## Finding Limits Graphically and Numerically

1. Consider the function $\frac{x^{2}-3 x+2}{x-1}$. To sketch the graph, we need to know what is going on at $x=1$.
(a) Using a Table: Using your TI: Press [TBLSET] (aka 2nd [WINDOW]) to make

TABLE SETUP
TblStart=0 $\Delta T b l=1$
Indpnt: Ruto Rsk
Depend: Ruto Ask and fill in the following

(b) Now type the function in your TI as $Y_{1}$, Use [ZOOM] 4. and [TRACE] to see what is happening around $x=1$.
(c) Write the Limit expression:
(d) We read this as "The $\qquad$ of $f(x)$ as $x$ approaches $\qquad$ is $\qquad$ "
2. Estimating a Limit Numerically:

$$
\lim _{x \rightarrow 0} \frac{x}{\sqrt{x+1}-1}
$$

| $x$ | -0.1 | -0.01 | -0.001 | -0.0001 | 0 | 0.0001 | 0.001 | 0.01 | 0.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ |  |  |  |  |  |  |  |  |  |

3. Estimating a Limit Numerically:

$$
\lim _{x \rightarrow 3} \frac{\left(x^{2}-9\right)(x+1)}{x-3}
$$

| $x$ | 2.9 | 2.99 | 2.999 | 2.9999 | 3 | 3.0001 | 3.001 | 3.01 | 3.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ |  |  |  |  |  |  |  |  |  |

In general, even if $f(c) \neq L$, if $f(x)$ becomes arbitrarily close to a single number $L$ as $x$ approaches $c$ from either side, we say that the limit of $f(x)$ as $x$ approaches $c$ is $L$. This limit is written:
4. One sided limits: Let $g(x)= \begin{cases}x^{2} & \text { for } x<1 \\ x+2 & \text { for } x>1\end{cases}$
(a) $g(0)=$
(b) $g(4)=$
(c) $g(1)=$
(d) $\lim _{x \rightarrow 1^{-}} g(x)=$
(e) $\lim _{x \rightarrow 1^{+}} g(x)=$
(f) $\lim _{x \rightarrow 1} g(x)=$
5. Finding a limit Graphically
(a) $\lim _{x \rightarrow-3} f(x)=$
(b) $\lim _{x \rightarrow-7^{-}} f(x)=$
(c) $\lim _{x \rightarrow-7^{+}} f(x)=$
(d) $\lim _{x \rightarrow-7} f(x)=$
(e) $f(-6)=$
(f) $\lim _{x \rightarrow-6} f(x)=$
(g) $\lim _{x \rightarrow 4} f(x)=$
(h) $\lim _{x \rightarrow 5} f(x)=$
(i) For what values of $a$ is $\lim _{x \rightarrow a} f(x)=1$ ?
6. Examples of 3 types of Limits that Fail to exist
(a) $\lim _{x \rightarrow 0} \sin \left(\frac{1}{x}\right)$ why?
(b) $\lim _{x \rightarrow 2}\left(\frac{1}{|x-2|}\right)$ why?
(c) If $f(x)=\left\{\begin{array}{cc}-2, & \text { if } x \leq 3 \\ 3, & \text { if } x>3\end{array}\right.$, the $\lim _{x \rightarrow 3} f(x)=D . N . E$. why?

