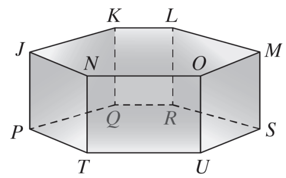
Mini-Lesson 11.1
Solids and Cross Sections

Learning Objectives:

1. Recognize polyhedra and their parts.
2. Visualize cross sections of solids.
3. Visualize solids formed by revolving a region about a line.
4. Key vocabulary: *polyhedron, face, edge, vertex, polyhedra, net, cross section, topographic map, contour map*

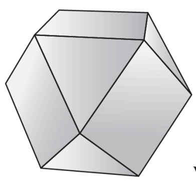
Key Examples:

1. a) How many vertices, edges, and faces are in the polyhedron? List them.
   b) Is $\overline{QU}$ an edge? Explain why or why not.



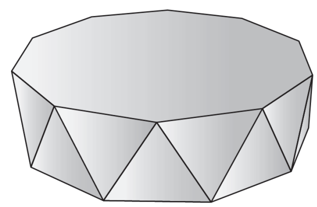
2. For each polyhedron, use Euler's Formula to find the missing number.

a)



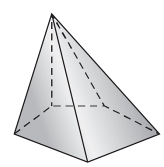
faces: 14
edges: _____
vertices: 12

b)

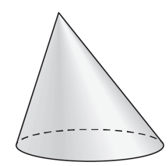


faces: 22
edges: 40
vertices: _____

3. Which of the two solids shown is a polyhedron?



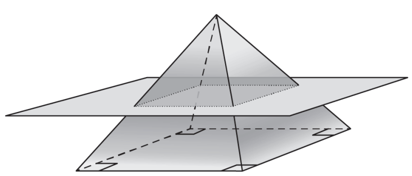
Pyramid



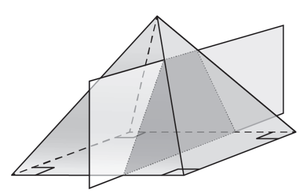
Cone

4. What is the cross section formed by the planes and the solid shown?

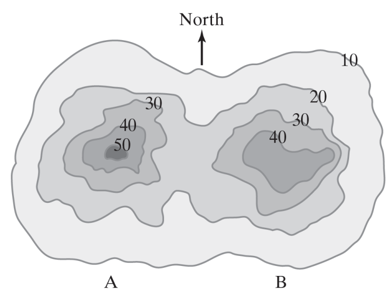
a)



b)



5. Use the contour map to answer the following questions.
   a) What is the elevation of Hill A?
   b) Which side of Hill B—north, east, south, or west, is steepest?



Elevation in meters.

Answers: 1) *See Additional Answers at end of Mini-Lessons*. 2a) 24 2b) 20 3) the pyramid 4a) square 4b) trapezoid 5a) 50 m 5b) east 6) ring

Mini-Lesson 11.1

Solids and Cross Sections

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Geometry
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eText, Section 11.1 (MML)

Interactive Lecture Video
Section 11.1

Interactive Lecture Video
Objective 1

Interactive Lecture Video
Objective 2

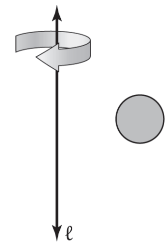
Interactive Lecture Video
Objective 3

Video Organizer
Section 11.1 (print)

Video Organizer
Section 11.1 (MML)

PowerPoints, Section 11.1

6. Describe the solid of revolution obtained by rotating the given plane region about line ℓ.



Teaching Notes:

- Many people find it difficult to visualize three-dimensional objects from two-dimensional drawings. Throughout this chapter, give students the opportunity to look at real three-dimensional objects and models and, if possible, to work with them hands-on.
- Students may find it particularly difficult to visualize cross sections and solids of revolution from two-dimensional drawings. Demonstrate these concepts with computer animations.
- To help students to learn or review the basic types of solids, bring in a set of plastic or wooden models that includes prisms, cylinders, pyramids, cones, and spheres.

ERROR PREVENTION

- Some students will have difficulty visualizing the solid that would be created by folding a net. To give them hands-on three-dimensional experience, provide a page of nets that they can cut out, fold, and tape to create the solids.

