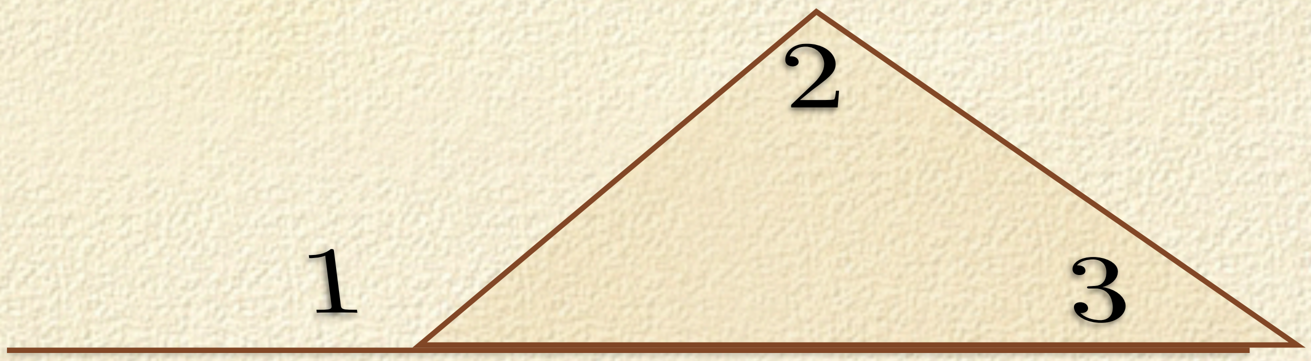
Chapter 6: Inequalities in Geometry

Fr Chris Thiel
St. Francis High School

Exterior Angle Theorem

Euclid's Elements, Book 1 Proposition 16

The exterior angle of a triangle is greater than either remote interior angle



$$\angle 1 > \angle 2$$

$$\angle 1 > \angle 3$$

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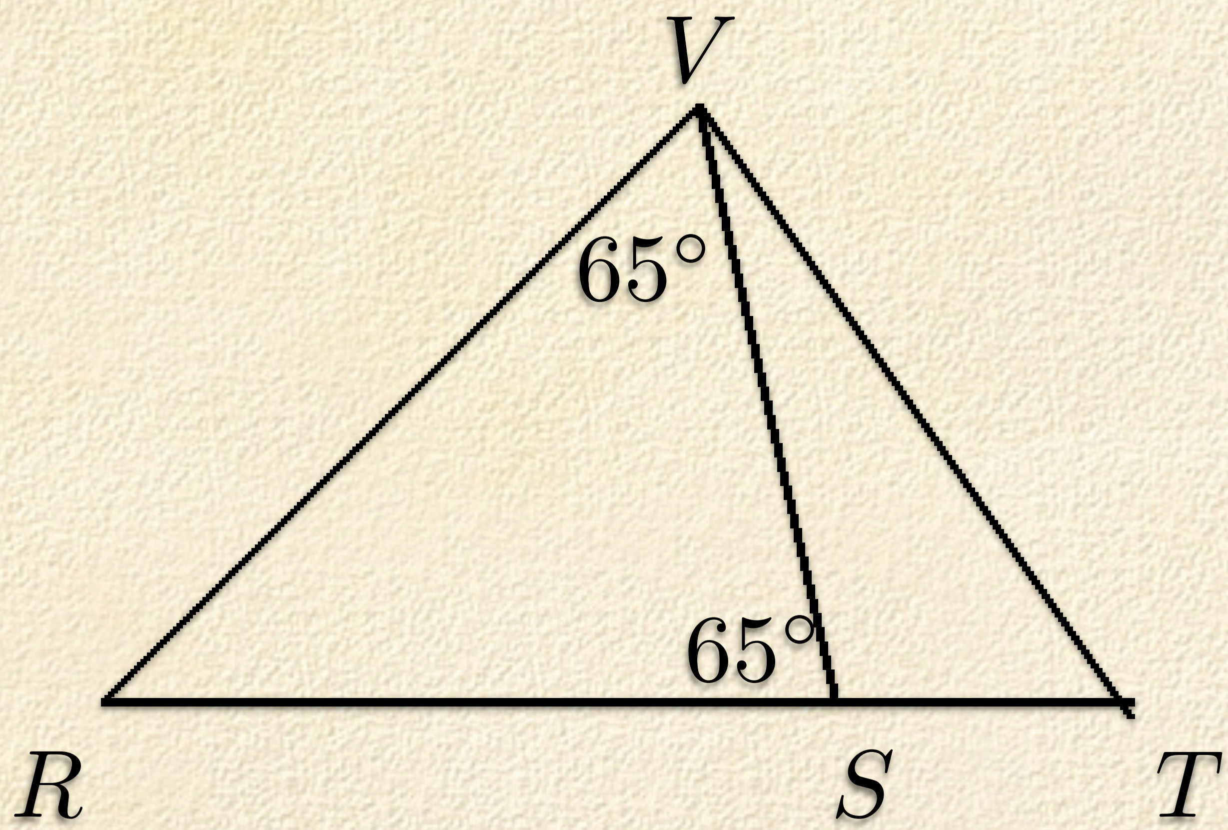


