Congruent Triangles

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Triangles

- 3 Sides
- 3 Angles
- If congruent, ALL 6 match!

CPCT

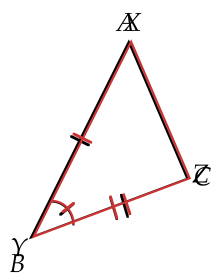
- Corresponding Parts of Congruent Triangles are Congruent

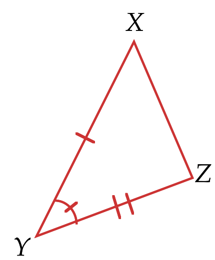
$$\triangle ABC \cong \triangle XYZ$$

means:

$$\angle A \cong \angle X$$
$$\angle B \cong \angle Y$$
$$\angle C \cong \angle Z$$
$$\overline{AB} \cong \overline{XY}$$
$$\overline{BC} \cong \overline{YZ}$$
$$\overline{AC} \cong \overline{XZ}$$

Euclid's Elements,
Book 1, Proposition 4
Side-Angle-Side
(SAS)

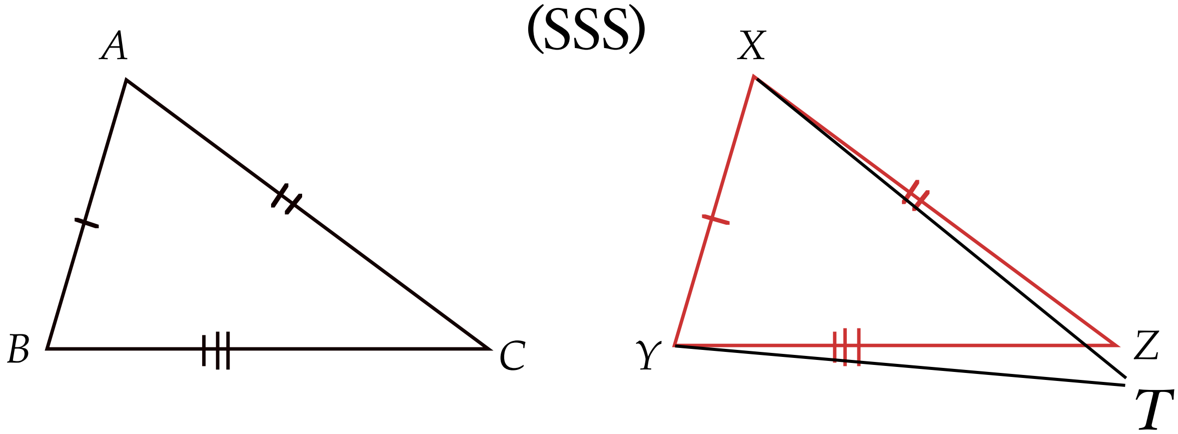




Book 1, Proposition 8

Side-Side-Side

(SSS)



Copy $\angle ABC$ so that $\angle ABC \cong \angle XYT$
and $\overline{BC} \cong \overline{YT}$

So $\triangle ABC \cong \triangle XYT$ (SAS)

$\overline{BC} \cong \overline{YT}$(CPCT) $\cong \overline{YZ}$(Trans)

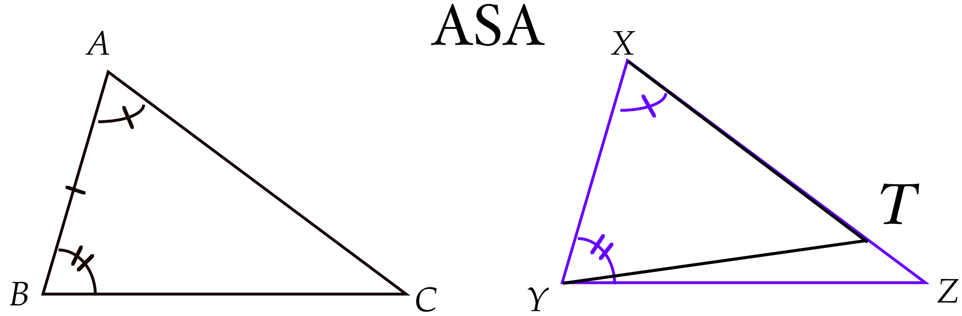
$\overline{AC} \cong \overline{XT}$(CPCT) $\cong \overline{XZ}$(Trans)

