Definition of Extrema
Let f be defined on an interval I containing c.
1) $f(c)$ is the **minimum** of f on I if $f(c) \le f(x)$ for all x in I.
2) $f(c)$ is the **maximum** of f on I if $f(c) \ge f(x)$ for all x in I.
The minimum and maximum of a function on an interval are the extreme values or extrema of the function on the interval. The minimum and maximum of a function on an interval are also called the **absolute minimum** and **absolute maximum**, or the **global minimum** and **global maximum**, on the interval.

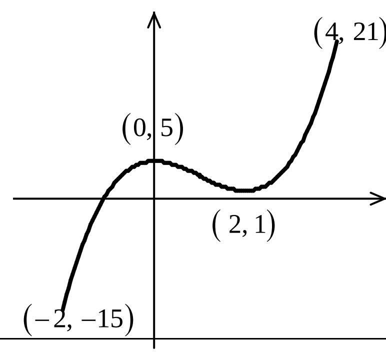
Definition of Relative Extrema
1) If there is an open interval containing c on which $f(c)$ is a maximum, then $(c, f(c))$ is called a **relative maximum** of f, or you can say that f has a **relative maximum** at $(c, f(c))$.
2) If there is an open interval containing c on which $f(c)$ is a minimum, then $(c, f(c))$ is called a **relative minimum** of f, or you can say that f has a **relative minimum** at $(c, f(c))$.
The relative maximum and relative minimum points are sometimes called **local maximum** and **local minimum points**, respectively.

In the figure on the right, on the interval $[-2, 4]$,
f has an absolute maximum at ________________

f has an absolute minimum at ________________

f has a relative maximum at ________________

f has a relative minimum at ________________

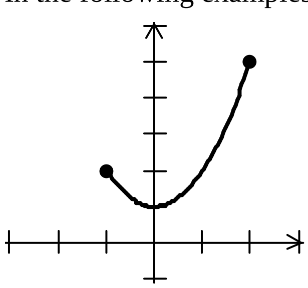


Definition of a Critical Number and a Critical Point
Let f be defined at c. If $f'(c) = 0$ or if f is not differentiable at c, then c is a **critical number** of f and the point $(c, f(c))$ is a **critical point** of f.

Theorem
Relative extrema occur only at critical numbers.

Ex. In the following examples, name the maximum and minimum points if possible.

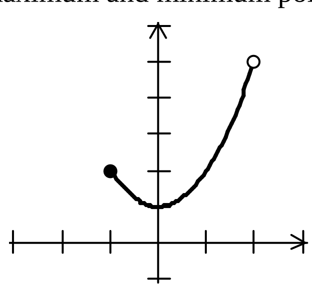
(a)



Minimum at _____________

Maximum at _____________

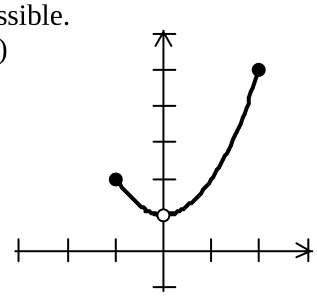
(b)



Minimum at _____________

Maximum at _____________

(c)



Minimum at _____________

Maximum at _____________

What conditions are necessary to guarantee that there will be a maximum and a minimum?

<u>Extreme Value Theorem</u> (EVT)

If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ for some numbers c and d in $[a, b]$.

<u>Guidelines for Finding Extrema on a Closed Interval</u> – **<u>Candidates Test</u>**

To find the extrema of a continuous function f on a closed interval $[a, b]$, use the following steps:

1) Find $f'(x)$ and the critical numbers of f in $[a, b]$.
2) Exaluate f at each critical number in (a, b).
3) Evaluate f at each endpoint in $[a, b]$.
4) The least of these values is the minimum. The greatest is the maximum.

<u>Ex</u>. Find the absolute maximums and minimums of f on the given closed interval, and state where these values occur.

(a) $f(x) = 3x^2 - 24x - 1 \quad [-1, 5]$

(b) $f(x) = 6x^3 - 6x^4 + 5 \quad [-1, 2]$

(c) $f(x) = 3x^{2/3} - 2x + 1 \quad [-1, 8]$

(d) $f(x) = \sin^2 x + \cos x \quad [0, 2\pi]$