Mini-Lecture 9.1

The Pythagorean Theorem and Its Converse

Learning Objectives:

1. Use the Pythagorean Theorem.
2. Use the Converse of the Pythagorean Theorem.
3. Key vocabulary: *Pythagorean triple*

Key Examples:

1. a) The legs of a right triangle have lengths 12 and 16. Find the length of the hypotenuse.
   b) Check to see that the side lengths in part a) form a Pythagorean triple.

2. The hypotenuse of a right triangle has length 14. One leg has length 6. Find the length of the other leg. Write the answer in simplest radical form.

3. A painter leans a 12-foot ladder against a wall. The ladder reaches 10 feet up the wall. To the nearest tenth of a foot, how far is the base of the ladder from the wall?

4. A triangle has side lengths 25, 45, and 50. Is the triangle a right triangle? Explain.

5. Is a triangle with side lengths 11, 14, and 18 acute, obtuse, or right?

Answers: 1a) 20 1b) $12^2 + 16^2 = 144 + 256 = 400 = 20^2$ 2) $4\sqrt{10}$ 3) 6.6 ft
4) No; $25^2 + 45^2 = 625 + 2025 = 2650$, and $50^2 = 2500$, so $25^2 + 45^2 \neq 50^2$. 5) obtuse

Mini-Lesson 9.1

The Pythagorean Theorem and Its Converse

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PowerPoints, Section 9 .1

Teaching Notes:

- When asked what the Pythagorean Theorem is (or says), many students will respond "*a*-squared plus *b*-squared equals *c*-squared," but will not be able to tell you what that means. Make sure that every student can state the theorem in words and illustrate it by drawing a diagram and showing the appropriate calculations with specific numbers.
- For enrichment, introduce Euclid's method for generating Pythagorean triples:

  If m and n are positive integers with $m > n$, then the integers $a = m^2 - n^2$, $b = 2mn$, and $c = m^2 + n^2$ form a Pythagorean triple.

  Students could research other methods for generating Pythagorean triples as a project or an extra-credit assignment.

ERROR PREVENTION

- When given the lengths of two sides of a right triangle and asked to find the length of the third side, some students will always add the squares of the two given sides. To help them understand and remember that when the lengths of one leg and the hypotenuse are given they must subtract rather than add, ask them to write out the theorem in words and underline the words *legs* and *hypotenuse.*

Closure Questions:

- If you are given the lengths of the three sides of any triangle, how can you determine whether it is an acute, right, or obtuse triangle?

  Let c be the length of the longest side of the triangle and a and b be the lengths of the other two sides in either order. Then, if $c^2 < a^2 + b^2$, the triangle is acute, if $c^2 = a^2 + b^2$, the triangle is right, and if $c^2 > a^2 + b^2$. the triangle is obtuse.

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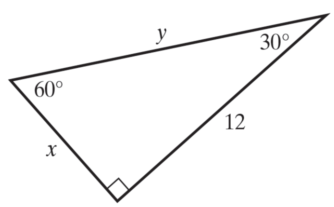
Mini-Lesson 9.2
Special Right Triangles

Learning Objectives:

1. Use the properties of 45°-45°-90° triangles.
2. Use the properties of 30°-60°-90° triangles.

Key Examples:

1. Find the length of the hypotenuse of a 45°-45°-90° triangle with leg length $13\sqrt{2}$.
2. The length of the hypotenuse of a 45°-45°-90° triangle is 22. Find the length of one leg.
3. A playground sits on a square lot with a perimeter of 600 feet. Two diagonal paths cross the lot. To the nearest foot, how long is each path?
4. Find the values of x and y. If an answer involves a radical, write it in simplest radical form.



5. A road sign has the shape of an equilateral triangle. If the sides of the sign measure 25 in., what is the height of the sign to the nearest tenth of an inch?

