Mini-Lesson 7.1
Ratios and Proportions

Learning Objectives:

1. Write ratios as fractions.
2. Write ratios in simplest form.
3. Understand and work with extended ratios.
4. Solve proportions.
5. Key vocabulary: *ratio, extended ratio, proportion, extremes, means, cross products*

Key Examples:

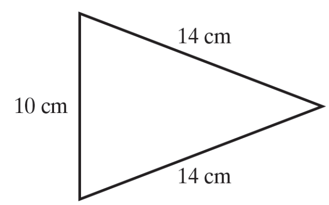
1. Write the ratio of 14 to 31 using fractional notation.

2. Write each ratio as a fraction in simplest form.

   a) \$30 to \$18 b) 7 lb to 28 oz c) 6500 m to 26 km

3. Given the isosceles triangle shown:

   a) Find the ratio of the length of the base to the length of a leg.
   b) Find the ratio of the length of a leg to the perimeter of the triangle.



4. The lengths of the sides of a triangle are in the extended ratio 3:8:9. The perimeter is 80 cm. What are the lengths of the sides?

5. Solve each proportion for the variable.

   a) $\frac{7}{4}=\frac{n}{12}$ b) $\frac{8}{y+4}=\frac{6}{y}$

Answers: 1) $\frac{14}{31}$ 2a) $\frac{5}{3}$ 2b) $\frac{4}{1}$ 2c) $\frac{1}{4}$ 3a) $\frac{5}{7}$ 3b) $\frac{7}{19}$ 4) 12 cm, 32 cm, 36 cm 5a) $n = 21$ 5b) $y = 12$

Mini-Lesson 7.1

Ratios and Proportions

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Geometry **Multimedia Lesson Resources**

eText, Section 7.1 (MML)

Interactive Lecture Video Section 7.1

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Interactive Lecture Video Objective 3

Interactive Lecture Video Objective 4

Video Organizer Section 7.1 (print)

Video Organizer Section 7.1 (MML)

PowerPoints, Section 7.1

Teaching Notes:

- This section should be a review for students. Concentrate on helping students apply what they already know about ratios of numbers to ratios in geometry.
- Most students are able to recite, "The product of the means equals the product of the extremes." Ask them to create their own examples to demonstrate that they understand this statement and how to use cross products to solve proportions.

ERROR PREVENTION

- Students often make errors when working with ratios in which the units are not the same. To help them understand that in this case they must convert one of the units so that the units in the numerator and denominator match. Use a simple everyday example that requires a conversion that everybody knows, for example, "Write the ratio $\frac{2 \text{ weeks}}{7 \text{ days}}$ in simplest form." If a student writes $\frac{2}{7}$, ask follow-up questions: "Which is longer, 2 weeks or 7 days? How many times as long?"

Closure Questions:

- What is the difference between a *ratio* and a *proportion*?

  A ratio is the quotient of two quantities. A proportion is an equation stating that two ratios are equal.

Mini-Lesson 7.2

Proportion Properties and Problem Solving

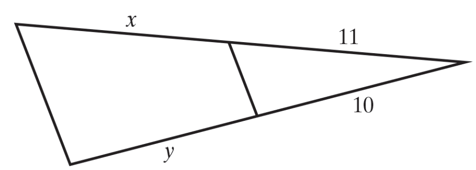
Learning Objectives:

1. Use properties of proportions to write equivalent proportions.
2. Solve problems by writing proportions.

Key Examples:

1. Use the Properties of Proportions to write three proportions equivalent to $\frac{x}{9}=\frac{y}{13}$.

2. In the diagram, $\frac{x}{11}=\frac{y}{10}$. What ratio completes the equivalent proportion? Justify your answer.



   a) $\frac{y}{x}=\text{—}$

   b) $\frac{x+11}{11}=\text{—}$

3. On a particular map in a road atlas, a distance of 70 miles corresponds to 4 cm. How many miles correspond to 11 cm?

4. A cookie recipe requires 8 tablespoons of butter. If this recipe is used to bake 30 cookies, how much butter is needed 75 cookies?

