Product and Quotient Rules and Higher Order Derivatives

## Slightly Less Basic Derivative Rules

1. The Product Rule: $\frac{d}{d x}[f(x) \cdot g(x)]=$
2. The Quotient Rule: $\frac{d}{d x}\left[\frac{f(x)}{g(x)}\right]=$
3. The Chain Rule : $\frac{d}{d x}[f(g(x))]=$

More on this next time....

1. Now use your words....
(a) $\frac{d}{d x}[f(x) \cdot g(x)]=$ The $\qquad$ of the $1^{\text {st }}$ function $\qquad$ the $2^{\text {nd }}$ $\qquad$
the derivative of the $\qquad$ times the $\qquad$
(b) $\frac{d}{d x}\left[\frac{f(x)}{g(x)}\right]=$ The $\qquad$ of the $1^{\text {st }}$ function $\qquad$ the $2^{\text {nd }}$ $\qquad$ the derivative of the $\qquad$ times the $\qquad$ ALL DIVIDED BY $\qquad$ SQUARED.
2. If $h(x)=\left(3 x-2 x^{2}\right)(5+4 x)$, find $h^{\prime}(x)$

|  | $f$ | $g$ |
| :---: | :---: | :---: |
| Function |  |  |
| Derivative |  |  |

$$
h^{\prime}(x)=
$$

3. If $y=\frac{5 x-2}{x^{2}+1}$, find $y^{\prime}$

|  | $f$ | $g$ |
| :---: | :---: | :---: |
| Function |  |  |
| Derivative |  |  |

$y^{\prime}=$
4. If $y=3 x^{2} \sin x$, find $y^{\prime}$
5. If $y=2 x \cos x-2 \sin x$, find $y^{\prime}$
6. $\frac{d}{d x}\left[\frac{3-\frac{1}{x}}{x+5}\right]=$
7. Given $g(2)=3, g^{\prime}(2)=-4, h(2)=-1, h^{\prime}(x)=5$, answer the following:
(a) If $f(x)=g(x) h(x)$, find $f^{\prime}(2)$
(b) If $q(x)=\frac{g(x)}{h(x)}$, find $q^{\prime}(2)$
8. $f(x)=\frac{x^{2}}{x^{2}+1}$. Find the point(s) at which the graph of $f$ has a horizontal tangent line.
9. Now we can do the rest of the trig functions!
(a) $\frac{d}{d x}(\tan x)=\frac{d}{d x}\left[\frac{\sin x}{\cos x}\right]=$
(b) $\frac{d}{d x}(\sec x)=\frac{d}{d x}\left[\frac{1}{\cos x}\right]=$
10. "If all you have is a hammer, then everything looks like a nail." These all these are easier WITHOUT using the quotient rule:
(a) $\frac{d}{d x}\left[\frac{x^{2}+3 x}{6}\right]=$
(b) $\frac{d}{d x}\left[\frac{5 x^{4}}{8}\right]=$
(c) $\frac{d}{d x}\left[\frac{-3\left(3 x-2 x^{2}\right)}{7 x}\right]=$
(d) $\frac{d}{d x}\left[\frac{9}{5 x^{2}}\right]=$

Higher Order Derivatives Recall from last time:
(a) Position function:
(b) Velocity function:
(c) Acceleration function:

Taking a derivative of a derivative can just keep going and going...

## Notation of Higher Order derivatives:

If $f$ is a $n$-times differentiable function, we write:

1. First derivative:
2. Second derivative:
3. Third derivative:
4. Fourth derivative:
5. $n^{\text {th }}$ derivative:
6. Because the moon has no atmosphere, a falling object on the moon encounters no air resistance. In 1971, Apollo 15 astronaut David Scott demonstrated that a feather and a hammer fall at the same rate on the moon (https://youtu.be/oYEgdZ3iEKA). The position function for each of these falling objects is given by: $s(t)=-0.8128 t^{2}+2$, where $s(t)$ is measured in meters, and $t$ is the time in seconds. What is the acceleration due to gravity on the moon? How many times greater is the Earth's acceleration due to gravity?
