Law of Cosines

Fr Chris - St Francis HS

The proof requires 3 things:

The distance between (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

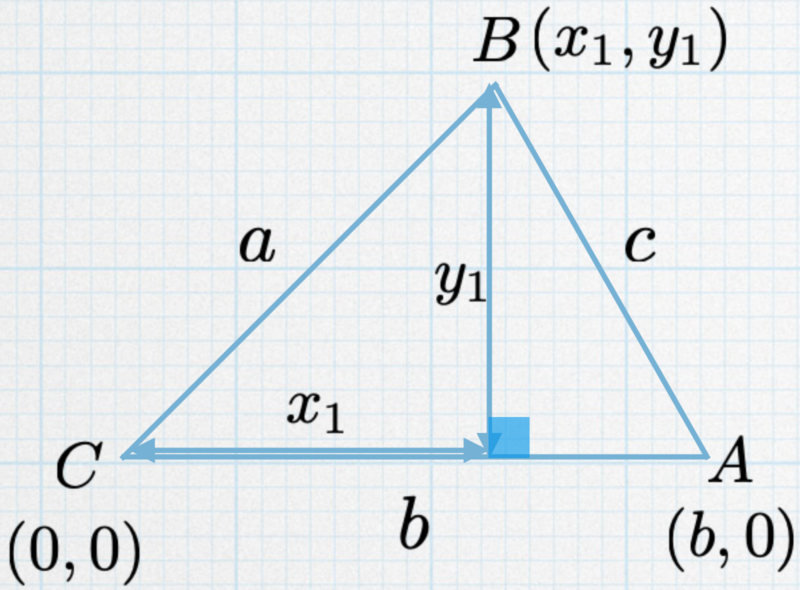
For any Right Triangle with sides $a, b,$ and c :

$$a^2 + b^2 = c^2$$

"SOH-CAH-TOA"

$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Proof



$$\cos C = \frac{x_1}{a} \qquad \sin C = \frac{y_1}{a}$$

$$x_1 = a\cos C \qquad y_1 = a\sin C$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$c = AB = \sqrt{(b - a\cos C)^2 + (0 - a\sin C)^2}$$

$$c^2 = (b - a\cos C)^2 + (0 - a\sin C)^2$$

$$c^2 = (b^2 - 2ab\cos C + (a\cos C)^2) + (a^2(\sin C)^2)$$

$$c^2 = (b^2 - 2ab\cos C + a^2(\cos C)^2) + (a^2(\sin C)^2)$$

$$c^2 = b^2 - 2ab\cos C + a^2\left[\left(\frac{x_1}{a}\right)^2 + \left(\frac{y_1}{a}\right)^2\right]$$

$$c^2 = b^2 - 2ab\cos C + a^2\left[\frac{x_1^2 + y_1^2}{a^2}\right]$$

$$c^2 = b^2 - 2ab\cos C + a^2\left[\frac{a^2}{a^2}\right] = a^2 + b^2 - 2ab\cos C$$