$RT > RS$?

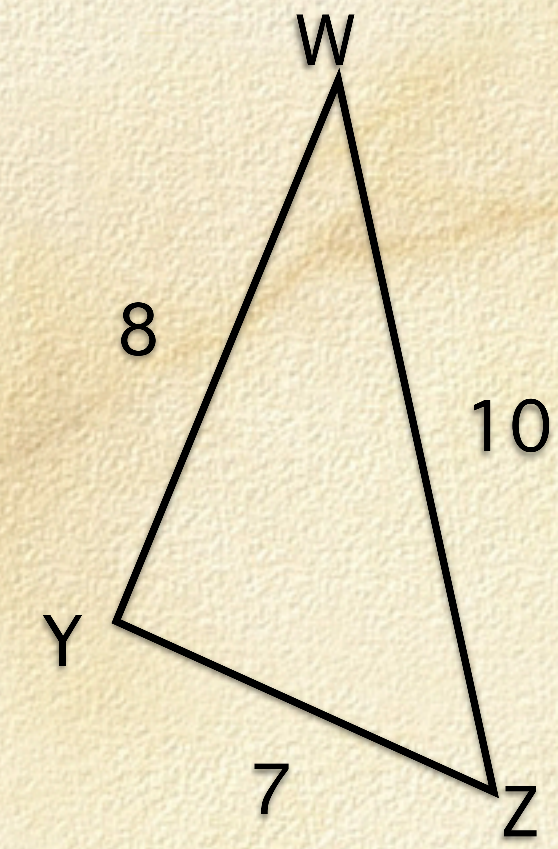
$RT > RV$?

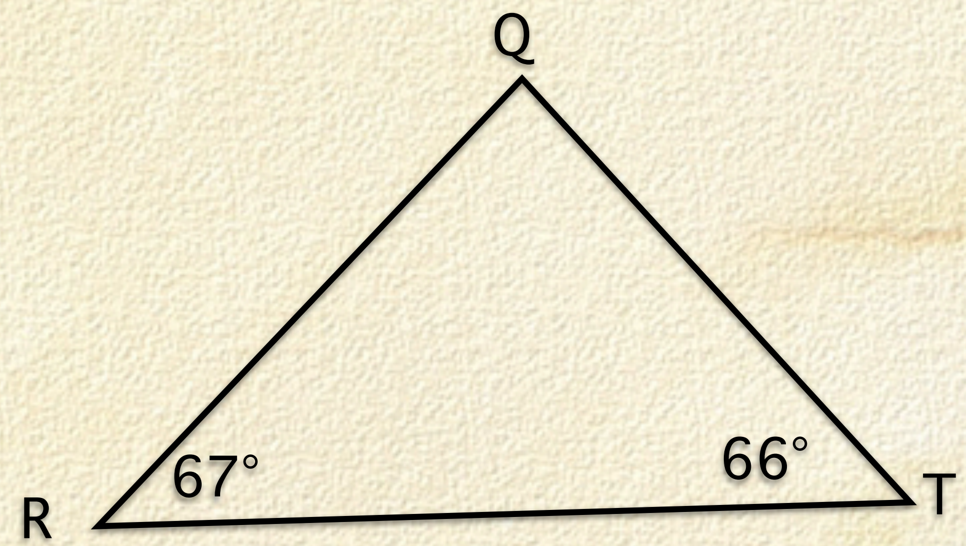
$RS > ST$?