***T* must be *Z*!**

Book 1, Proposition 26

Angle-Side-Angle

ASA



Find T along $\overline{XZ}$ so that $\overline{XT} \cong \overline{AC}$

Then $\triangle ABC \cong \triangle XYT$(SAS)

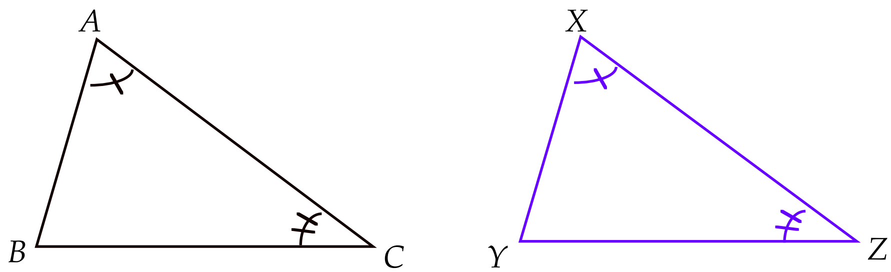
$\angle XYT \cong \angle ABC$(CPCT) $\cong \angle XYZ$(Trans.)

So *T* is *Z*!

Book I, Proposition 26

Angle-Angle-Side

(AAS or SAA)

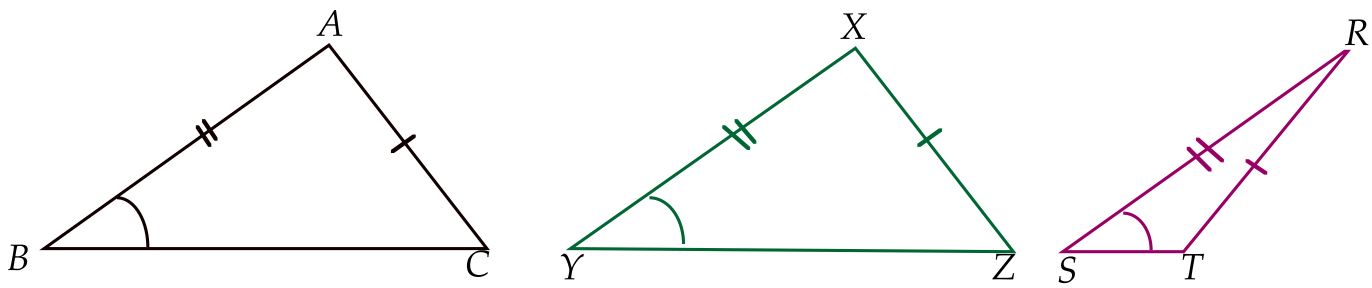


By subtraction from 180°

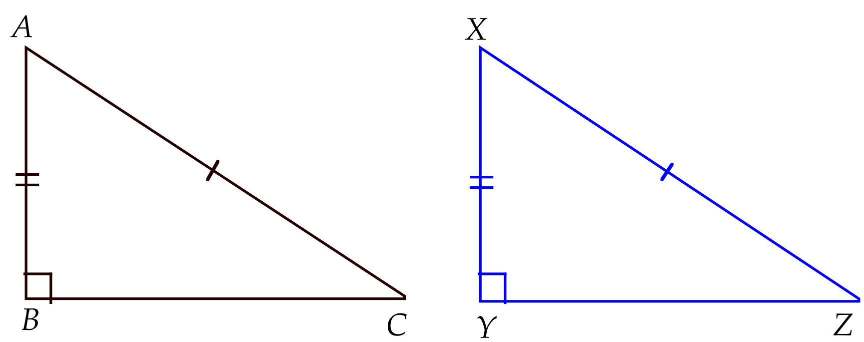
$$\angle B \cong \angle Y$$

$$\triangle ABC \cong \triangle XYZ (\text{ASA})$$

So what about SSA (or ASS)?

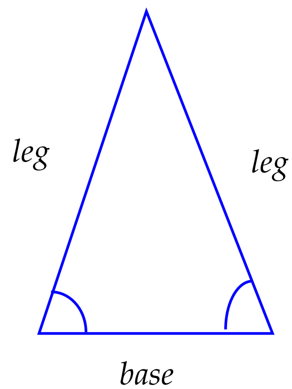


Unless the angle is a right Angle!