Answers: 1) 26 2) $11\sqrt{2}$ 3) 212 ft 4) $x = 4\sqrt{3}$, $y = 8\sqrt{3}$ 5) 21.7 in.

Mini-Lesson 9.2
Special Right Triangles

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Interactive Lecture Video Section 9.2

Interactive Lecture Video Objective 1

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PowerPoints, Section 9.2

Teaching Notes:

- Some students may remember the properties of a 45°-45°-90° triangle better if you call it an *isosceles right triangle* to emphasize that the two legs are congruent.
- Students often have trouble remembering the relationships between the lengths of the three sides in a 30°-60°-90° triangle. Remind them that if they know that the shorter leg is half as long as the hypotenuse, they can always find the length of the third leg by the Pythagorean Theorem.

ERROR PREVENTION

- Some students may think that because the ratio of angle measures in a 30°-60°-90° triangle is 1:2:3, the sides are in the same ratio. Show them that this is not possible because of sum of the lengths of any two sides of a triangle is always greater than the length of the third side (the Triangle Inequality Theorem).

Closure Questions:

- How can you use an equilateral triangle to find the lengths of the sides in a 30°-60°-90° triangle?

  In an equilateral triangle, each altitude is also an angle bisector and a median. Draw one such segment to divide the equilateral triangle into two congruent 30°-60°-90° triangles. The shorter leg of either 30°-60°-90° triangle is half the length of the side of the equilateral triangle, the hypotenuse is equal to the side of the equilateral triangle, and the length of the longer leg is the length of the altitude of the equilateral triangle.

Mini-Lesson 9.3

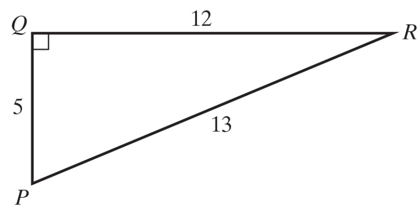
Trigonometric Ratios

Learning Objectives:

1. Use the sine, cosine, and tangent ratios to determine side lengths in right triangles.
2. Use the sine, cosine, and tangent ratios to determine angle measures in right triangles.
3. Key vocabulary: *trigonometric ratios, sine, cosine, tangent*

Key Examples:

1. What are the sine, cosine, and tangent ratios for $\angle R$? Give the exact value and a four-decimal place approximation.

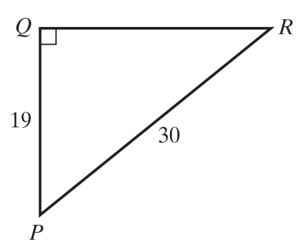


2. In right triangle XYZ, $\angle Z$ is the right angle and $\sin X = \sin Y$. What are $m\angle X$ and $m\angle Y$?

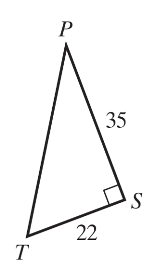
3. An airplane rises from takeoff and flies at an angle of 12° with the horizontal runway. When it has flown 3500 feet, how far, to the nearest foot, is it above the ground?

4. What is $m\angle P$ to the nearest degree?

a)



b)



Answers: 1) $\sin R = \frac{5}{13} \approx 0.3846$, $\cos R = \frac{12}{13} \approx 0.9231$, $\tan R = \frac{5}{12} \approx 0.4167$ 2) $m\angle X = m\angle Y = 45°$ 3) 728 ft 4a) 51° 4b) 32°

Mini-Lesson 9.3

Trigonometric Ratios

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PowerPoints, Section 9.3

Teaching Notes:

- Emphasize that the values of the trigonometric ratios are the same for all triangles with the same angle measures because they are similar by the AA Similarity Postulate. Also remind students that in similar triangles, corresponding sides are proportional, so each of these ratios will have the same values in any two triangles with the same angle measures.

ERROR PREVENTION

- Some students have trouble distinguishing between the sine and cosine of an acute angle in a right triangle. Provide extra practice in identifying the leg *opposite* an acute angle the leg *adjacent* to it, pointing out the adjacent side is one of the sides of the angle, while the opposite side is not.