Closure Questions:

- What is Euler's formula, and what does it mean?

  Euler's formula is $F+V=E+2$. *It means that in any polyhedron, the sum of the number of faces (F) and the number of vertices (V) is two more than the number of edges (E).*

- Which solids among prisms, cylinders, pyramids, cones, and spheres are polyhedra? Explain.

  Prism and pyramids are polyhedra because all of their faces of polygons. Cylinders, cones, and spheres are not polyhedra because they contain curved surfaces.

Mini-Lesson 11.2

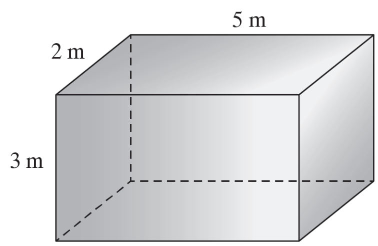
Surface Areas of Prisms and Cylinders

Learning Objectives:

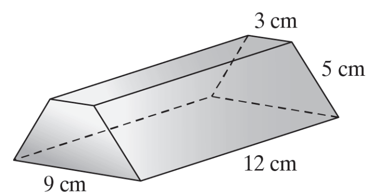
1. Find the surface area of a prism.
2. Find the surface area of a cylinder.
3. Key vocabulary: *surface area, prism (base, lateral face, altitude, height, lateral area), right prism, oblique prism, cylinder (base, altitude, height, lateral area, surface area), right cylinder, oblique cylinder*

Key Examples:

1. What is the surface area of the prism? Use a net.



2. The prism in the figure has bases that are isosceles trapezoids. Find the surface area of the prism by answering parts a–d.
   a) What is the perimeter of a base?
   b) What is the lateral area of the prism?
   c) What is the area of a base?
   d) What is the surface area of the prism?



3. A cylinder has a radius of 5 in. and a height of 14 in. What is the surface area of the cylinder in terms of π?

4. A tank at an aquarium is used for small jellyfish. The tank is in the shape of a cylinder with a radius 0.5 m and a height of 2.0 m. Find the surface area of this tank, including the area of the top base. Use $\pi = 3.14$.

Answers: 1) 62 sq m 2a) 22 cm 2b) 264 sq cm 2c) 24 sq cm 2d) 312 sq cm 3) 190π sq in. 4) 7.85 sq m

Mini-Lesson 11.2

Surface Areas of Prisms and Cylinders

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eText, Section 11.2 (MML)

Interactive Lecture Video Section 11.2

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 11.2 (print)

Video Organizer Section 11.2 (MML)

PowerPoints, Section 11.2

Teaching Notes:

- Although the figures at the top of p.495 illustrate that the lateral surface of a cylinder is really a rectangle, students will be able to see and remember this more clearly if you demonstrate this with a real soup can, cutting off the label vertically and laying it out flat.
- Students may have difficulty remembering all of the formulas in this chapter. Grouping similar formulas will cut down on the number of individual formulas to remember. Show students that the formulas for the surface area of prisms and cylinders are really the same: The surface area is the sum of the areas of the two bases and the lateral area, which is the product of distance around the base (perimeter for a prism, circumference for a cylinder) and the height.

ERROR PREVENTION

- Some students may think that, because they are working with three-dimensional objects, the surface area should be measured in cubic units units. Drawing a net is a good way to show that the surface area is the sum of the areas of plane figures, so is measured in square units.

Closure Questions:

- How are prisms and cylinders similar? How are they different?

  Similar: Both have two parallel congruent faces. Different: The bases of a prism are polygons, while the bases of a cylinder are circles. The lateral faces of a prism are rectangles, while the lateral surface of a cylinder is a single curved surface.

- What is the difference between the lateral area and the surface area of a prism or cylinder?

  The lateral area is the sum of the areas of all the lateral faces, which are all faces except the bases. The surface area is the sum of all the faces, including both the lateral faces and the two bases.

