

Name _____

Date _____

Polar & Parametric Equations**Polar Coordinates - Examples**

Identify which point is graphed at the given polar coordinate: (radius, angle).

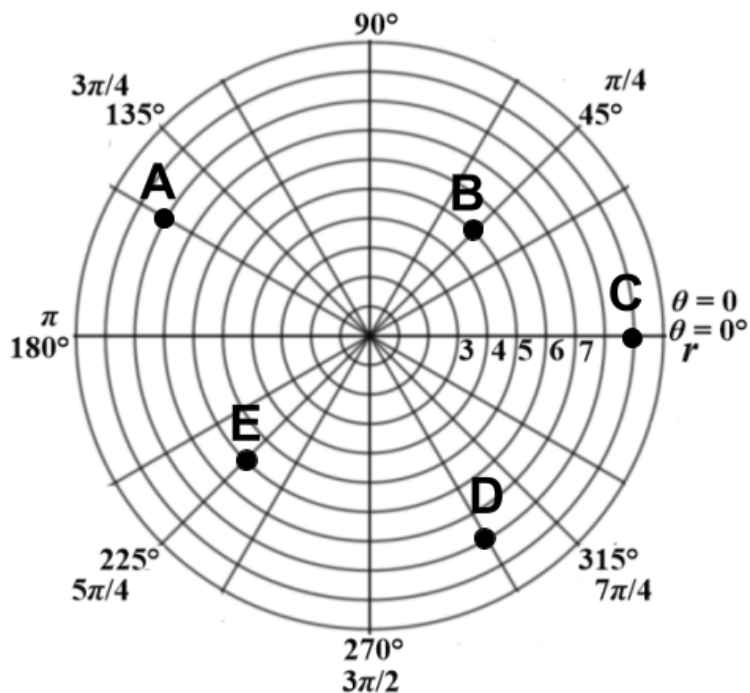
a. $(9, 0): C$

b. $\left(8, \frac{5\pi}{6}\right): A$

c. $\left(8, \frac{5\pi}{3}\right): D$

d. $\left(-6, \frac{\pi}{4}\right): E$

e. $\left(5, \frac{\pi}{4}\right): B$

Rewrite the given polar coordinate using either by adding n circles, or a negative radius with the angle in the opposite direction.

f. $\left(1, \frac{2\pi}{3}\right)$ could be $\left(1, \frac{8\pi}{3}\right)$ or $\left(-1, -\frac{\pi}{3}\right)$.

g. $\left(4, \frac{7\pi}{6}\right)$ could be $\left(4, \frac{31\pi}{6}\right)$ or $\left(-4, \frac{\pi}{6}\right)$

Polar Coordinates - Exercises

Identify which point is graphed at the given polar coordinate.

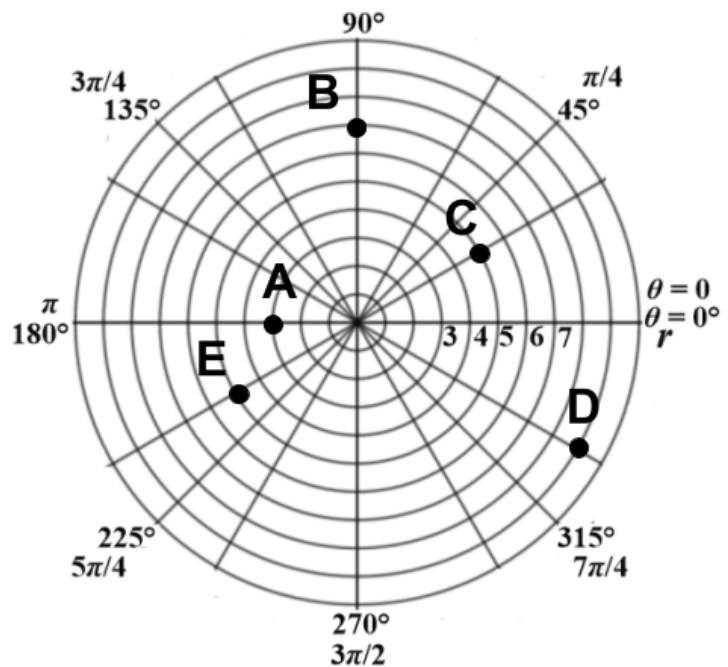
1. $\left(-5, \frac{\pi}{6}\right)$

2. $\left(5, \frac{\pi}{6}\right)$

3. $\left(9, \frac{11\pi}{6}\right)$

4. $(-3, 0)$

5. $\left(7, -\frac{3\pi}{2}\right)$



Rewrite the given polar coordinate using either by adding n circles, or a negative radius with the angle in the opposite direction

6. $\left(-2, \frac{3\pi}{4}\right)$.