Answers: 1) Property (1): $\frac{9}{x}=\frac{13}{y}$; Property (2): $\frac{x}{y}=\frac{9}{13}$; Property (3): $\frac{x+9}{9}=\frac{y+13}{13}$ 2a) $\frac{10}{11}$; Property (2)

2b) $\frac{y+10}{10}$; Property (3) 3) 192.5 miles 4) 20 tablespoons

Mini-Lesson 7.2

Proportion Properties and Problem Solving

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Geometry
Multimedia Lesson Resources

eText, Section 7.2 (MML)

Interactive Lecture Video Section 7.2

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 7.2 (print)

Video Organizer Section 7.2 (MML)

PowerPoints, Section 7.2

Teaching Notes:

- This section should be easy for most students because it is a review of work with proportions from their algebra course. Examples 3 and 4 provide a good review of the four-step problem-solving process.
- Emphasize applications of proportions that are involve lengths and distances, such as scale models, blueprints, and scale drawings, as these will lead nicely into the work with proportional sides of similar polygons in Section 7.3.

ERROR PREVENTION

- Students sometimes make errors in writing equivalent ratios. For example, they might invert one ratio in a proportion, but not the other. Remind them that they can always check whether two ratios are equivalent by cross-multiplication.

Closure Questions:

- When writing a proportion to solve an application, how can you make sure that you have written the proportion correctly?

  Write the four quantities in the proportion with units attached, making sure that the numerators have matching units and the denominators have matching units. Then drop the units when solving the proportion by using cross products.

Mini-Lesson 7.3

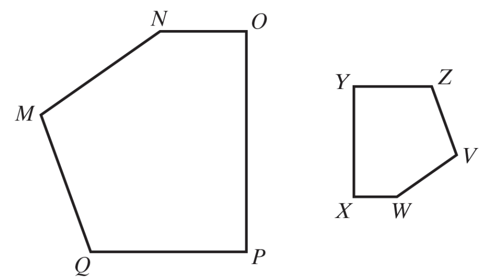
Similar Polygons

Learning Objectives:

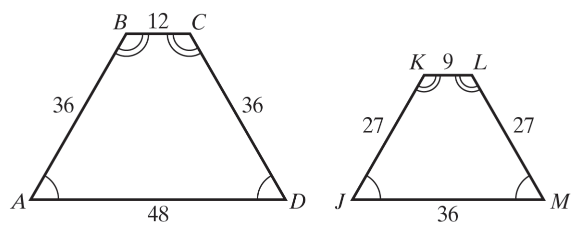
1. Identify similar polygons.
2. Use similar polygons to solve applications.
3. Key vocabulary: *similar figures, similar polygons, extended proportion, scale factor*

Key Examples:

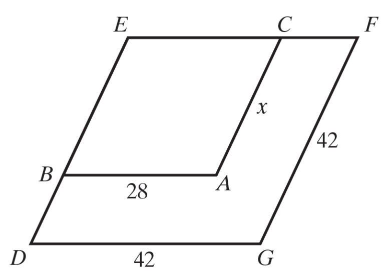
1. In the figure, $MNOPQ \sim VWXYZ$.
   a) What are the pairs of congruent angles?
   b) What is the extended proportion for the ratios of the lengths of corresponding sides?



2. Are the polygons similar? If they are, write a similarity statement and give the scale factor.



3. Use the figure to find the value of x.



4. A rectangular billboard image is 5 ft wide and 4 ft high. If the available billboard space is 25 ft by 15 ft, what are the dimensions of the largest image that can fit into the space?

Answers: 1a) $\angle M \cong \angle V$, $\angle N \cong \angle W$, $\angle O \cong \angle X$, $\angle P \cong \angle Y$, $\angle Q \cong \angle Z$ 1b) $\frac{MN}{VW} = \frac{NO}{WX} = \frac{OP}{XY} = \frac{PQ}{YZ} = \frac{QM}{ZV}$

2) similar; $ABCD \sim JKLM$; $\frac{4}{3}$ or 4:3 3) $x = 28$ 4) 18.75 ft by 15 ft

Mini-Lesson 7.3
Similar Polygons

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Geometry
Multimedia Lesson Resources

eText, Section 7.3 (MML)

Interactive Lecture Video Section 7.3

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 7.3 (print)

Video Organizer Section 7.3 (MML)

PowerPoints, Section 7.3

Teaching Notes:

- Emphasize that while congruent figures are the same shape and the same size, similar figures are the same shape but *not necessarily* the same size. This means that congruence is a special case of the more general concept of similarity.

