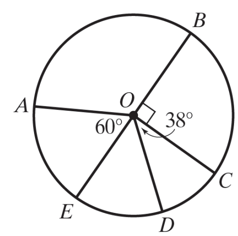
Mini-Lecture 12.1
Circle Review and Tangent Lines

Learning Objectives:

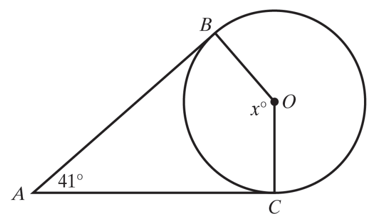
1. Review circles and arcs.
2. Use properties of a tangent line to a circle.
3. Key vocabulary: *tangent to a circle, point of tangency, tangent ray, tangent segment, common tangent, line of centers, tangent circles*

Key Examples:

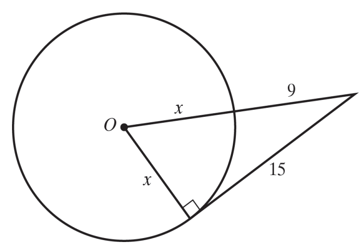
1. The radius of $\odot O$ is 12 cm.
   a) Find $m\widehat{AC}$.
   b) Find $m\widehat{CDB}$.
   c) Find the length of $\widehat{ABC}$.



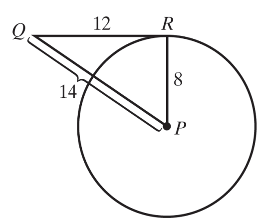
2. $\overline{AB}$ and $\overline{AC}$ are tangent to $\odot O$.
   What is the value of x?



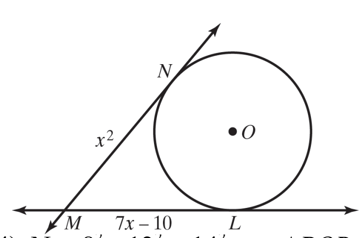
3. What is the radius of $\odot O$?



4. Is $\overline{QR}$ tangent to $\odot P$? Explain.



5. If $\overleftrightarrow{MN}$ and $\overleftrightarrow{ML}$ are tangents to $\odot O$, find the value of x.



Answers: 1a) 150° 1b) 270° 1c) 14π cm 2) $x = 139$ 3) 8 4) No; $8^2 + 12^2 \neq 14^2$, so ΔPQR is not a right triangle and $\overline{PR}$ is not perpendicular to $\overline{QR}$. 5) $x = 2$ or $x = 5$ 6) 18 cm

Mini-Lesson 12.1

Circle Review and Tangent Lines

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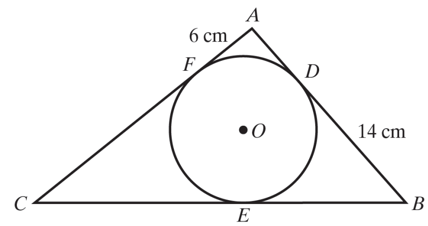
Video Organizer
Section 12.1 (MML)

PowerPoints, Section 12.1

6. $\odot O$ is inscribed in ΔABC, which has a perimeter of 76 cm. What is the length of $\overline{CE}$?



Teaching Notes:

- This section contains a lot of material, but much or all of it should be review. Begin with the vocabulary by asking students to list all the circle terms they know and to illustrate them on a circle.
- Students may find it confusing that the textbook defines a tangent as a *line,* but then illustrates and describes a tangent ray and a tangent segment. (See p.537.) Explain that a tangent is in fact a line, but that when only part of the line is relevant to a problem, we may only show a ray or a segment in the diagram.

ERROR PREVENTION

- Students may make errors in writing major arcs or finding their degree measures. Emphasize that, for any two points on a circle, there are both a minor arc and major arc with the same endpoints, so we need to be able to tell them apart: When we write just two letters (the endpoints), we mean the minor arc, so if we want to write the major arc, we must include a third point somewhere along the arc.

Closure Questions:

- How many common tangents can two tangent circles have? How many of these are internal tangents and how many are external tangents?

  There are two possibilities: Two externally tangent circles have two common external tangents and two common internal tangents. Two internally tangent circles have one common external tangent, but no common internal tangents.