Mini-Lesson 11.3

Surface Areas of Pyramids and Cones

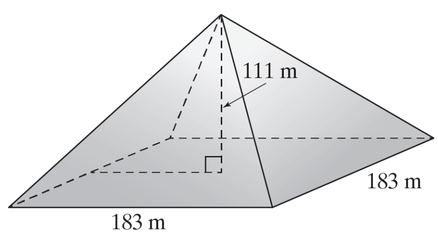
Learning Objectives:

1. Find the surface area of a pyramid.
2. Find the surface area of a cone.
3. Key vocabulary: *pyramid (base, lateral face, vertex, altitude, height, slant height, lateral area, surface area), regular pyramid, cone (base, altitude, vertex, height, slant height, lateral area, surface area), right cone*

Key Examples:

1. A square pyramid has base edges of 9 ft and a slant height of 7 ft. What is the surface area of the pyramid?

2. The hotel Luxor in Las Vegas, Nevada, is a glass-plated square pyramid with base edges that are 183 m long, and an altitude of 111 m. To the nearest whole number, what is the surface area of the hotel?



3. The radius of the base of a cone is 17 cm. Its slant height is 11 cm. What is the surface area in terms of π?

4. The paper hats for a child's birthday party have the shape of a cone. If the diameter of the base is 4 in. and the height is 6.2 in., how much paper was needed to make each hat? Round to the nearest whole number.

Answers: 1) 207 sq ft 2) 86,138 sq m 3) 476π sq cm 4) 41 sq in.

Mini-Lesson 11.3

Surface Areas of Pyramids and Cones

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eText, Section 11.3 (MML)

Interactive Lecture Video Section 11.3

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 11.3 (print)

Video Organizer Section 11.3 (MML)

PowerPoints, Section 11.3

Teaching Notes:

- Some students may only be familiar with square pyramids. Use models to demonstrate that any polygon can be the base of a pyramid, while any regular polygon can be the base of a regular pyramid.

ERROR PREVENTION

- Students are likely to confuse the *slant height* with the *height* of a pyramid or cone and therefore to make errors in calculations that involve these measurements. Use the second figure on p.500 of the textbook to show both of these in the same square pyramid. Point out that the slant height is the length of the hypotenuse of a right triangle in which the height is the length of one of the legs, so the slant height will always be greater than the height.

Closure Questions:

- Why does the formula for the surface area of a regular pyramid contain the slant height of the pyramid rather than the height?

  The surface area of a pyramid is the sum of the area of the base and the areas of the triangular lateral faces. The slant height of a regular pyramid is the length of the altitudes of the triangular faces, so is used to calculate their area.

- How are pyramids and cones similar? How are they different?

  Similar: Both have one face and a vertex. Both have slant heights.
  Different: The base of a pyramid is a polygon, while the base of cone is a circle. The lateral faces of a pyramid are triangles, while the lateral surface of a cone is single curved surface.

Mini-Lesson 11.4

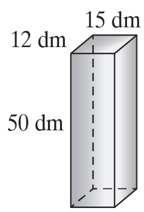
Volume of Prisms and Cylinders and Cavalieri's Principle

Learning Objectives:

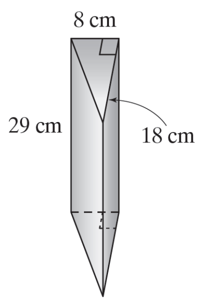
1. Find the volume of a prism
2. Find the volume of a cylinder.
3. Find the volume of composite solids.
4. Key vocabulary: *volume, composite solid*

Key Examples:

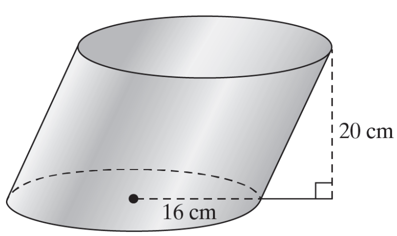
1. What is the volume of the rectangular prism?



2. What is the volume of the triangular prism ?



3. What is the volume of the cylinder in terms of π?



Answers: 1) 9000 cu dm 2) 2088 cu cm 3) 5120π cu cm 4) 32,544 cu m

Mini-Lesson 11.4

Volume of Prisms and Cylinders and Cavalieri's Principle

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eText, Section 11.4 (MML)

Interactive Lecture Video Section 11.4

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Interactive Lecture Video Objective 3

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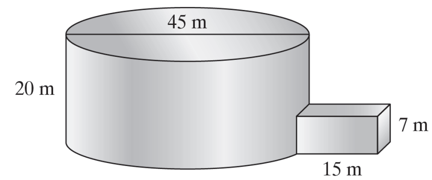
PowerPoints, Section 11.4

4. A certain building can be described by a large cylinder and an adjacent square prism. The dimensions of both of these are shown in the figure. What is the approximate volume of the building to the nearest cubic meter?



