CollegeBoard
Advanced Placement Program

AP® Calculus
Multiple-Choice Question Collection 1969–1998

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For more information about equity and access in principle and practice, please send an email to apequity@collegeboard.org.

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About This Collection

Multiple-choice questions from past AP Calculus Exams provide a rich resource for teaching topics in the course and reviewing for the exam each year. Over the years, some topics have been added or removed, but almost all of the old questions still offer interesting opportunities to investigate concepts and assess student understanding. Always consult the most recent Course Description on AP Central® for the current topic outlines for Calculus AB and Calculus BC.

Please note the following:

- The solution to each multiple-choice question suggests one possible way to solve that question. There are often alternative approaches that produce the same choice of answer, and for some questions such multiple approaches are provided. Teachers are also encouraged to investigate how the incorrect options for each question could be obtained to help students understand (and avoid) common types of mistakes.
- Scientific (nongraphing) calculators were required on the AP Calculus Exams in 1993.
- Graphing calculators have been required on the AP Calculus Exams since 1995. In 1997 and 1998, Section I, Part A did not allow the use of a calculator; Section I, Part B required the use of a graphing calculator.
- Materials included in this resource may not reflect the current AP Course Description and exam in this subject, and teachers are advised to take this into account as they use these materials to support their instruction of students. For up-to-date information about this AP course and exam, please download the official AP Course Description from the AP Central Web site at apcentral.collegeboard.com.

1969 AP Calculus AB: Section I

90 Minutes—No Calculator

Note: In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

1. Which of the following defines a function f for which $f(-x) = -f(x)$?

(A) $f(x) = x^2$ (B) $f(x) = \sin x$ (C) $f(x) = \cos x$

(D) $f(x) = \log x$ (E) $f(x) = e^x$

---

2. $\ln(x-2) < 0$ if and only if

(A) $x < 3$ (B) $0 < x < 3$ (C) $2 < x < 3$

(D) $x > 2$ (E) $x > 3$

---

3. If $\begin{cases} f(x) = \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & \text{for } x \neq 2, \\ f(2) = k \end{cases}$ and if f is continuous at $x = 2$, then $k =$

(A) 0 (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{3}$ (D) 1 (E) $\dfrac{7}{5}$

---

4. $\displaystyle\int_0^8 \frac{dx}{\sqrt{1+x}} =$

(A) 1 (B) $\dfrac{3}{2}$ (C) 2 (D) 4 (E) 6

---

5. If $3x^2 + 2xy + y^2 = 2$, then the value of $\dfrac{dy}{dx}$ at $x = 1$ is

(A) –2 (B) 0 (C) 2 (D) 4 (E) not defined

6. What is $\lim_{h\to 0} \dfrac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist.

(E) It cannot be determined from the information given.

---

7. For what value of *k* will $x+\dfrac{k}{x}$ have a relative maximum at $x=-2$?

(A) –4 (B) –2 (C) 2 (D) 4 (E) None of these

---

8. If $p(x)=(x+2)(x+k)$ and if the remainder is 12 when $p(x)$ is divided by $x-1$, then $k=$

(A) 2 (B) 3 (C) 6 (D) 11 (E) 13

---

9. When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is

(A) $\frac{1}{4\pi}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\pi}$ (D) 1 (E) π

---

10. The set of all points (e^t, t), where *t* is a real number, is the graph of $y=$

(A) $\frac{1}{e^x}$ (B) $e^{\frac{1}{x}}$ (C) $x e^{\frac{1}{x}}$ (D) $\frac{1}{\ln x}$ (E) $\ln x$