Closure Questions:

- How do you know that the values of sin A and cos A must be less than 1 for any acute angle A?

  The sine and cosine ratios are ratios of the lengths of the legs of a right triangle to the length of the hypotenuse. Because the hypotenuse is the longest side of a right triangle, these ratios must always be less than 1.

Mini-Lesson 9.4
Solving Right Triangles

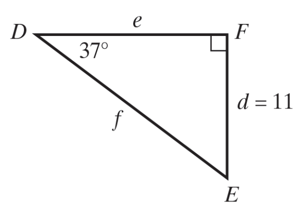
Learning Objectives:

1. Solve right triangles.
2. Use angles of elevation and depression to solve problems.
3. Key vocabulary: *solving the triangle, angle of elevation, angle of depression*

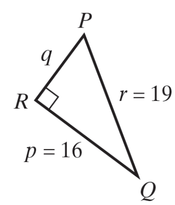
Key Examples:

1. Solve the right triangle. If needed, round any answers to one decimal place.

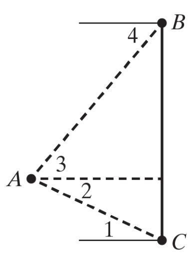
a)



b)



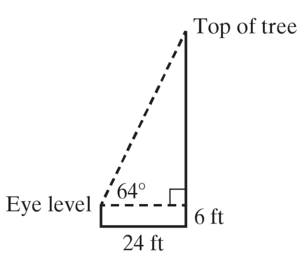
2. An observer standing on a platform at point A is making measurements to the top of a tower at point B and to the base of the tower at point C.



What is a description of the angle as it relates to this situation?

a) $\angle 1$ b) $\angle 2$ c) $\angle 3$ d) $\angle 4$

3. Jason is standing 24 ft from the base of a tree. From his eye level, 6 ft above the ground, the angle of elevation of the top of the tree is 64°. Find the height of the tree to the nearest foot.



4. Meghan, who is 65 inches tall, casts a shadow 42 inches long. Find the angle of elevation of the sun to the nearest degree.

Answers: 1a) $m\angle E = 53°$, $e \approx 14.6$, $f \approx 18.3$ 1b) $q = 10.2$, $m\angle P = 57.4°$, $m\angle Q = 32.6°$ 2a) angle of elevation from base of tower to observer 2b) angle of depression from observer to base of tower 2c) angle of elevation from observer to top of tower 2d) angle of depression from top of tower to observer 3) 55 ft 4) 57°

Mini-Lesson 9.4
Solving Right Triangles

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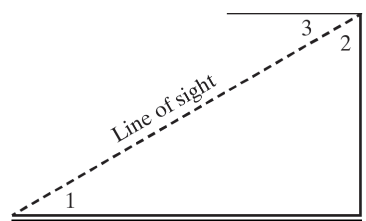
PowerPoints, Section 9.4

Teaching Notes:

- Students should learn to solve a right triangle when the known measurements are given without a figure. Provide practice in sketching and labeling their own figures for various combinations of given measurements (lengths of both legs, lengths of one leg and hypotenuse, length of one leg and one acute angle, length of one hypotenuse and one acute angle).
- Before starting to do any calculations for solving a right triangle, students should make two lists: the measurements they know and the measurements they need to find.

ERROR PREVENTION

- Many students incorrectly identify the angle of depression in an application. In this figure, they will identify $\angle 2$ as the angle of depression rather than $\angle 3$.



To avoid this error, remind students that both the angle of elevation and angle of depression are always measured between the line of sight and a *horizontal* line, never a vertical line. Also, point out the angle of depression looking down is always congruent to the angle of elevation looking up between the same two locations.