7. $\left(4, \frac{5\pi}{6}\right)$.

8. $\left(6, \frac{7\pi}{4}\right)$ if $n = -2$.

9. $\left(-3, \frac{11\pi}{6}\right)$ if $n = 3$.

Converting Between Rectangular and Polar Coordinates - Examples

Convert each polar coordinate to rectangular form.

a. $\left(4, \frac{2\pi}{3}\right) = \left(4 \cos \frac{2\pi}{3}, 4 \sin \frac{2\pi}{3}\right) = (-2, 2\sqrt{3})$

b. $(2, \pi) = (2 \cos \pi, 2 \sin \pi) = (-2, 0)$

c. $(-2, 30^\circ) = (-2 \cos 30^\circ, -2 \sin 30^\circ) = (-\sqrt{3}, -1)$

Convert each rectangular coordinate to polar form.

d. $(2\sqrt{3}, 2) = (\sqrt{12+4}, \arctan \frac{2}{2\sqrt{3}}) = (4, \frac{\pi}{6})$

e. $(-2\sqrt{3}, -2) = (4, \frac{7\pi}{6})$

f. $(-2, 2) = (2\sqrt{2}, \frac{3\pi}{4})$

Convert each rectangular equation to polar form.

g. $y = 4$. Substitute then solve for r: $r \sin(\theta) = 4$ becomes $r = \frac{4}{\sin \theta}$

h. $y = 2x^2$ Substitute: $r \sin \theta = 2r^2 \cos^2 \theta$ Solve for r: $r = \frac{\sin \theta}{\cos^2 \theta} = \tan \theta \sec \theta$

i. $x^2 = 36 - y^2$ Answer: because $x^2 + y^2 = r^2$, we have $r = 6$

Convert each polar equation to rectangular form.

j. $6r \cos \theta = 12$ Answer: because $x = r \cos \theta$ this is the vertical line $x = 2$

k. $r = 2 + 3 \cos \theta$ Answer: multiply both sides by r, substitute then solve for r: $x^2 + y^2 \pm \sqrt{x^2 + y^2} - 3x = 0$

l. $r = \frac{1}{2 + \sin \theta}$ is the same as $4x^2 + 3y^2 + 2y - 1 = 0$

Converting Between Rectangular and Polar Coordinates - Homework

Convert each polar coordinate to rectangular form.

10. $\left(2, \frac{7\pi}{6}\right)$

11. $\left(1, \frac{5\pi}{6}\right)$

12. $\left(-2, \frac{\pi}{6}\right)$

13. $\left(4, \frac{7\pi}{4}\right)$

14. $\left(3, \frac{3\pi}{2}\right)$

Convert each rectangular coordinate to polar form.

15. $(4, 4)$

16. $(-4, 4)$

17. $(-4, -4)$

18. $(4, -4)$

19. $(4, 4\sqrt{3})$

Convert each rectangular equation to polar form.

20. $x = 2$

21. $x = 3y^2$

22. $y^2 = 3x$

23. $4x^2 = 100 - 4y^2$

Convert each polar equation to rectangular form.