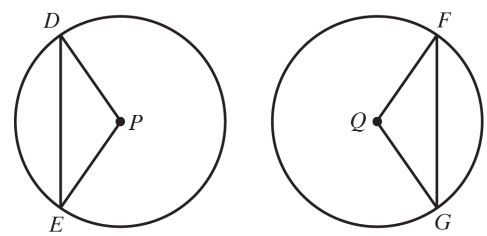
Mini-Lesson 12.2
Chords and Arcs

Learning Objectives:

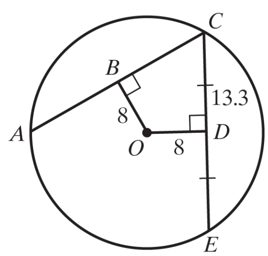
1. Use congruent chords, arcs, and central angles.
2. Use perpendicular bisectors to chords.
3. Key vocabulary: *chord*

Key Examples:

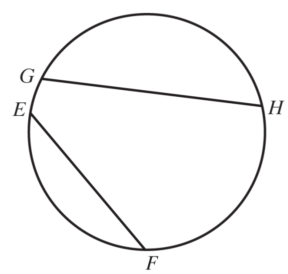
1. In the diagram, $\odot P \cong \odot Q$. Given that $\widehat{DE} \cong \widehat{FG}$, what can you conclude?



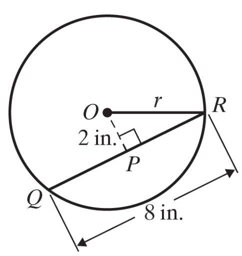
2. What is the length of $\overline{AC}$? Justify your answer.



3. Find the center of the circle with chords $\overline{EF}$ and $\overline{GH}$.



4. Find the value of r to the nearest tenth.



Answers: 1) $\angle DPE \cong \angle FQG$ because, within congruent circles, congruent arcs have congruent central angles (Converse of Theorem 12.2.-1 (Theorem 12.2-2)). $\overline{DE} \cong \overline{FG}$ because, within congruent circles, congruent arcs have congruent chords (Converse of Theorem 12.2-5 (Theorem 12.2-6)). 2) 26.6; Within a circle, chords equidistant from the center are congruent (Theorem 12.2-7). 3) *See Additional Answers at end of Mini-Lessons.* 4) $r \approx 4.5$ in.

Mini-Lesson 12.2

Chords and Arcs

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Interactive Lecture Video Objective 1

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Video Organizer Section 12.2 (MML)

PowerPoints, Section 12.2

<u>Teaching Notes:</u>

- This section contains 11 theorems, which can be an overwhelming amount of new material for students to absorb. Remind students that if a conditional statement and its converse are both true, they can combined into a single biconditional ("if and only if"). Rewriting the pairs of theorems that are converses in this way provides a way to cut down on the number of separate theorems to remember.
- The Helpful Hint on p.545 of the textbook is a great way to summarize and remember the content of the six theorems presented on that page.
- For Example 3, you may need to review the compass-and-straight edge construction of the perpendicular bisector of a segment from Section 1.8.

ERROR PREVENTION

- Some students may be confused by Theorems 12.2-7 and 12.2-8 about chords equidistant from the center of a circle because there are think that the distance would depend on which point you choose on the chord. Remind them that the distance from a point to a line (or segment) is *defined* as the perpendicular distance, which is length of the shortest segment that connects a point to a line or segment.

<u>Closure Questions:</u>

- How can you locate the center of any circle?

  Draw any two chords that are not parallel. Use a compass and straight edge to construct their perpendicular bisectors. The intersection point of the two chords is the center.

Mini-Lesson 12.3

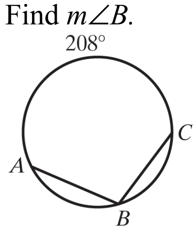
Inscribed Angles

Learning Objectives:

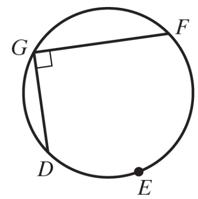
1. Find the measure of an inscribed angle.
2. Find the measure of an angle formed by a tangent and a chord.
3. Key vocabulary: *inscribed angle, intercepted arc*

Key Examples:

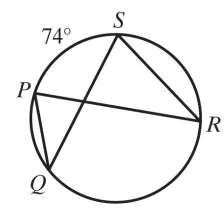
1. a) Find $m\angle B$.



