2.6 (p158-160) #21-27 odd, 42-43

(21)

① Given

$\frac{dx}{dt} = 2$ ft/sec

(a) Find $\left.\frac{dy}{dt}\right|_{x=7}$

$x^2 + y^2 = 25^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-x\frac{dx}{dt}}{y}$

$\left.\frac{dy}{dt}\right|_{x=7} = \frac{-7(2)}{\sqrt{25^2-7^2}} = \frac{-14}{24}$ ft/sec.

sliding down @ a rate of $-\frac{7}{12}$ ft per second

Find $\left.\frac{dy}{dt}\right|_{x=15}$

$= \frac{-15(2)}{\sqrt{25^2-15^2}}$

$= \frac{-30}{20}$

$= -\frac{3}{2}$ ft/sec

Find $\left.\frac{dy}{dt}\right|_{x=24}$

$= \frac{-24(2)}{\sqrt{25^2-24^2}}$

$= \frac{-48}{7}$ ft/sec

(b) Find $\left.\frac{dA}{dt}\right|_{x=7 \text{ ft}}$

$A = \frac{1}{2} \times y$

$\frac{dA}{dt} = \frac{1}{2} \times \frac{dy}{dt} + y\frac{1}{2}\cdot\frac{dx}{dt}$

$= \frac{1}{2}(7)\left(-\frac{7}{12}\right) + \frac{24}{2}(2)$

$= \frac{527}{24}$ ft^2/sec

(c) Find $\left.\frac{d\theta}{dt}\right|_{x=7\text{ ft}}$

$\tan\theta = \frac{24}{7}$

$\tan\theta = \frac{y}{x}$

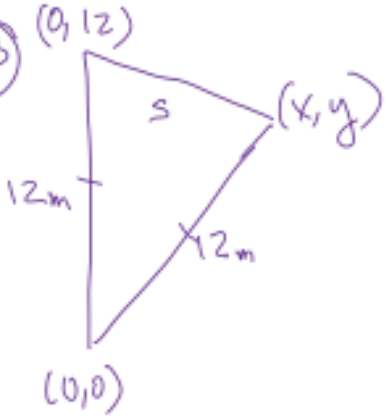
$\sec^2\theta \frac{d\theta}{dt} = \frac{x\left(\frac{dy}{dt}\right) - y\left(\frac{dx}{dt}\right)}{x^2}$

$\left(\frac{25}{7}\right)^2 \frac{d\theta}{dt} = \frac{7\left(\frac{-7}{12}\right) - 24(2)}{49}$

$\frac{d\theta}{dt} = \left(\frac{49}{625}\right)\left(\frac{-\frac{49}{12} - 48}{49}\right)$

$\frac{d\theta}{dt} = \frac{1}{12}$ rad/sec

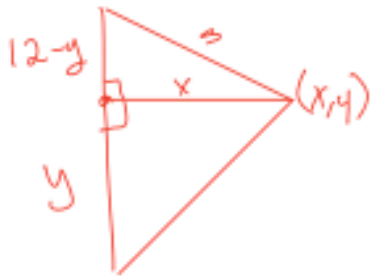
(23)



$\frac{ds}{dt} = -\frac{1}{5}$ m/sec

find $\left.\frac{dy}{dt}\right|_{y=6} = ?$ $\left.\frac{dx}{dt}\right|_{y=6} = ?$

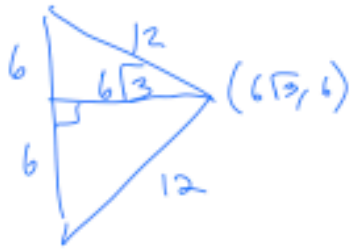
$\left(-\frac{1}{5}\text{ m/sec}\right)$ $\left(\frac{-\sqrt{3}}{15}\text{ m/sec}\right)$