Teaching Notes:

- Some students may only be familiar with rectangular prisms (boxes). Emphasize that the bases of a prism can be any congruent polygons.
- Emphasize that the same formula gives the volume of both a prism and a cylinder: $V = Bh.$ The fewer separate formulas that students need to memorize, the easier it will be learn and remember them.

ERROR PREVENTION

- Some students may associate the term "base" with "bottom" and think that the bases of a prism or cylinder are always at the top and bottom of a figure as it is drawn on paper. To correct this misunderstanding, provide extra practice in identifying the bases of prisms and cylinders in different orientations, such as the triangular prisms in Exercises 5–8 and the cylinder in Exercise 10 on p.510 of the textbook.

Closure Questions:

- A rectangular prism (or box) has three pairs of opposite congruent faces that are rectangles. Which of these should be used as the bases to calculate the surface area and volume? Explain.

  Any of the three pairs of opposite faces can be chosen as the bases. Any of these three choices will give the same results for the surface area and volume of the prism.

Mini-Lesson 11.5

Volumes of Pyramids and Cones

Learning Objectives:

1. Find the volume of a pyramid.
2. Find the volume of a cone.

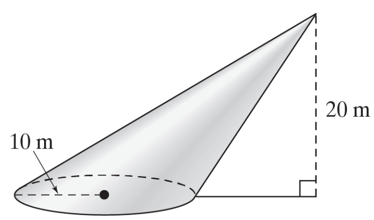
Key Examples:

1. A glass paperweight has the shape of a square pyramid. If an edge of the base is 3.5 in. and the height is 3 in., what is the volume of the paperweight?

2. What is the volume of a square pyramid with base edges 16 cm and slant height 17 cm?

3. What is the volume of a waffle cone with a diameter of 3 in. and a height of 6.5 in.? Round to the nearest tenth.

4. What is the volume of the oblique cone in terms of π and rounded to the nearest cubic meter?



Answers: 1) 12.25 cu in. 2) 1280 cu cm 3) 15.3 cu in. 4) $\frac{2000}{3}\pi$ cu m $\approx$ 2094 cu m

Mini-Lesson 11.5

Volumes of Pyramids and Cones

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Interactive Lecture Video Section 11.5

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 11.5 (print)

Video Organizer Section 11.5 (MML)

PowerPoints, Section 11.5

Teaching Notes:

- Use Example 2 to reinforce the distinction between *height* and *slant height* that was introduced in Section 11.3.

ERROR PREVENTION

- Students will often think that the volume of the cone is one-half the volume of a cylinder with the same radius and height rather than one-third the volume of the cylinder. The demonstration that it takes three cones full of rice to fill the cylinder illustrated on p.514 will be much more effective and the result better remembered if you have a student assist you in carrying out this demonstration with real objects, pouring a substance such as rice, sand, or water.

Closure Questions:

- What single formula can be used to calculate the volume of both a pyramid and a cone?

  $$V = \frac{1}{3}Bh$$

- A *tetrahedron* is a triangular pyramid all of whose faces are congruent equilateral triangles. How many faces does a tetrahedron have, and which one should be used to calculate its surface area and volume?

  A tetrahedron has four faces. Any of these four faces can be used as the base, as all choices will give the same results for the surface area and volume.

Mini-Lesson 11.6

Surface Areas and Volumes of Spheres

Learning Objectives:

1. Find the surface area and volume of a sphere.
2. Key vocabulary: *sphere, center of a sphere, radius of a sphere, diameter of a sphere, circumference of a sphere, great circle, hemisphere*

Key Examples:

1. What is the surface area of a sphere with a radius of 28 cm? Give your answer in terms of π and rounded to the nearest square centimeter.
2. What is the surface area of a beach ball with circumference 50 in.? Round your answer to the nearest square inch.
3. A sphere has a diameter of 46 cm. What is its volume to the nearest cubic centimeter?
4. The volume of a sphere is 1800 cubic meters. What is its surface area of the nearest tenth?