ERROR PREVENTION

- Students often make errors in writing the extended proportions for the lengths of corresponding sides correctly. Remind them of the process of identifying corresponding vertices and corresponding sides in congruent triangles. When working with similar triangles, the side lengths of one polygon will be the numerators and the corresponding sides lengths of the other the denominators.

Closure Questions:

- How are the definitions of congruent and similar polygons alike? How are they different?

  In both congruent and similar polygons, corresponding angles are congruent. In congruent polygons, corresponding sides are also congruent, while in similar polygons, they are proportional.

- What is the scale factor between two congruent polygons?

  $\frac{1}{1}$ *or 1:1*

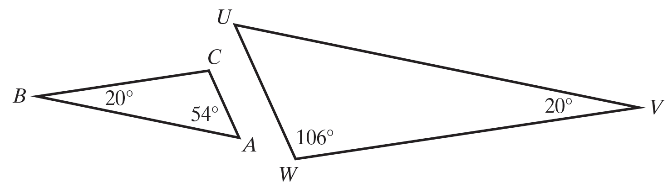
Mini-Lesson 7.4
Proving Triangles Are Similar

Learning Objectives:

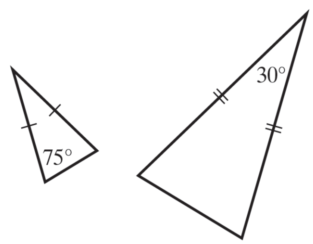
1. Use the AA ~ Postulate and the SAS ~ and SSS ~ Theorems.
2. Use similarity to find indirect measurements.
3. Key vocabulary: *indirect measurement*

Key Examples:

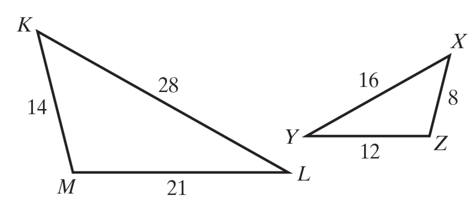
1. Determine whether the two triangles in the figure are similar. Explain.



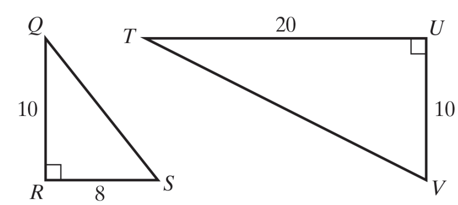
2. Determine whether the two triangles in the figure are similar. Explain.



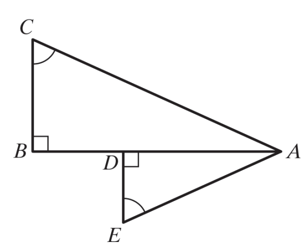
3. Determine whether the triangles are similar. If they are, write a similarity statement for them. Explain how you know that the triangles are similar. Use the AA ~ Postulate, SAS ~ Theorem, or the SSS ~ Theorem.



4. Determine whether the triangles are similar. If they are, write a similarity statement for them. Explain how you know that the triangles are similar. Use the AA ~ Postulate, SAS ~ Theorem, or the SSS ~ Theorem.



5. **Given:** $\angle C \cong \angle E$
   $\overline{CB} \perp \overline{BA}$, $\overline{ED} \perp \overline{DA}$
   Prove: $\Delta ABC \sim \Delta ADE$



Answers: 1) The acute angles in ΔABC measure 20° and 54°, while in ΔUVW they measure 20° and 180° – 20° – 106° = 54°, so the triangles are similar. 2) Each triangle is isosceles, so the base angles in each triangle must be 75°. The triangles are similar by the AA ~ Postulate. 3) $\Delta KLM \sim \Delta XYZ$ by the SSS ~ Theorem. 4) The triangles are not similar because the ratios of the lengths of the legs of the right triangles are not equal. 5) *See Additional Answers at end of Mini-Lessons*. 6) 553 m

Mini-Lesson 7.4

Proving Triangles Are Similar

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Interactive Lecture Video Section 7.4