$x^2 + (12-y)^2 = s^2$

$2x\frac{dx}{dt} + 2(12-y)(-1)\frac{dy}{dt} = 2s\frac{ds}{dt}$

When $y = 6$



Also $x^2 + y^2 = 12^2$

$2x\frac{dx}{dy} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$

Putting Red & Blue together

$x\frac{dx}{dt} + (y-12)\left(-\frac{x}{y}\frac{dx}{dt}\right) = s\frac{ds}{dt}$

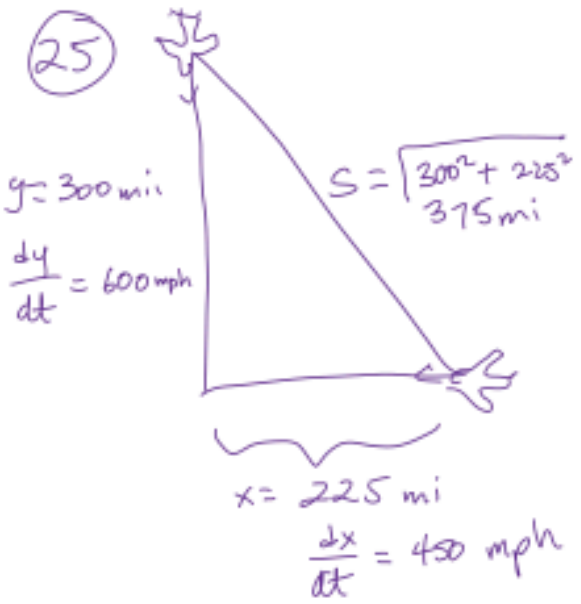
$6\sqrt{3}\frac{dx}{dt} + (6-12)\left(-\frac{x\sqrt{3}}{6}\cdot\frac{dx}{dt}\right) = 12\cdot\frac{-1}{5}$

$\frac{dx}{dt}\left(6\sqrt{3}+6\sqrt{3}\right) = -\frac{12}{5}$ m/sec

$\frac{dy}{dt} = \frac{-12}{5(12\sqrt{3}} = -\frac{1}{5\sqrt{3}}$

$\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = \left(-\frac{6\sqrt{3}}{6}\right)\left(\frac{-1}{5\sqrt{3}}\right) = \frac{1}{5}$ m/sec

(25)



(a) find $\frac{ds}{dt}$

$$x^2 + y^2 = s^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2s\frac{ds}{dt}$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = s\frac{ds}{dt}$$

$$225(-450) + 300(600) = 375\frac{ds}{dt}$$

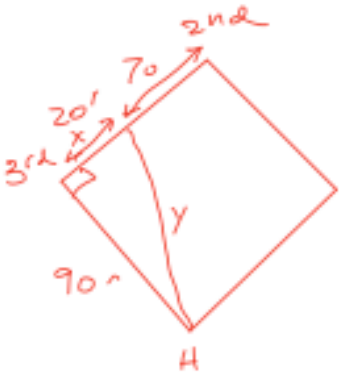
$$\frac{ds}{dt} = -750 \text{ mph}$$

(b) Find the time when $s = 0$

$$750 \cdot t = 375$$

$$t = \frac{1}{2} \text{ hour}$$

(27)



x = dist. player runs ft

$\frac{dx}{dt} = -25$ ft/sec

find $\left.\frac{dy}{dt}\right|_{x=20} = ?$

$90^2 + x^2 = y^2$

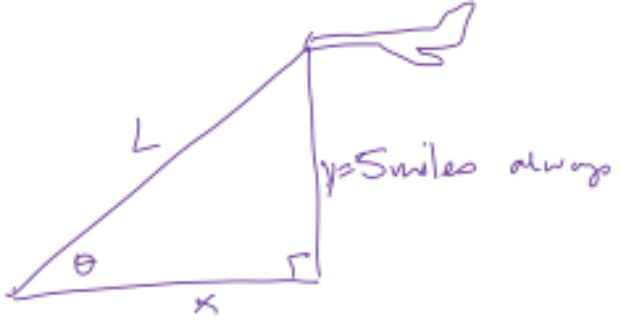
$0 + 2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

$(20)(25) = \sqrt{90^2+20^2}\,\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{(20)(25)}{10\sqrt{85}} = \frac{-50}{\sqrt{85}}$

≈ -5.423 ft/sec

(42) p160



Given $\frac{dx}{dt} = -600$ mph

find

(a) $\left.\frac{d\theta}{dt}\right|_{\theta=30} = ?$

$\tan 30 = \frac{1}{\sqrt{3}} = \frac{5}{x}$

$x = 5\sqrt{3}$

$\tan\theta = \frac{5}{x} = 5x^{-1}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{-5}{x^2} \cdot \frac{dx}{dt}$