Closure Questions:

- Which combinations of measurements (in addition to the right angle) are needed in order to solve a right triangle?

  the lengths of two sides or the length of one side and the measure of one acute angle

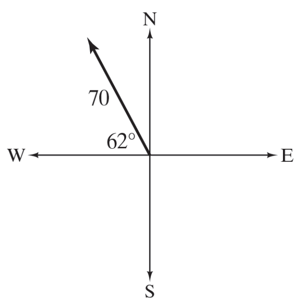
Mini-Lesson 9.5
Vectors

Learning Objectives:

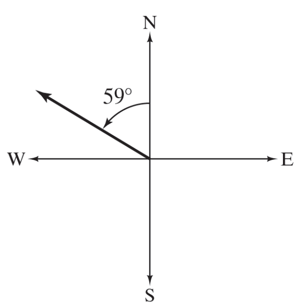
1. Describe vectors.
2. Solve problems involving vector addition.
3. Key Vocabulary: *vector, magnitude, initial point, resultant, component form*

Key Examples:

1. What is the component form of the vector in the figure? Round the coordinates to the nearest tenth.



2. a) Describe the direction of the vector in the figure using compass directions.
   b) Is there more than one way to describe the direction of this vector? Explain.



3. An airplane lands 85 km east and 130 km south from where it took off. What are the approximate magnitude and direction of the flight vector?

4. What is the resultant of $\langle -6, 8 \rangle$ and $\langle -2, -3 \rangle$?

5. A small airplane has a speed of 250 mph in still air. The plane heads directly north and encounters a 45-mph wind blowing due east. Find the resulting speed and direction of the plane. Round the speed to the nearest whole number and the angle to the nearest degree.

Answers: 1) $\langle -32.9, 61.8 \rangle$ 2a) 59° west of north 2b) Yes; it can also be described as 31° north of west. 3) about 155 km at 57° south of east 4) $\langle -8, 5 \rangle$ 5) 254 mph at 10° east of north

Mini-Lesson 9.5

Vectors

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PowerPoints, Section 9.5

Teaching Notes:

- Make sure students understand the new vocabulary and notation in this lesson before you start discussing the examples.
- Emphasize the distinction between the notation for writing a vector in component form with angle brackets, $\langle x, y \rangle$, and writing the coordinates of a point in the plane with parentheses, (x, y).

ERROR PREVENTION

- Some students may try to find the magnitude of the sum of two vectors by adding the magnitudes of the two vectors. Draw some simple diagrams to show that this is not true unless the two vectors have the same direction.

Closure Questions:

- What are the two characteristics that are required for a quantity to be a vector?

  magnitude and direction

- What is the magnitude of a vector $\langle x, y \rangle$?

  $\sqrt{x^2 + y^2}$

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Extension Mini-Lesson

The Law of Sines

Learning Objectives:

1. Use the Law of Sines to solve oblique triangles.
2. Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.
3. Find the area of an oblique triangles using the sine function.
4. Key Vocabulary: *oblique triangle, Law of Sines, ambiguous case*

Key Examples:

1. Solve triangle *ABC* with $A = 98°$, $B = 23°$, and $b = 10$ inches. Round lengths of sides to the nearest tenth.

2. Solve triangle *ABC* with $B = 72°$, $C = 36.3°$. and $a = 18$. Round measures to the nearest tenth.

3. Solve triangle *ABC* with $A = 108°$, $a = 41$, and $b = 27$. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.

4. Solve triangle *ABC* with $B = 68°$, $b = 15$, and $c = 18$.

5. Solve triangle *ABC* with $C = 47°$, $b = 31$, and $c = 26$. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.

6. Find the area of a triangle having two sides of lengths 32 centimeters and 45 centimeters and an included angle of 118°. Round to the nearest square centimeter.