Interactive Lecture Video Objective 1

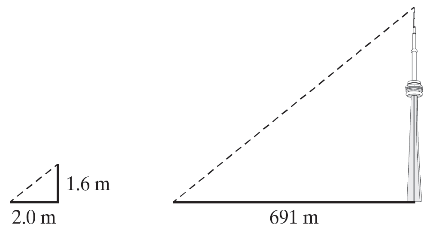
Interactive Lecture Video Objective 2

Video Organizer Section 7.4 (print)

Video Organizer Section 7.4 (MML)

PowerPoints, Section 7.4

6. The CN Tower in Toronto, Canada, is the tallest free-standing structure in the Western Hemisphere. Approximate the tower's height to the nearest meter.



Teaching Notes:

- Begin this section by reviewing the triangle congruence postulates and theorems from Chapter 4: the SSS Postulate, SAS Postulate, ASA Postulate, AAS Theorem, and H-L Theorem. Explain that we now want to determine which combinations of congruent angles and proportional sides will guarantee that two triangles are similar.

ERROR PREVENTION

- As in work with congruent triangles, students often make errors in matching up corresponding vertices and sides in similar triangles, especially when the triangles are oriented differently or are overlapping triangles. Remind students to use the given information and the figure carefully to identify corresponding vertices and sides before writing a similarity statement or starting a similarity proof.

Closure Questions:

- Some geometry textbooks use an AAA Similarity Postulate, rather than the AA Similarity Postulate, and then call the AA condition a corollary. Why is that approach equivalent to just using the AA Postulate, as in your textbook?

  By the Third Angles Theorem, if two triangles have two pairs of congruent angles, the third angles will also be congruent, so AA is equivalent to AAA.

- What is the difference between the SSS Congruence Theorem and the SSS Similarity Postulate?

  In the SSS Congruence Postulate, all three pairs of corresponding sides must be congruent, while in the SSS Similarity Theorem, corresponding sides are proportional.

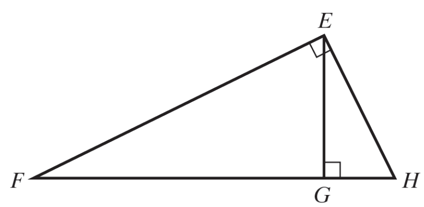
Mini-Lesson 7.5
Geometric Means and Similarity in Right Triangles

Learning Objectives:

1. Use altitudes of right triangles to prove similarity.
2. Find the geometric mean of the lengths of segments in a right triangle.
3. Solve applications involving right triangles.
4. Key Vocabulary: *geometric mean*

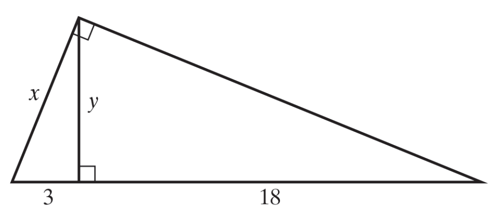
Key Examples:

1. What similarity statement can you write relating the three triangles in the diagram?

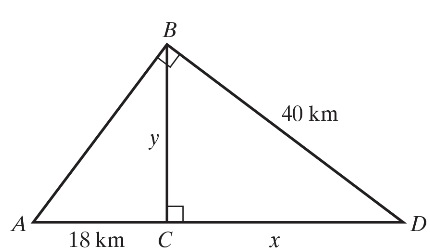


2. What is the geometric mean of 5 and 36? Write your answer as a simplified radical.

3. What are the values of x and y? Write your answer as a simplified radical.



4. A highway between points *A* and *B* has been closed for repairs. An alternate route between there two locations is to travel between *A* and *C*, and then from *C* to *B*. What is the value of *y*, and what is the total distance from *A* to *C* to *B*?