24. $8r\sin\theta = 24$

25. $r = 8\cos\theta$

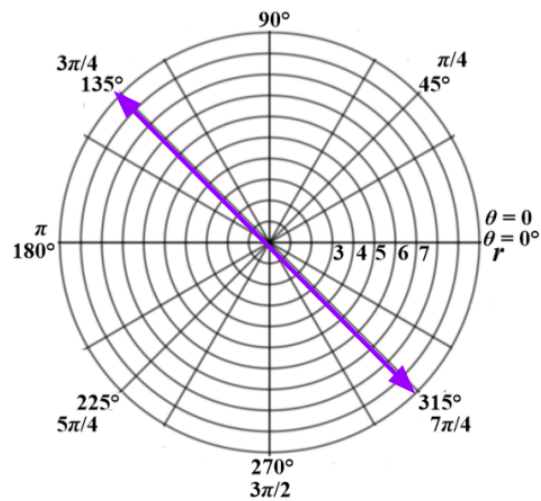
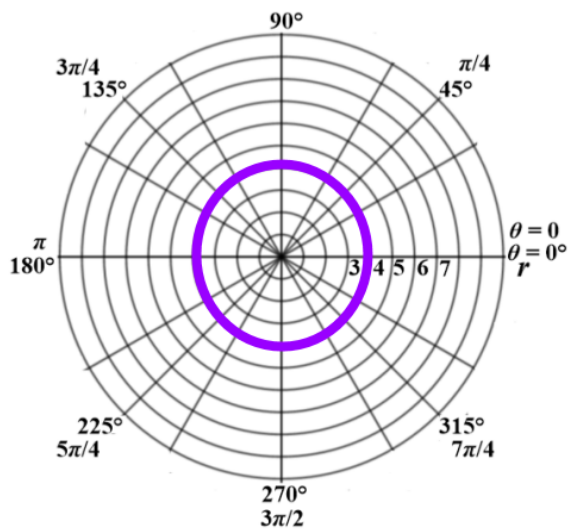
26. $r = 2 + 3\sin\theta$

27. $r = \frac{1}{1+\cos\theta}$

Polar Equations and Graphs - Examples

Write a polar equation for the given graph.

a. $r = 4$



b. $\theta = \frac{3\pi}{4}$

Find the graph of the given equation.