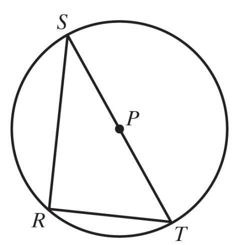
b) Find $m\widehat{DEF}$.



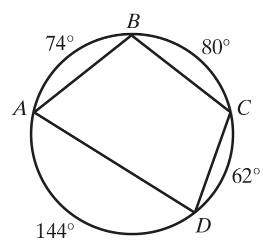
c) Find $m\angle Q$ and $m\angle R$.



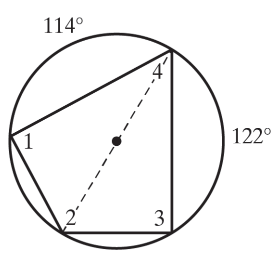
2. a) In $\odot P$, what is $m\angle R$?



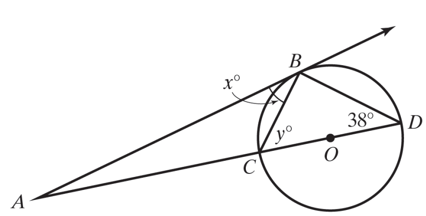
b) What are $m\angle A$, $m\angle B$, $m\angle C$, and $m\angle D$?



3. What is the measure of each numbered angle in the diagram?



4. In the diagram, $\overline{AB}$ is tangent to $\odot O$. What are the values of x and y?



Answers: 1a) 104° 1b) 180° 1c) $m\angle Q = m\angle R = 37°$ 2a) 90° 2b) $m\angle A = 71°$, $m\angle B = 103°$, $m\angle C = 109°$, $m\angle D = 77°$ 3) $m\angle 1 = 90°$, $m\angle 2 = 118°$, $m\angle 3 = 90°$, $m\angle 4 = 62°$ 4) $x = 38$, $y = 52$

Mini-Lesson 12.3

Inscribed Angles

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Interactive Lecture Video Section 12.3

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Interactive Lecture Video Objective 2

Video Organizer Section 12.3 (print)

Video Organizer Section 12.3 (MML)

PowerPoints, Section 12.3

Teaching Notes:

- The demonstration of Theorem 12.3-5 on p.554 is a good way to show the similarity between an inscribed angle and an angle formed by a tangent and chord. This will be even more effective is students are able to view the process dynamically with real-life objects (such as using two rulers and rotating one of them) or a computer animation.

ERROR PREVENTION

- Some students may make errors in a problem like Practice 2a on p.553 because they have trouble identifying the intercepted arc for each of the angles of the quadrilateral. To avoid this problem, show them that by extending the sides of the angle outside the circle they can more easily see the intercepted arc and its measure. For example, in the figure for Practice 2a, extending chords AB and AD outside the circle will show students that they should ignore point C and that the measure of the arc intercepted by $\angle A$ is $100° + 90° = 190°$, so $m\angle A = 95°$.

Closure Questions:

- Corollary 2 of the Inscribed Angle Theorem states that an angle inscribed in a semicircle is a right angle. What can you say about an angle inscribed in a major arc and an angle inscribed in a minor arc? Explain.

  An angle inscribed in a major arc intercepts a minor arc. Because the measure of a minor arc is between 0° and 180°, the measure of the inscribed angle will be between 0° and 90°, so it is an acute angle. An angle inscribed in a minor arc intercepts a major arc. Because the measure of a major arc is between 180° and 360°, the measure of the inscribed angle will be between 90° and 1800°, so it is an obtuse angle.

Mini-Lesson 12.4

Additional Angle Measures and Segment Lengths

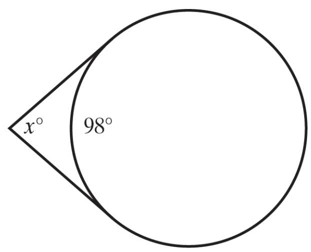
Learning Objectives:

1. Find measures of angles formed by chords, secants, and tangents
2. Find the lengths of segments associated with circles.
3. Key vocabulary: *secant*

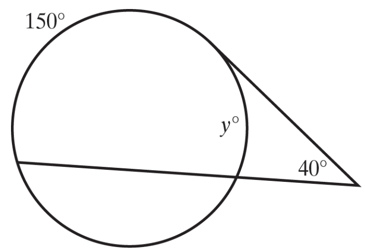
Key Examples:

1. What is the value of each variable?

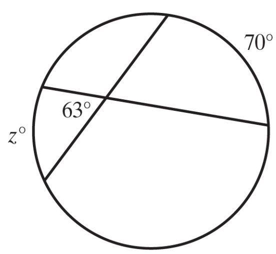
a)



b)



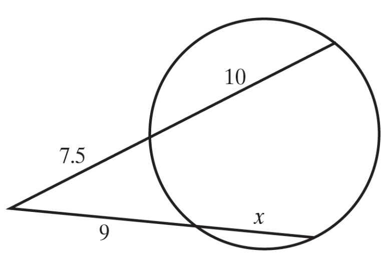
c)



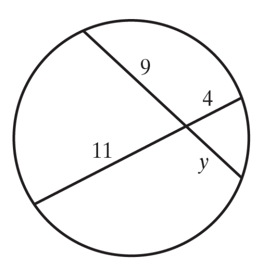
2. A departing space probe is sending back pictures of the Earth as it crosses the Earth's equator. The angle formed by the two tangents to the equator is 21.5°. Find the measure of the arc of the equator that is visible from to the satellite.

3. What is the value of the variable to the nearest tenth?

a)



b)



Answers: 1a) $x = 82$ 1b) $y = 70$ 1c) $z = 56$ 2) 158.5° 3a) $x \approx 5.6$ 3b) $y \approx 4.9$

Mini-Lesson 12.4

Additional Angle Measures and Segment Lengths

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Interactive Lecture Video Objective 1

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PowerPoints, Section 12.4

Teaching Notes:

- After discussing Theorems 12.4-1 and 12.4-2, have students prepare, individually, in small groups, or as a class, a summary of all the relationships between angle measures and arc measures from Sections 12.3 and 12.4.
- When discussing Theorem 12.4-1, emphasize that one rule works with all three situations: two secants, a secant and a tangent, and two tangents.
- Students often have trouble understanding and applying Theorem 12.4-3 about segment products. Use the Helpful Hint on p.561 to help students to identify the segment lengths to be multiplied in each of the three cases and then provide extra practice in identifying these segments with other diagrams.

ERROR PREVENTION

- In problems involving angles formed by intersecting lines and circles, errors arise when students forget whether to find half the sum of the intercepted arcs or half the difference. An easy way to remember this is illustrated in Example 1 on p.559: *inside: sum; outside: difference.*

Closure Questions:

- What is a way to state the rule for the measure of an angle formed by intersecting chords without using the words *add, addition,* or *sum*?

  The measure of an angle formed by two chords intersecting inside a circle is the average of the measures of the two intercepted arcs.

Mini-Lesson 12.5

Coordinate Planes—Circles

Learning Objectives:

1. Find an equation of a circle.
2. Find the center and radius of a circle written in standard form.
3. Complete the square to find the center and radius of a circle.
4. Key Vocabulary: *standard form of an equation of a circle, standard equation of a circle*

Key Examples:

1. What is the standard equation of each circle?
   a) center (–4, 3); radius 11
   b) center (5, −1), radius $\sqrt{7}$

2. What is the standard equation of the circle with center (–2, 1) that passed through the point (–6, –4)?

3. What is the center and radius of the circle with equation $(x+1)^2+(y+3)^2=4$? Graph the circle.

4. Graph $x^2+y^2=9$.

5. Graph $x^2+(y-3)^2=16$.

6. Find the center and radius; then graph $x^2+y^2-8x+4y=-16$.

Answers: 1a) $(x+4)^2+(y-3)^2=121$ 1b) $(x-5)^2+(y+1)^2=7$ 2) $(x+2)^2+(y-1)^2=41$
3)–6) *See Additional Answers at end of Mini-Lessons.*

Mini-Lesson 12.5

Coordinate Planes—Circles

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Interactive Lecture Video Objective 3

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PowerPoints, Section 12.5

Teaching Notes:

- The equation of a circle is based on the distance formula. Review this formula and provide as much practice in using it as your students need.
- Many students find completing the square to be the most difficult method for solving quadratic equations. Even if they have learned this method successfully in an algebra course, they may have forgotten it, so a review will be helpful. Explain that to find the center and radius of a circle, the basic idea is the same, but while before we only completed the square on one variable, *x,* now we will be completing the square on both *x* and *y*. It might be helpful to call this process "completing the squares."