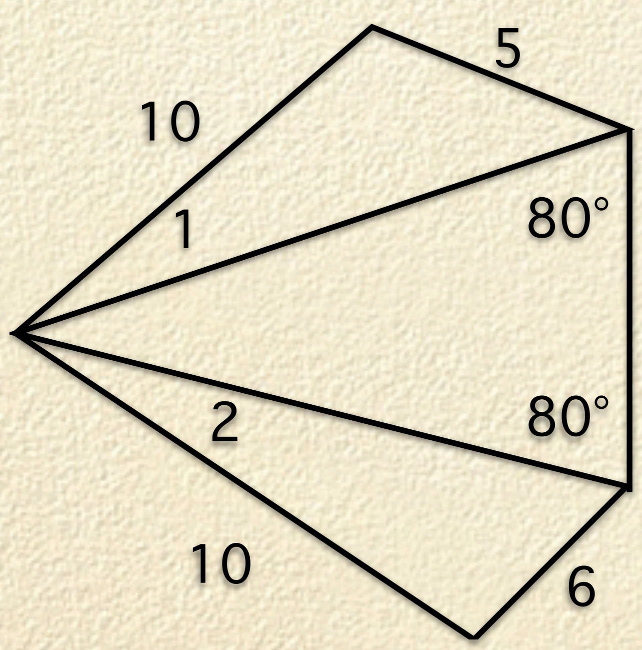
$VT < RS$?

Within a triangle...

- Sides opposite big angles are bigger (Book 1, proposition 18)
- Sides opposite small angles are smaller
- Angles opposite big sides are bigger (Book 1, proposition 19)
- Angles opposite small sides are smaller