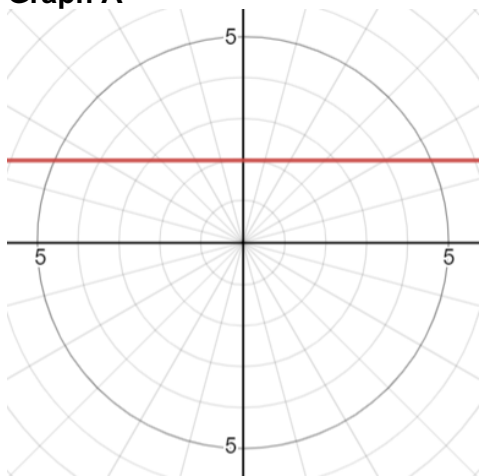
c. $r = 4\cos\theta$ is Graph C

d. $r = 2\csc\theta$ is Graph A

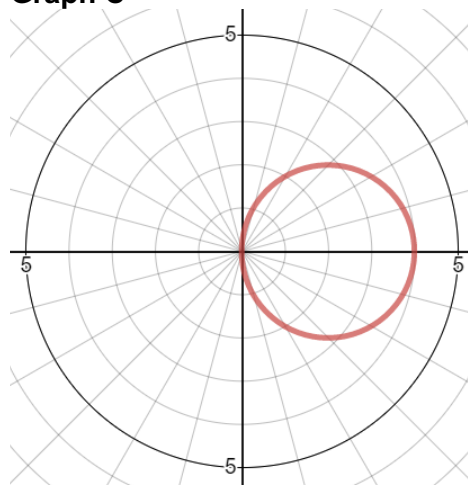
e. $r\cos\theta = 4$ is Graph D

f. $\frac{1}{4}r = \sin\theta$ is graph is B

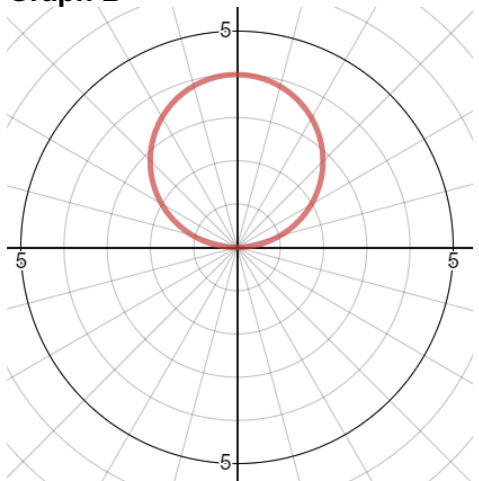
Graph A



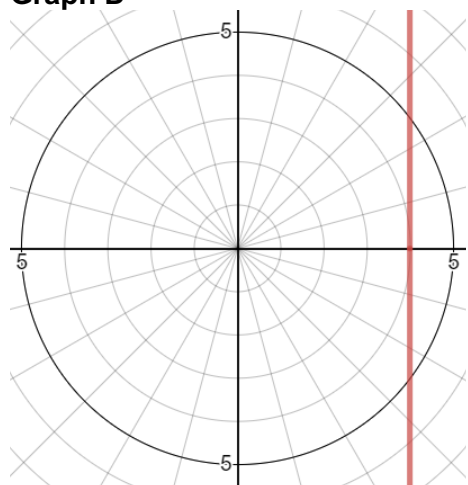
Graph C



Graph B



Graph D



Use your calculator to answer the following questions:

- g. What is the maximum radius of the 'petals' in the graph of $r = 3\cos(6\theta)$? Answer: 3
- h. How many 'petals' does the graph of $r = 3\cos(6\theta)$ have? Answer: 12
- i. What is the maximum radius of the 'petals' in the graph of $r = 4\sin(5\theta)$? Answer: 4
- j. How many 'petals' does the graph of $r = 4\sin(5\theta)$ have? Answer: 5

Choose the graph/equation that matches the described function.