ERROR PREVENTION

- The most common errors in writing or interpreting the equation of a circle are sign errors, particularly when one or both coordinates of the center are negative numbers. When reading an equation such as $(x+4)^2+(y+5)^2=36,$ a student may think that the coordinates of the center are (4, 5) rather than (–4, –5). To avoid this error, have students write $(x+4)^2$ as $[x-(-4)]^2$, or ask them what value of *x* will make *x* + 4 be equal to 0.

Closure Questions:

- When would you use the process of completing the square when graphing a circle?

  If the equation of the circle is not given in standard form, completing the square can be used to rewrite the equation in standard form in order to determine its center and radius.

Mini-Lesson 12.6
Locus

Learning Objectives:

1. Draw and describe a locus.
2. Key vocabulary: *locus*

Key Examples:

1. Sketch and describe the locus of all points in a plane that are equidistant from the four vertices of a square.

2. Sketch the locus of points in the coordinate plane that satisfy these conditions:
   - the points that are 3 units from the origin
   - the points that are 2 units from the point (1, −2)

3. a) What is the locus of points in space that are 2.5 feet from segment $\overline{RS}$?
   b) What is the locus of points in space that are 6 units from a sphere with radius 3 units?

Answers: 1) and 2) *See Additional Answers at end of Mini-Lessons.* 3a) The locus is the lateral surface of a cylinder with radius 2.5 ft and centerline $\overline{RS}$. 3b) The locus a sphere with the same center and radius 9 units.

Mini-Lesson 12.6

Locus

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PowerPoints, Section 12.6

Teaching Notes:

- The concept of *locus* sounds more complicated than it really is. Tell students the word *locus* means "place" and has the same root and similar meaning to "location." If they interpret the directions "Find the locus" as "Find the location," they may find the work in this section less confusing.

ERROR PREVENTION

- Students are likely to have more trouble with visualizing and describing loci in space than in a plane. Help them by illustrating the situations with real three-dimensional objects or with computer animations.

Closure Questions:

- How does the locus of points at a given distance from a given point differ depending on whether you are considering geometry in a plane or in space? What names are used for the given point and the given distance?

  In a plane, the locus is a circle, while in a space it is a sphere. In both cases, the given point is the center and the given distance is the radius.

Extension Mini-Lesson

Parabolas

Learning Objectives:

1. Find an equation of a parabola given the focus and directrix.
2. Key vocabulary: *conic section, parabola, focus, directrix*

Key Examples:

1. Find the equation of the parabola with focus $(2, -4)$ and directrix $y = -1$. Write the equation so that it is solved for y.

Answers: 1) $y = -\frac{1}{6}x^2 + \frac{2}{3}x - \frac{19}{6}$

Extension Mini-Lesson

Parabolas

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(MML)

Interactive Lecture Video
Chapter 12
Extension—Parabolas

Interactive Lecture Video
Objective 1

Video Organizer
Chapter 12
Extension—Parabolas
(print)

Video Organizer
Chapter 12
Extension—Parabolas
(MML)

Teaching Notes:

- For more thorough coverage of conic sections, follow this discussion of parabolas with Appendix B.1: Ellipses and Hyperbolas.
- Example 1 will be more meaningful if you display the graph of the parabola or ask students to graph it on their graphing calculators and locate the vertex, which is (4, 3). Show or have students discover that the vertex is the same distance (2 units) from the focus and directrix. Because the vertex is a point on the parabola, this will help students understand the locus definition of a parabola stated on p.574.

ERROR PREVENTION

- Students may make errors in the algebraic work needed to writing the equation of the parabola so it is solved for *y.* When this occurs, help students to identify and fix their errors by following this checklist:
  1) Did you correctly identify the values of *a, b,* and *c*?
  2) Did you expand the binomial correctly?
  3) Did you make any sign errors in your work?

Closure Questions:

- Is the focus of a parabola part of the parabola? Explain.

  No; the focus is not a point on the parabola. The focus is "inside" the parabola. It is above a parabola opening upward and below a parabola opening downward.

- Is the directrix of a parabola the same line as the axis of symmetry? Explain.

  No; the directrix is not the same line as the axis of symmetry. For a parabola opening upward or downward, the axis of symmetry is a vertical line, while the directrix is a horizontal line.