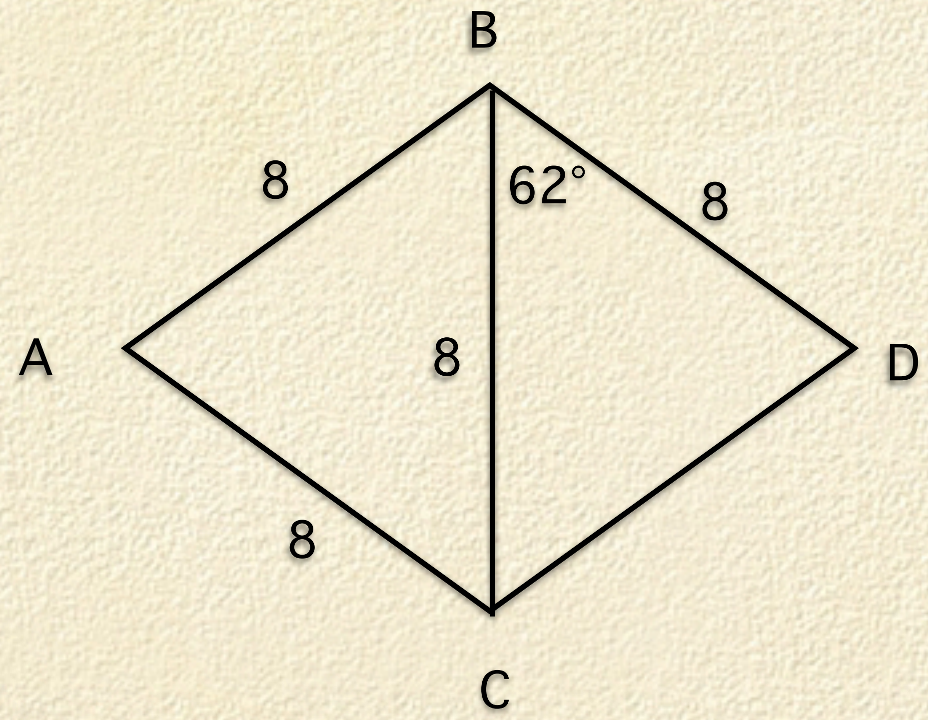

W
8
10
Y
7
Z


Q
67°
66°
R
T


5
10
1
80°
2
80°
10
6

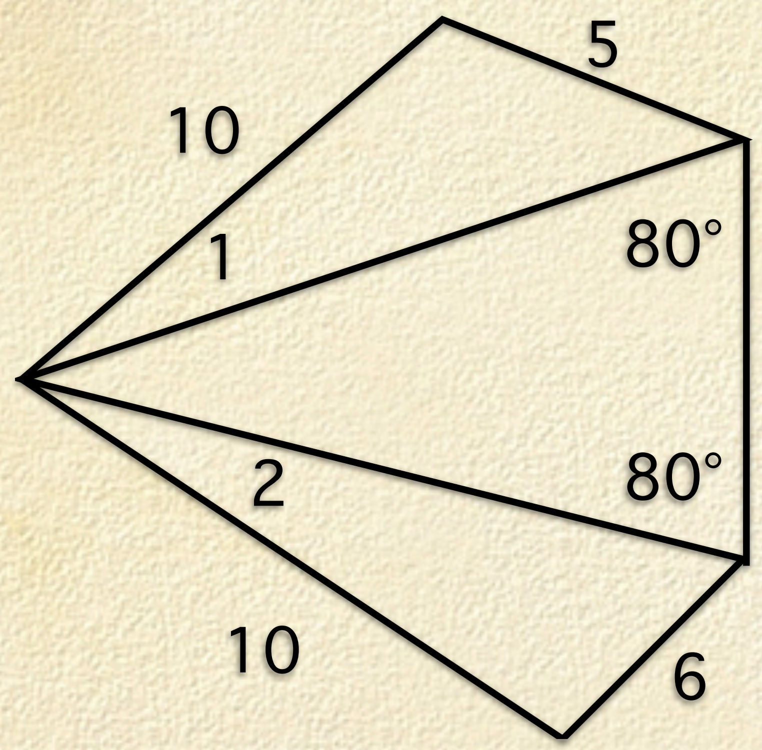
SAS Inequality Theorem

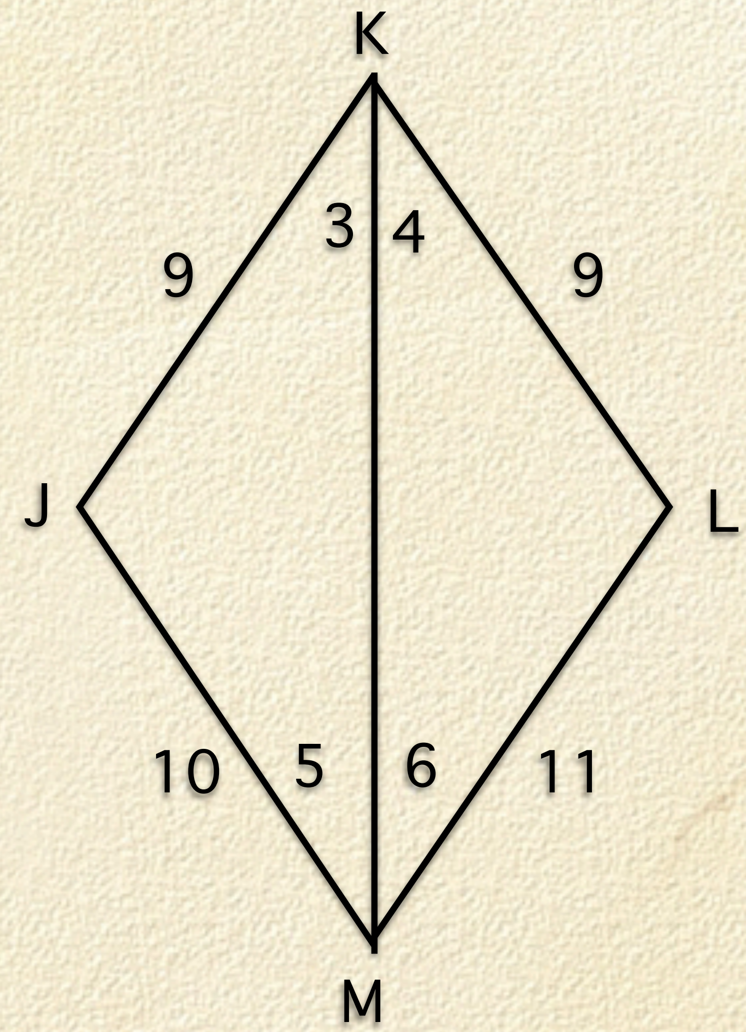
Book I, Proposition 24



SSS Inequality Theorem

Book I, Proposition 25





Indirect Proofs
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- Suppose the conclusion is not true
- Demonstrate that there is a contradiction
- Conclude the conclusion must be true

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If n^2 is an odd integer, then n is odd