But we call this
Hypotenuse-Leg (HL)

To Prove the HL Theorem, we first need to prove
the Isosceles Triangle Theorem



Isosceles Triangle Theorem

Iff the base angles of an Isosceles Triangle is congruent, then the legs are congruent

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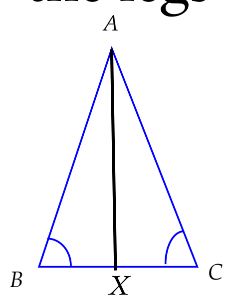
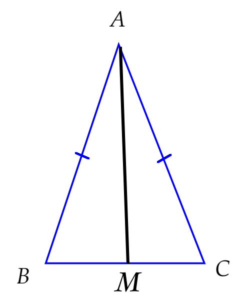
Given $\angle B \cong \angle C$
Prove $\overline{AB} \cong \overline{AC}$



Find X, so that $\overline{AX}$ bisects $\angle BAC$

$\angle B \cong \angle C$
$\angle BAX \cong \angle CAX$
$\overline{AX} \cong \overline{AX}$
$\triangle BAX \cong \triangle CAX$
$\overline{AB} \cong \overline{AC}$

Given $\overline{AB} \cong \overline{AC}$
Prove $\angle B \cong \angle C$



Find M, the midpoint of $\overline{BC}$

$\overline{AB} \cong \overline{AC}$
$\overline{AM} \cong \overline{AM}$
$\overline{BM} \cong \overline{CM}$
$\triangle BAM \cong \triangle CAM$
$\angle B \cong \angle C$

Proof of the
Hypotenuse-Leg Theorem
(HL)

Given:

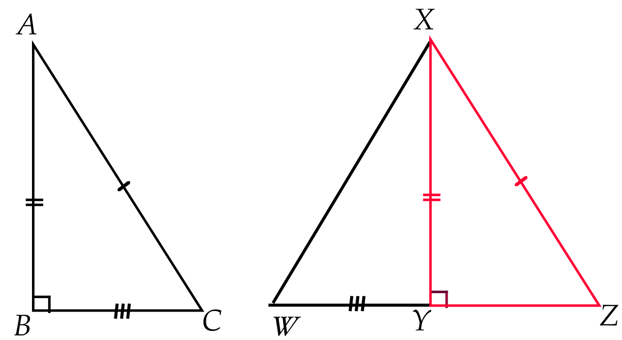
$\overline{AB} \cong \overline{XY}$

$\overline{BC} \cong \overline{YZ}$

$m\angle B = m\angle Y = 90^\circ$

Prove:

$\triangle ABC \cong \triangle XYZ$



Construct W along $\overline{ZY}$ so that $\overline{BC} \cong \overline{YW}$

$m\angle ABC = m\angle XYW = 90^\circ$

$\overline{AB} \cong \overline{XY}$

$\triangle ABC \cong \triangle XYW$

$\overline{XW} \cong \overline{AC}$

$\overline{XW} \cong \overline{XZ}$

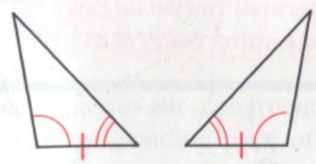
$\angle W \cong \angle Z$

$m\angle XYW = m\angle XYZ = 90^\circ$

$\triangle ABC \cong \triangle XYZ$

Can the two triangles be proved congruent? If so, what postulate can be used?

4.



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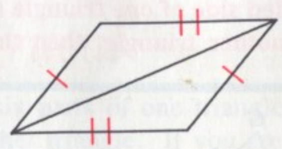

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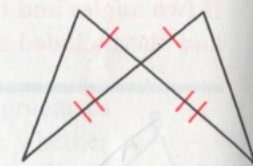

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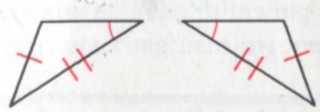

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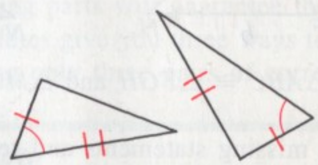

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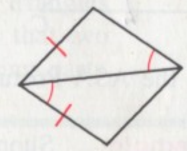
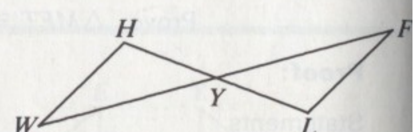
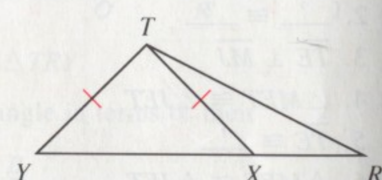