k. Lemniscates Answer: Graph B

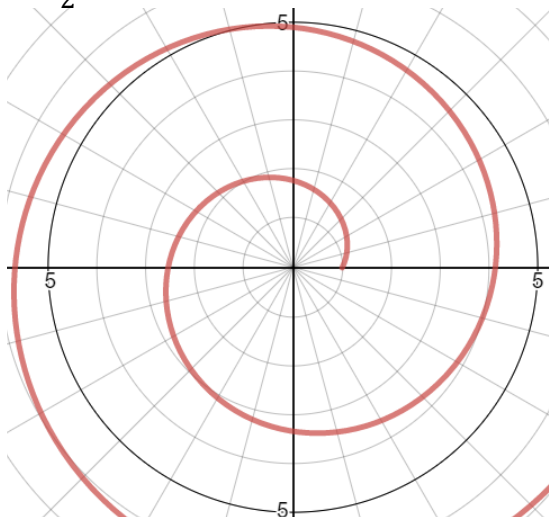
l. Rose Curve Answer: Graph D

m. Archimedean Spiral Answer: A

n. Logarithmic Spiral Answer C

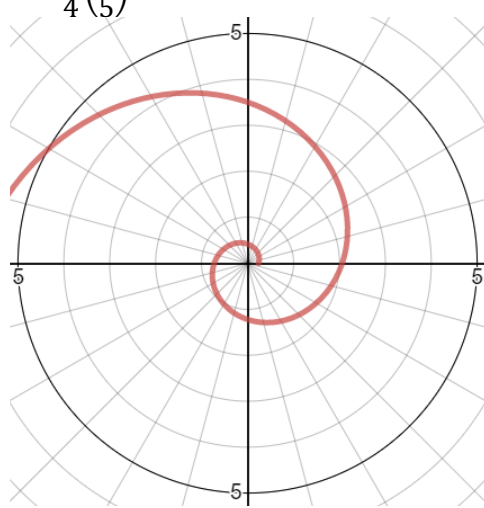
Graph A

$$r = \frac{1}{2}\theta + 1$$



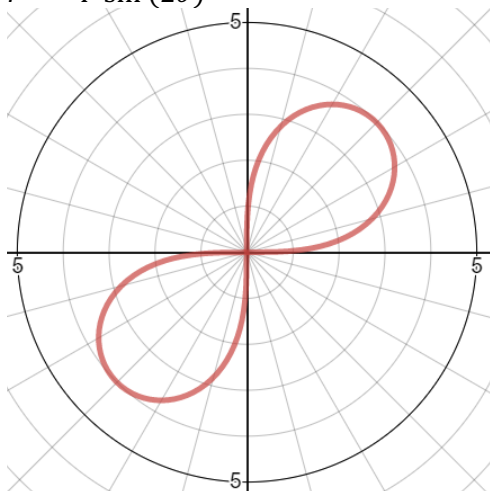
Graph C

$$r = \frac{1}{4}\left(\frac{7}{5}\right)^\theta$$



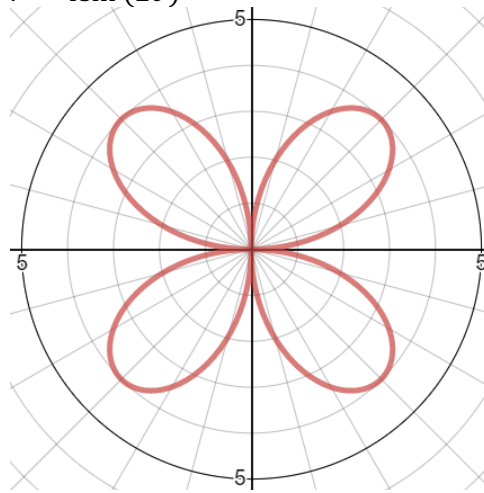
Graph B

$$r^2 = 4^2 \sin(2\theta)$$



Graph D

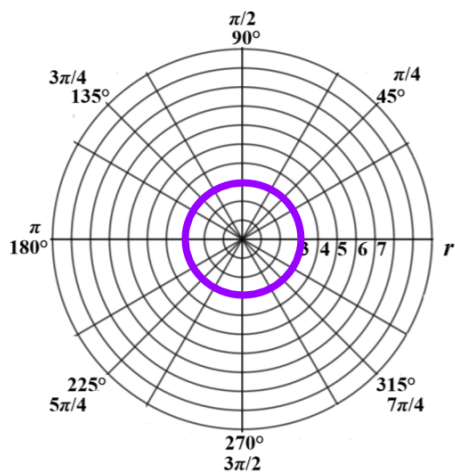
$$r = 4 \sin(2\theta)$$



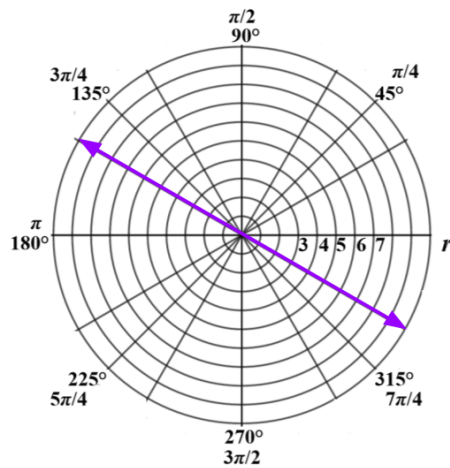
Polar Equations and Graphs - Homework

Write a polar equation for the given graph.

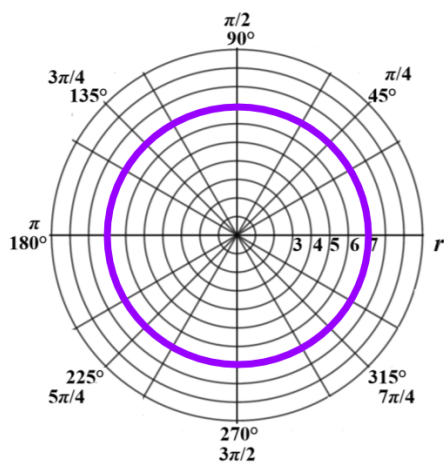
28.



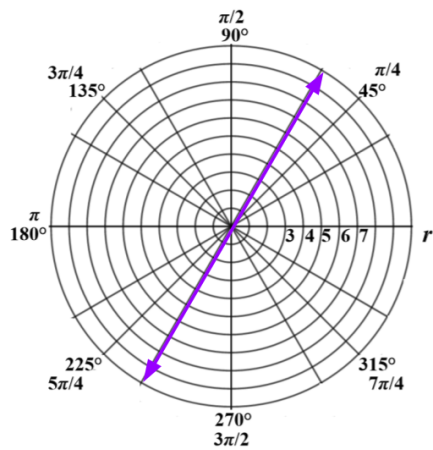
29.



30.



31.



Find the graph of the given equation.