$\frac{d\theta}{dt} = \cos^2\theta \left(\frac{-5}{x^2}\right)(-600)$

$\frac{d\theta}{dt} = \cos^2\theta \left(\frac{-5}{x^2}\right)(-600)$

$\left.\frac{d\theta}{dt}\right|_{\theta=30} = \left(\frac{\sqrt{3}}{2}\right)^2 \left(\frac{3000}{(5\sqrt{3})^2}\right)$

$\left.\frac{d\theta}{dt}\right|_{\theta=30} = \frac{3}{4} \cdot \frac{3000}{75} = 30$ rad/hr

or $\frac{1}{2}$ rad/min

(b) $\left.\frac{d\theta}{dt}\right|_{\theta=60}$

$\tan 60 = \sqrt{3} = \frac{5}{x}$

$x = \frac{5}{\sqrt{3}}$ or $\frac{5\sqrt{3}}{3}$

$\frac{d\theta}{dt} = (\cos 60^\circ)^2 \left(\frac{3000}{x^2}\right)$

$\left(\frac{1}{2}\right)^2 \left(\frac{3000}{\frac{25}{3}}\right)$

$= \frac{1}{4}\left(\frac{9000}{25}\right)$

$= 90$ rad

$= 60$ rad/hr

or $\frac{3}{2}$ rad/min

(c) $\left.\frac{d\theta}{dt}\right|_{\theta=75^\circ}$

$\tan 75 = \frac{5}{x}$

$x = \frac{5}{\tan 75}$

$\frac{d\theta}{dt} = (\cos^2 75^\circ)\left(\frac{3000}{x^2}\right)$

$= \cos^2 75^\circ \left(\frac{3000}{\frac{25}{\tan^2 75}}\right)$

$= 120 \cos^2 75^\circ \tan^2 75$

$= 120 \sin^2 75$

$= 111.9615$ rad/hr

$= 1.866$ rad/min

Clever Alt. Way

$\cos^2\theta = \frac{x^2}{L^2}$

$\sin\theta = \frac{5}{L}$

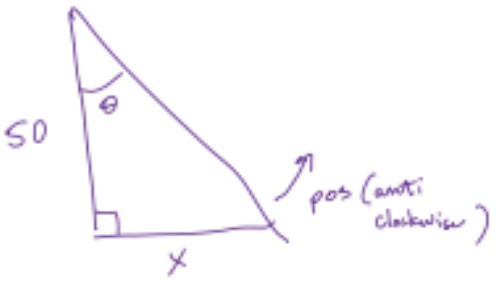
$L = \frac{5}{\sin\theta}$

$\frac{1}{L^2} = \frac{\sin^2\theta}{25}$

$\frac{d\theta}{dt} = \left(\frac{\sin^2\theta}{25}\right)\left(\frac{-5}{x^2}\right)(-600)$

$= 120 \sin^2\theta$

(43) (a) $\frac{200\pi}{3}$ ft/sec (b) 200π ft/sec (c) $\approx 427.43\pi$ ft/sec



given 30 revs / min

$= 30(2\pi)$ deg./min

$= 60\pi$ deg/min or π rad/sec

find

(a) $\left.\frac{dx}{dt}\right|_{\theta = 30° = \frac{\pi}{6}}$

$\tan\theta = \frac{x}{50}$

$x = 50\tan\theta$

$\frac{dx}{dt} = 50\sec^2\theta \frac{d\theta}{dt}$

$\left.\frac{dx}{dt}\right|_{\theta=\frac{\pi}{6}} = 50\left(\frac{2}{\sqrt{3}}\right)^2(\pi)$

$= \frac{200\pi}{3}$ ft/sec

(b) $\left.\frac{dx}{dt}\right|_{\theta=\frac{\pi}{3}} = 50\left(\frac{2}{1}\right)^2\pi$

$= 200\pi$ ft/sec

(c) $\left.\frac{dx}{dt}\right|_{\theta=70} = \frac{50\pi}{(\cos 70)^2}$

$= 427.\,Y3160855\pi$ rad/sec

≈ 1342.816 rad/sec