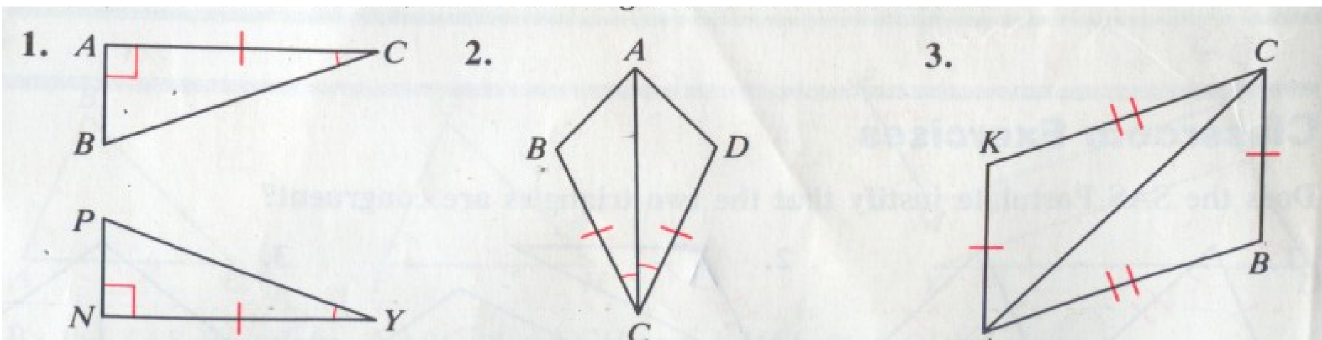
10. Explain how you would prove the following.
Given: $\overline{HY} \cong \overline{LY}$;
$\overline{WH} \parallel \overline{LF}$
Prove: $\triangle WHY \cong \triangle FLY$



11. a. List two pairs of congruent corresponding sides and one pair of congruent corresponding angles in $\triangle YTR$ and $\triangle XTR$.

b. Notice that, in each triangle, you listed two sides and a *nonincluded* angle. Do you think that SSA is enough to guarantee that two triangles are congruent?



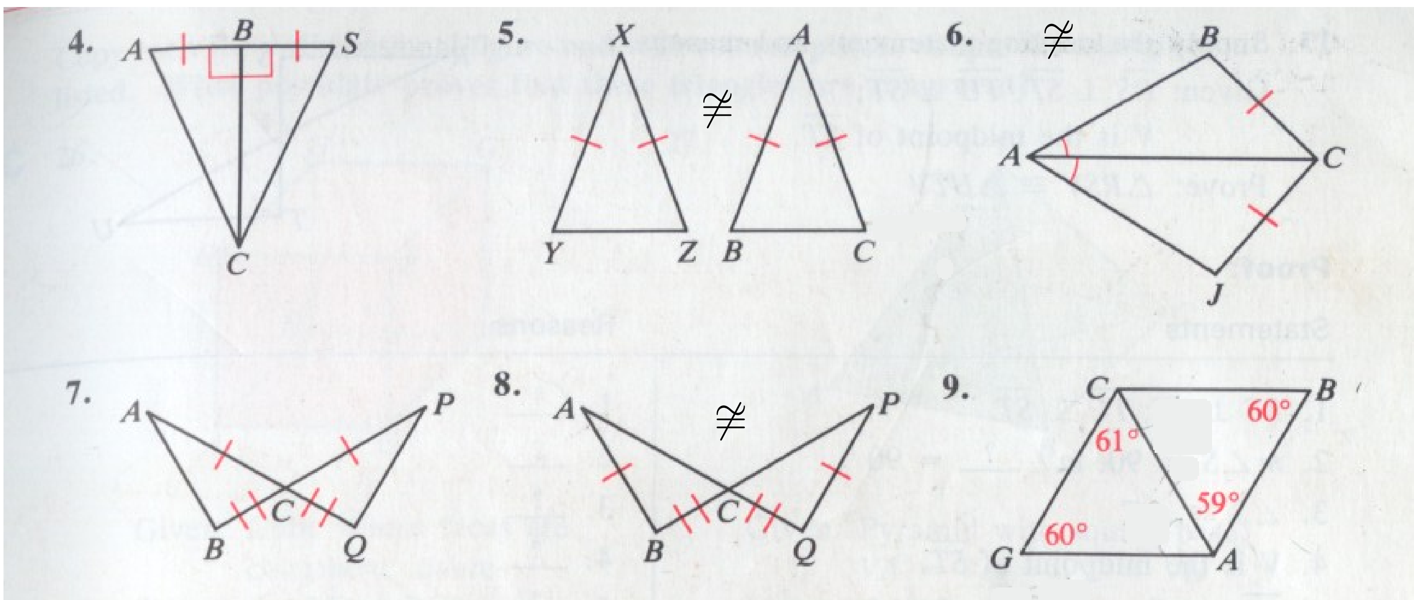


$\triangle ABC \cong \triangle ADC$(SAS)