Answers: 1) $C = 59°$, $a \approx 25.3$ inches, $c \approx 21.9$ inches 2) $A = 71.7°$, $b \approx 18.0$, $c \approx 11.2$ 3) $B \approx 39°$, $C \approx 33°$, $c \approx 23.6$ 4) no triangle 5) $A_1 \approx 72°$, $B_1 = 61°$, $a_1 \approx 33.9$ and $A_2 \approx 14°$, $B_2 = 119°$, $a_2 \approx 8.4$ 6) 636 sq cm

Extension Mini-Lesson

The Law of Sines

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Interactive Lecture Video Objective 1

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Interactive Lecture Video Objective 3

Video Organizer Chapter 9 Extension—The Law of Sines (print)

Video Organizer Chapter 9 Extension—The Law of Sines (MML)

Teaching Notes:

- Begin this section by reviewing the triangle congruence postulates and theorems from Chapter 4. To avoid confusion, you might point out that the condition involving two angles and the non-included side was called AAS in Chapter 4, but is called SAA here; they are the same.

ERROR PREVENTION

- The ambiguous case can lead to many errors, especially when there are two solutions. Drawing the two triangles together and then each separately, as shown in the Solution for Example 5 on p.413 of the textbook is helpful. When the two triangles are shown together, outlining them in different colors will make the situation even clearer.

Closure Questions:

- When solving an oblique triangle, how many parts do you need to find? Explain.

  A triangle has 6 parts, 3 sides and 3 angles. In order to have enough information to solve the triangle, the measures of 3 parts must be given, so I need to find the measures of the other 3.

- When you are given the length of one side and the measure of any two angles in an oblique triangle, there is always one possible triangle, but if you are given the lengths of two sides and the measure of the non-included angle (SSA), there may be no triangle, one triangle, or two triangles. How do the triangle congruence postulates and theorems explain this difference?

  The ASA Postulate and AAS Theorem guarantee that there will be exactly one triangle when the measures of one side and any two angles are given, but there is no SSA postulate or theorem. This means that two triangles with this combination of congruent parts may not be congruent.

Extension Mini-Lesson

The Law of Cosines

Learning Objectives:

1. Use the Law of Cosines.
2. Key Vocabulary: *Law of Cosines*

Key Examples:

1. Solve triangle ABC with $B = 50°$, $a = 12$, and $c = 15$. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.

2. Solve triangle ABC with $a = 13$, $b = 25$, and $c = 33$. Round angle measures to the nearest degree.

Answers: 1) $b \approx 11.7$, $A \approx 52°$, $C \approx 78°$ 2) $C \approx 117°$, $A \approx 21°$, $B \approx 42°$

Extension Mini-Lesson

The Law of Cosines

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Interactive Lecture Video Objective 1

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Video Organizer Chapter 9 Extension—The Law of Cosines (MML)

Teaching Notes:

- Again, review the triangle congruence postulates and theorems. Have the class make a list of the combinations of given information for which you solve a right triangle (HL) and those for which the first step is to use the Law of Sines (ASA and AAS). Then ask students which combinations have not yet been accounted for (SAS and SSS) to motivate the need for the Law of Cosines.
- In the process of solving oblique triangles using the Law of Sines and the Law of Cosines, some observant students may wonder how we can talk about the sine or cosine of an obtuse triangle, or how the cosine of an angle can be negative since we have only defined the trigonometric ratios for acute angles. Tell them that the definitions of sine, cosine, and tangent will be extended in a trigonometry course.

ERROR PREVENTION

- Some students may be confused by the three forms of the Law of Cosines and not be sure which one to use in solving a particular problem. In the SAS case, where they are using the Law of Cosines to find the length of an unknown side, they should choose the form that has the unknown side length on the left.

Closure Questions:

- For which combinations of given triangle measurements would you use the Law of Cosines as the first step in solving a triangle?

  SAS and SSS

- Why is there no ambiguous case when using the Law of Cosines?

  The SAS and SSS Postulates tell us that given these combinations of parts, there is only one distinct triangle that can be constructed. (Any other triangle with the same measurements will be congruent to the original one.)