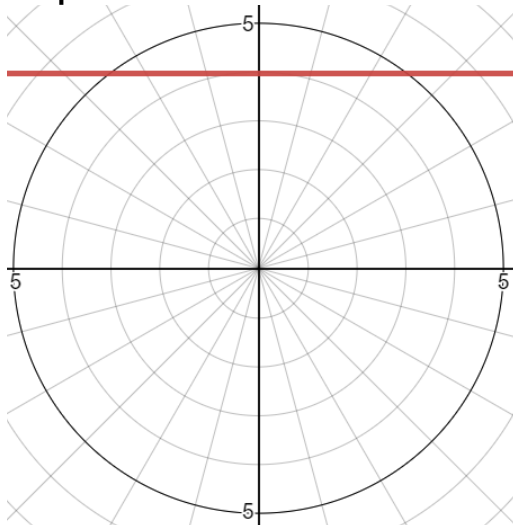
32. $r = 5\csc\theta$

33. $r = 3\sec\theta$

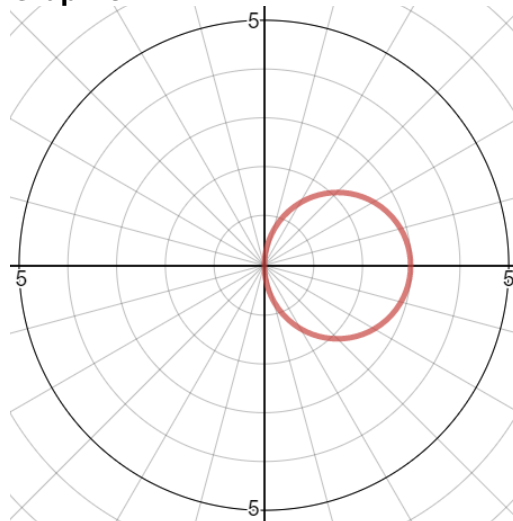
34. $r\sin\theta = 2$

35. $\frac{1}{3}r = \cos\theta$

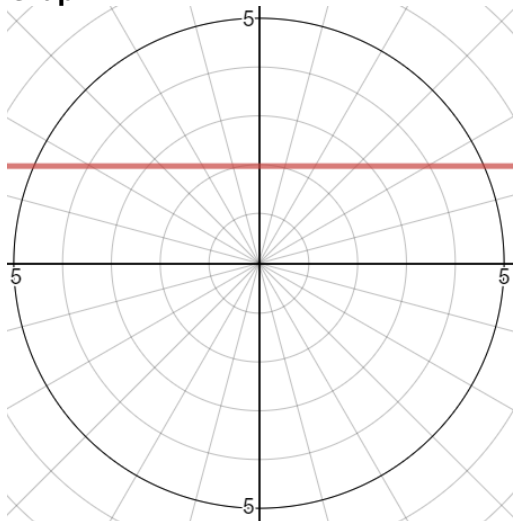
Graph A



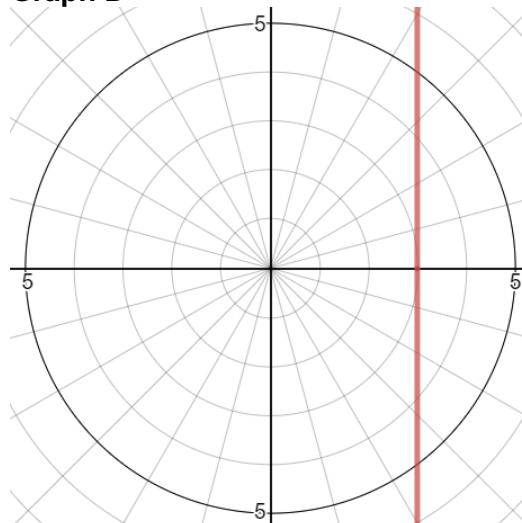
Graph C



Graph B



Graph D



Use a calculator to answer the following questions:

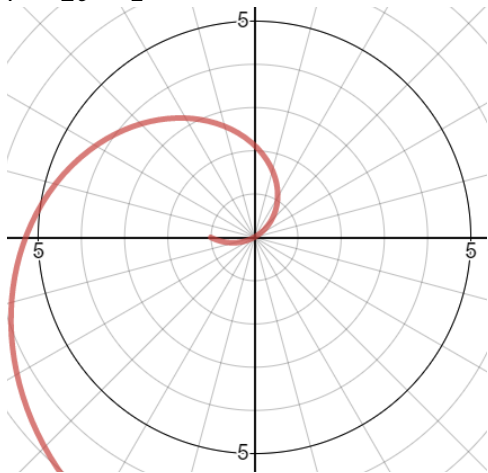
36. What is the maximum radius of the 'petals' in the graph of $r = 7\cos(4\theta)$?
37. How many 'petals' does the graph of $r = 7\cos(4\theta)$ have?
38. What is the maximum radius of the 'petals' in the graph of $r = 2\sin(11\theta)$?
39. How many 'petals' does the graph of $r = 2\sin(11\theta)$ have?

Choose the graph/equation that matches the described function.

40. Lemniscates
41. Rose Curve
42. Archimedean Spiral
43. Logarithmic Spiral

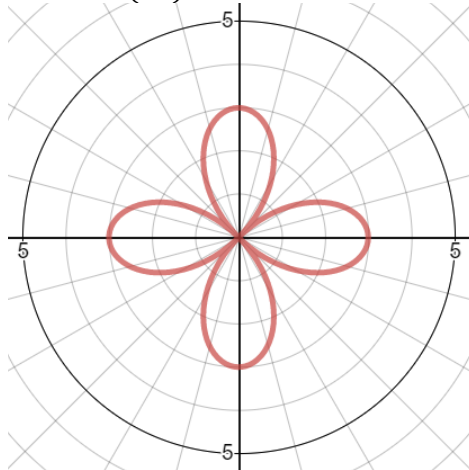
Graph A

$$r = 2\theta - 1$$



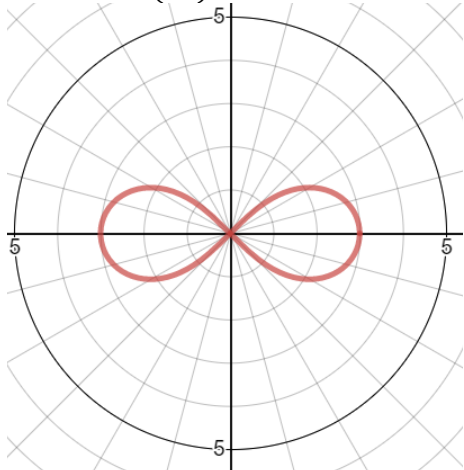
Graph C

$$r = 3\cos(2\theta)$$



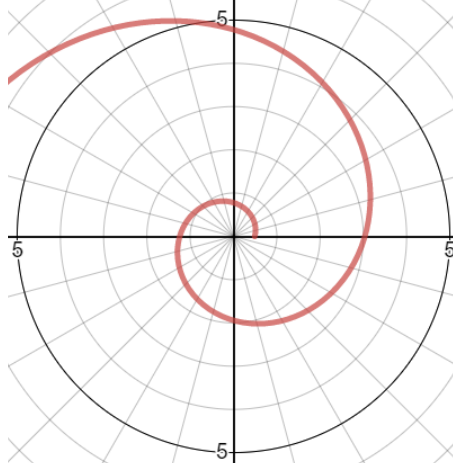
Graph B

$$r^2 = 3^2\cos(2\theta)$$



Graph D

$$r = \frac{1}{2}\left(\frac{1}{3}\right)^\theta$$



Parametric Equations and Curves - Examples

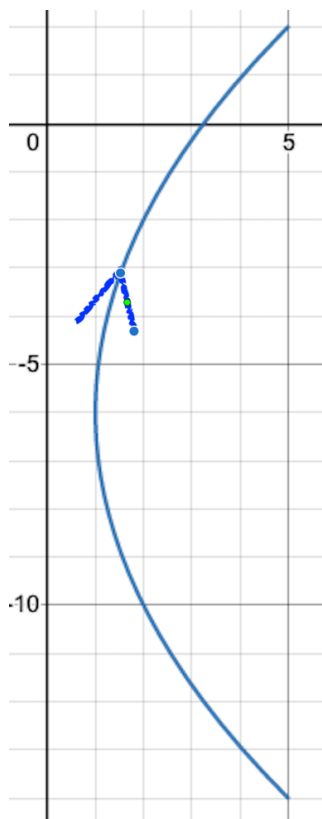
Create a table with the rectangular coordinates for the given parametric equations and then sketch the directed graph.