Answers: 1) 3136π sq cm ≈ 9852 sq cm 2) 796 sq in. 3) 50,965 cu cm 4) 715.6 sq m

Mini-Lesson 11.6

Surface Areas and Volumes of Spheres

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Interactive Lecture Video Objective 1

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PowerPoints, Section 11.6

Teaching Notes:

- Students may wonder where the formula for the surface area of a sphere comes from. While a formal derivation of this formula requires calculus, this formula can be demonstrated informally using orange peel. Students may enjoy watching the following video, or you could carry out the demonstration in class: http://www.youtube.com/watch?v=cAxHYFRx1Fs
- Conclude this section by having students, either individually or as a class, make a table listing the formulas for the surface area and volume of a prism, cylinder, pyramid, cone, and sphere.

ERROR PREVENTION

- Some students may have trouble distinguishing between the formulas for the surface area and volume of a sphere, leading to errors in calculating these measurements. Point out that surface area is measured in square units, while volume is measured in cubic units, so the surface area formula involves a factor of r^2, while the volume formula involves a factor of r^3.

Closure Questions:

- For which of the solids you have studied in this chapter do you need to use π in the calculation of the surface area and volume? What is an easy way to remember which ones these are?

  A cylinder, cone, and sphere involve circular bases and/or curve surfaces, so π is involved in calculating their surface areas and volumes. Prisms and pyramids are made up only of polygons, so π is not involved.

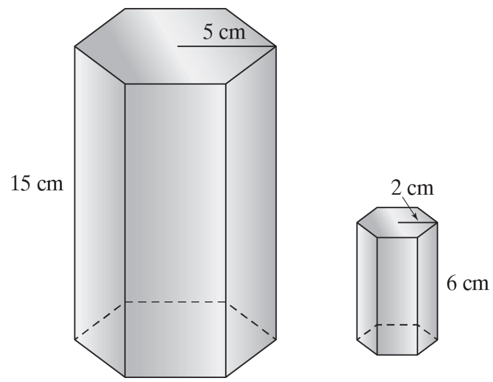
Mini-Lesson 11.7
Areas and Volumes of Similar Solids

Learning Objectives:

1. Compare and find the areas and volumes of similar solids.
2. Key vocabulary: *similar solids*

Key Examples:

1. Are the two hexagonal prisms similar? If so, what is the scale factor of the first figure to the second figure?



2. What is the scale factor of two similar prisms with surface areas 169 sq ft and 225 sq ft?

3. The volumes of two similar solids are 180 cu in. and 108 cu in. The surface area of the smaller solid is 75 sq in. What is the surface area of the larger solid to the nearest tenth?

4. A soup can holds 15 oz of soup. To the nearest ounce, how much soup will a similar can hold if each dimension is 1.2 times as large?

.

Answers: 1) yes; 5:2 or $\frac{5}{2}$ 2) $\frac{13}{15}$ or 13:15 3) 105.4 sq in. 4) 26 oz

Mini-Lesson 11.7

Areas and Volumes of Similar Solids

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Interactive Lecture Video Section 11.7

Interactive Lecture Video Objective 1

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Video Organizer Section 11.7 (MML)

PowerPoints, Section 11.7

Teaching Notes:

- Begin this section by reviewing the concept of similarity: "similar figures have the same shape, but not necessarily the same size," and the specific definition of similar polygons from Section 7.3. Using these ideas, ask students what they think it would mean for two solids to be similar.
- Also review the concept of *scale factor* from Section 7.3 and some real-life situations that involve scale factors with three-dimensional figures.

ERROR PREVENTION

- Some students may make errors with units in their answers to problems in this section. Review the distinction between (linear) units used to measure lengths, perimeter, and circumference, square units, used to measure area, and cubic units, used to measure volume.

Closure Questions:

- If you know the ratio of the volumes of two similar solids, how can you find the ratio of their surface areas?

  Take the cube root of the ratio of the volumes and then square it, or raise the ratio of the volumes to the 2/3 power.

- What must be true of two spheres for them to be similar? How does this differ from the requirements for other types of solids to be similar?

  All spheres are similar because they have the same shape. You do not have to show that their corresponding dimensions are proportional because spheres only have one dimension, the radius.