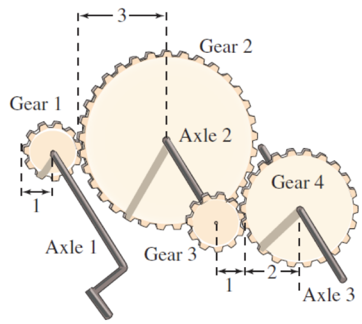
2.4 Notes and Examples

Name: Block: Seat:

The Chain Rule and the General Power Rule



Axle 1: y revolutions per minute
Axle 2: u revolutions per minute
Axle 3: x revolutions per minute

Because the first axle must make three revolutions to turn the second axle once, $\frac{dy}{du} =$______________

Because the second axle must make two revolutions to turn the third axle once, $\frac{du}{dx} =$______________

What is the rate of change of the first axle with respect to the third axile?

Derivative of the Composition of Functions

1. The Chain Rule : $\frac{dy}{dx} =$

   or $\frac{dy}{dt} =$

   or $\frac{dy}{dx} =$

   or $\frac{d}{dx}[f(g(x))] =$

1. Find the derivative of $f(x) = \sin 2x$

2. Find the derivative of $f(x) = \sqrt{3x^2 - x + 1}$

3. The order of the composition matters. Let's practice finding the inner and out functions:

Function $h(x)$	Outer function $f(u)$	Inner function $g(x)$	$h'(x) = f'(g(x)) \cdot g'(x)$ or $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
$\cos(x^2 + 1)$			
$\ln(\sec x)$			
e^{-x^2}			
$\sin(e^x)$			
$\cos^2(x)$			

4. (a) $\frac{d}{dx}[\sqrt{3x^2 + 4}] =$

(b) $\frac{d}{dx}[\sin^2 x] =$

(c) $\frac{d}{dx}[\sin x^2] =$

5. Composition of 3 functions

(a) $\frac{d}{dx}[\sin e^{x^2}] =$

The General Power Rule
...The power rule with the chain rule added....

If $y = [u(x)]^n$ then

$$\frac{dy}{dx} =$$

or, equivalently,

$$\frac{d}{dx}\left[u^n\right] =$$

6. Find the derivative of $f(x) = (3x - 2x^2)^3$

7. Find the derivative of $f(x) = \sqrt[3]{(x^2 - 1)^2}$

8. Find the derivative of $g(t) = \dfrac{-7}{(2t - 3)^2}$

9. Find the derivative of $f(x) = \dfrac{1}{x^2+1}$

10. Trig functions with the chain rule added:
If u is a function of x...
   (a) $\dfrac{d}{dx}[\sin u] =$
   (b) $\dfrac{d}{dx}[\cos u] =$
   (c) $\dfrac{d}{dx}[\tan u] =$
   (d) $\dfrac{d}{dx}[\sec u] =$
   (e) $\dfrac{d}{dx}[\cot u] =$
   (f) $\dfrac{d}{dx}[\csc u] =$

11. Find any equation of the tangent line to the graph of $f(x) = 2\sin x + \cos(2x)$ at the point $(\pi, 1)$. Then determine all the values of x in the interval $(0, 2\pi)$ at which the graph of f has a horizontal tangent.