Answers: 1) $\Delta EFH \sim \Delta GEH \sim \Delta GFE$ 2) $6\sqrt{5}$ 3) $x = 3\sqrt{7},\ y = 3\sqrt{6}$ 4) y = 24 km, total distance = 42 km

Mini-Lesson 7.5

Geometric Means and Similarity in Right Triangles

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Multimedia Lesson Resources

eText, Section 7.5 (MML)

Interactive Lecture Video Section 7.5

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Interactive Lecture Video Objective 3

Video Organizer Section 7.5 (print)

Video Organizer Section 7.5 (MML)

PowerPoints, Section 7.5

Teaching Notes:

- Theorem 7.5-1 and its proof can be very confusing to students because the same vertex or side plays different roles in the three right triangles. For example, in the figure in the theorem box on p.320 in the textbook, C is the right-angle vertex in ΔABC, the vertex of the smaller acute angle in ΔACD, and vertex of the larger acute angle in ΔCBD. Drawing the three right triangles separately, as shown in the theorem box on p.320 of in the solution for Example 1 on p.321 is very helpful.

ERROR PREVENTION

- Many students make errors in writing similarity statements for the three right triangles. To avoid this problem, they should identify the hypotenuse, short leg, and long leg in each of the three triangles and use this information to match up the corresponding vertices.

Closure Questions:

- Given two numbers, how can you calculate their arithmetic mean (average)? How do you calculate their geometric mean?

  To calculate the arithmetic mean, add the two numbers and divide the sum by 2. To calculate the geometric mean, multiply the two numbers and take the square root of the product.

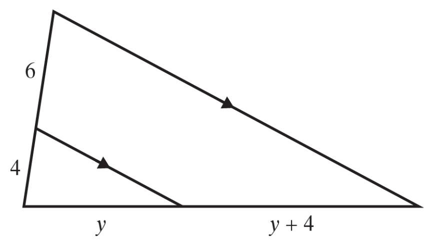
Mini-Lesson 7.6
Additional Proportions in Triangles

Learning Objectives:

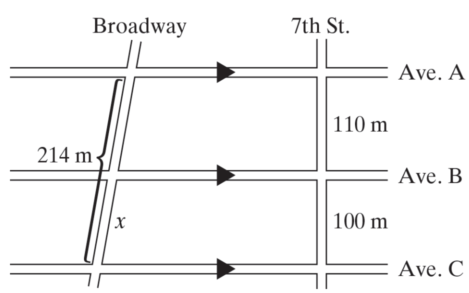
1. Use the Side-Splitter Theorem.
2. Use the Triangle-Angle-Bisector Theorem.

Key Examples:

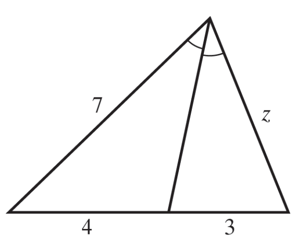
1. What is the value of y in the diagram?



2. Avenues A, B, and C, are parallel to each other, and are perpendicular to 7th Street. What is the length (x) of the block on Broadway between Avenues B and C to the nearest tenth?



3. What is the value of z in the diagram?



Answers: 1) $y = 8$ 2) 102 m 3) $z = 5.25$

Mini-Lesson 7.6

Additional Proportions in Triangles

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Geometry
Multimedia Lesson Resources

eText, Section 7.6 (MML)

Interactive Lecture Video Section 7.6

Interactive Lecture Video Objective 1

Interactive Lecture Video Objective 2

Video Organizer Section 7.6 (print)

Video Organizer Section 7.6 (MML)

PowerPoints, Section 7.6

Teaching Notes:

- The diagram for the Side-Splitter Theorem resembles that for the Triangle Midsegment Theorem. Ask students how these situations are alike and how they are different.

ERROR PREVENTION

- Some students may think that the Side-Splitter Theorem means that the two segments into which a side of a triangle is split are congruent to each other. Demonstrate that this is not necessarily the case by moving the segment that splits the sides into different positions, but always keeping it parallel to the base.

Closure Questions:

- In what situation will the Side-Splitter Theorem divide two sides of a triangle into four congruent segments?

  This will happen only when the triangle is isosceles and the segment parallel to the base splits the sides at its midpoints.

- In what situation will an angle bisector divide a triangle into two congruent right triangles?

  This will happen whenever the angle bisector bisects the vertex angle of an isosceles triangle.

- In what situation will an angle bisector divide a triangle into two congruent isosceles right triangles?

  This will happen whenever the angle bisector bisects the right angle of an isosceles right triangle.