1. Suppose n is not odd.
2. If n is not odd, then n is even
so there must be an integer k
where $n = 2k$

$$\text{so } n^2 = (2k)^2$$

$$\text{so } n^2 = 4k^2$$

but 2 is a factor of $4k^2$, so n^2 is even

but this contradicts our hypothesis that n is odd

3. Therefore n must be odd.

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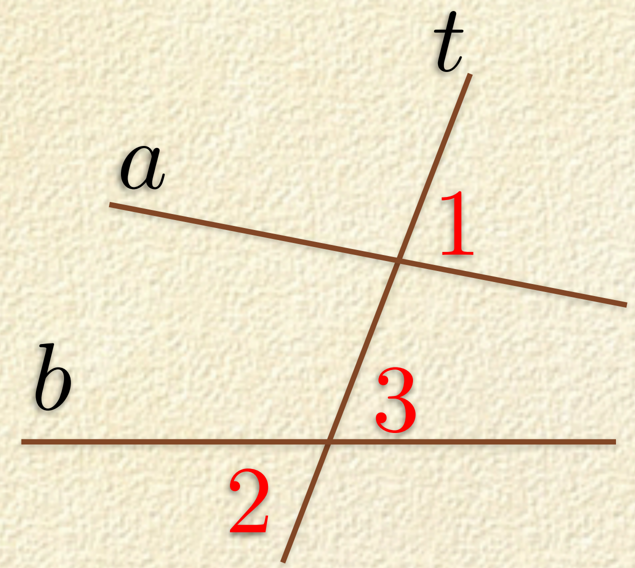


If $m\angle 1 \neq m\angle 2$, prove $a \nparallel b$

1. Suppose $a \parallel b$
2. Then $m\angle 1 = m\angle 3$

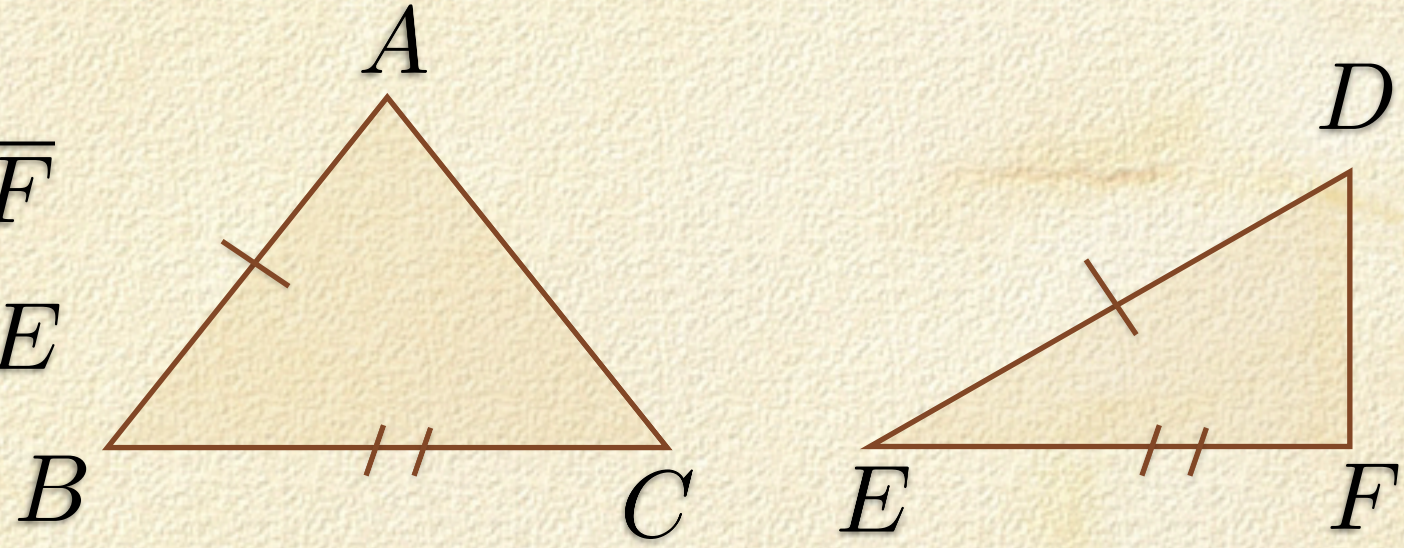
   $m\angle 3 = m\angle 2$

   $m\angle 1 = m\angle 2$ a contradiction!
3. Hence $a \nparallel b$

Now let's try this "indirect" way to prove the SSS Inequality Theorem

Given: $\overline{AC} > \overline{DF}$

Prove: $m\angle B > m\angle E$



1. Suppose $m\angle B \not> m\angle E$
2. Two cases:

   Case 1: If $\angle B \cong \angle E$, $\triangle ABC \cong \triangle DEF$ and $\overline{AC} \cong \overline{DF}$, a contradiction

   Case 2: If $m\angle B < m\angle E$, $AC < DF$ (by SAS Inequality Th), also a contradiction
3. Therefore $m\angle B > m\angle E$