$\triangle ABC \cong \triangle NPY$(ASA)

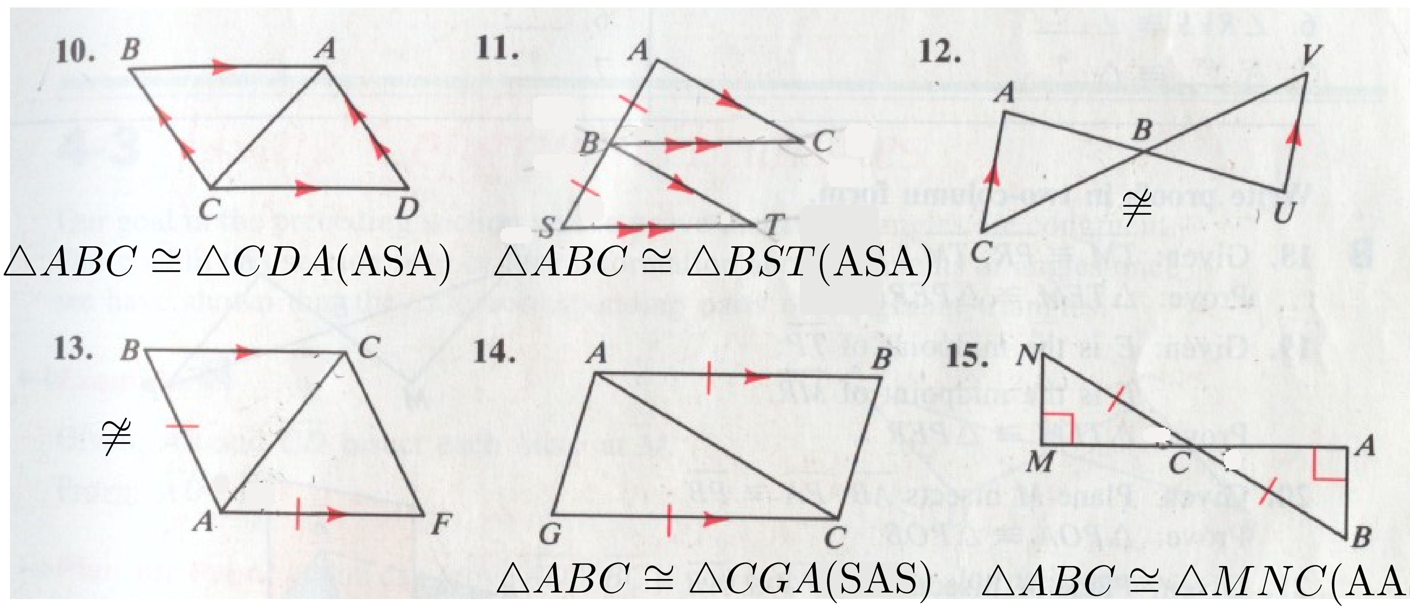
$\triangle ABC \cong \triangle CKA$(SSS)

$\triangle ABC \cong \triangle SBC$(SAS)



$\triangle ABC \cong \triangle PQC$(SAS)

$\triangle ABC \cong \triangle AGC$(ASA)



10. $\triangle ABC \cong \triangle CDA$(ASA)

11. $\triangle ABC \cong \triangle BST$(ASA)

12. $\ncong$

13. $\ncong$

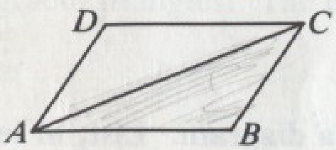
14. $\triangle ABC \cong \triangle CGA$(SAS)

15. $\triangle ABC \cong \triangle MNC$(AAS)

16. Supply the missing reasons.

Given: $\overline{AB} \parallel \overline{DC}$; $\overline{AB} \cong \overline{DC}$

Prove: $\triangle ABC \cong \triangle CDA$



Proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{DC}$	1. ?
2. $\overline{AC} \cong \overline{AC}$	2. ?
3. $\overline{AB} \parallel \overline{DC}$	3. ?
4. $\angle BAC \cong \angle DCA$	4. ?
5. $\triangle ABC \cong \triangle CDA$	5. ?