a.

$$x = t^2 + 1$$

$$y = 4t - 6$$

t	x	y
-2	5	-14
-1	2	-10
0	1	-6
1	2	-2
2	5	2



Rewrite the given parametric equations in rectangular form and provide the domain/range.

b.

$$x = \sqrt{t} + 2$$

$$y = t^2 - 3$$

Solve for t with one equation, then substitute in the other:

$$t = (x - 2)^2$$

$$y = (x - 2)^4 - 3$$

So the Domain is all reals greater than 2,

The Range is all reals greater than -3.

Create a pair of parametric equations for the given information.

c. A line with a slope of $\frac{1}{2}$ and passes through the point (4, 8). Answer: $x = 4 + t$, and $y = 8 + \frac{t}{2}$

d. Rectangular equation is $y = \frac{x^2}{9} + \frac{y^2}{16} = 1$. Answer: $x = 3 \cos t$ and $y = 4 \sin t$

e. Rectangular equation is $(x - 1)^2 + (y + 2)^2 = 36$. Answer: $x = 1 + \cos t$, and $y = -2 + 6 \sin t$

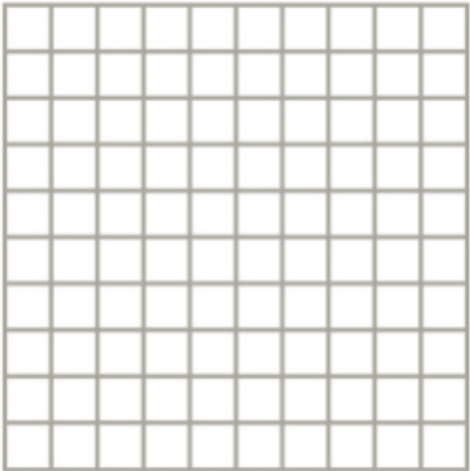
g. Rectangular equation is $y = x^2 + 3$. Answer: $x = t$, and $y = 3 + t^2$

Parametric Equations and Curves - Homework

Create a table with the rectangular coordinates for the given parametric equations and then sketch the graph.

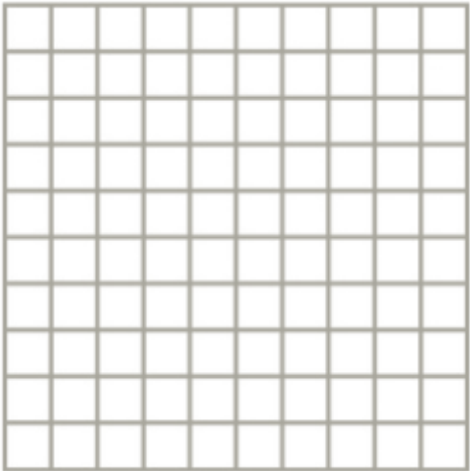
44.
 $x = 3 + t^2$
 $y = 2t + 5$

t	x	y
-2		
-1		
0		
1		
2		



45.
 $x = -2t + 5$
 $y = \frac{t^3}{2}$

t	x	y
-2		
-1		
0		
1		
2		



Rewrite the given parametric equations in rectangular form and provide the domain/range.

46.

$$x = 2\sqrt{t}$$

$$y = 2t - 2$$

47.

$$x = \frac{1}{3}t - 6$$

$$y = t^3 + 6$$

48.

$$x = \frac{1}{4}t^2$$

$$y = 4t^2$$

Create a pair of parametric equations for the given information.

49. A line with a slope of $\frac{1}{4}$ and passes through the point (6, 1).

50. A line with a slope of -1 and passes through the point (-4, 6).

51. Rectangular equation is $y = \frac{(x+2)^2}{25} + \frac{(y-2)^2}{9} = 1$.

52. Rectangular equation is $x^2 - 4x + y^2 + 6y = 23$.

53. Rectangular equation is $y = 3x^2 + 1$.