---

11. The point <u>on the curve</u> $x^2+2y=0$ that is nearest the point $\left(0,-\frac{1}{2}\right)$ occurs where *y* is

(A) $\frac{1}{2}$ (B) 0 (C) $-\frac{1}{2}$ (D) -1 (E) none of the above

12. If $f(x) = \dfrac{4}{x-1}$ and $g(x) = 2x$, then the solution set of $f\left(g(x)\right) = g\left(f(x)\right)$ is

(A) $\left\{\frac{1}{3}\right\}$ (B) $\{2\}$ (C) $\{3\}$ (D) $\{-1, 2\}$ (E) $\left\{\frac{1}{3}, 2\right\}$

---

13. The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \frac{\pi}{2}$, then $k =$

(A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

---

14. If the function f is defined by $f(x) = x^5 - 1$, then f^{-1}, the inverse function of f, is defined by $f^{-1}(x) =$

(A) $\dfrac{1}{\sqrt[5]{x}+1}$ (B) $\dfrac{1}{\sqrt[5]{x+1}}$ (C) $\sqrt[5]{x-1}$

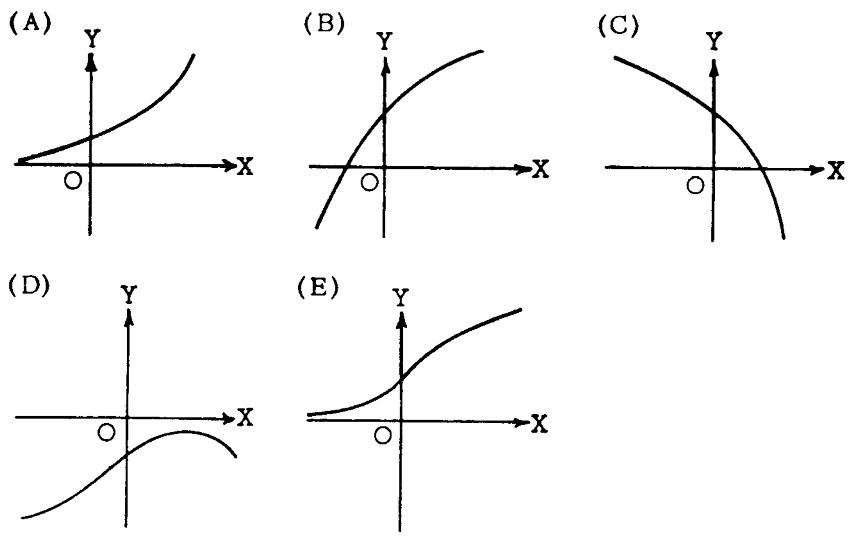
(D) $\sqrt[5]{x}-1$ (E) $\sqrt[5]{x+1}$

---

15. If $f'(x)$ and $g'(x)$ exist and $f'(x) > g'(x)$ for all real x, then the graph of $y = f(x)$ and the graph of $y = g(x)$

(A) intersect exactly once.
(B) intersect no more than once.
(C) do not intersect.
(D) could intersect more than once.
(E) have a common tangent at each point of intersection.

16. If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



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17. The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) $(0,0)$ only (B) $(3,162)$ only (C) $(4,256)$ only

(D) $(0,0)$ and $(3,162)$ (E) $(0,0)$ and $(4,256)$

---

18. If $f(x) = 2 + |x - 3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

(A) -1 (B) 0 (C) 1 (D) 2 (E) nonexistent

---

19. A point moves on the x-axis in such a way that its velocity at time t $(t > 0)$ is given by $v = \frac{\ln t}{t}$. At what value of t does v attain its maximum?

(A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for v.

20. An equation for a tangent to the graph of $y = \arcsin \frac{x}{2}$ at the origin is

(A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$ (D) $y = 0$ (E) $\pi x - 2y = 0$

---

21. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

---

22. $\frac{d}{dx}\left(\ln e^{2x}\right) =$

(A) $\frac{1}{e^{2x}}$ (B) $\frac{2}{e^{2x}}$ (C) $2x$ (D) 1 (E) 2

---

23. The area of the region bounded by the curve $y = e^{2x}$, the x-axis, the y-axis, and the line $x = 2$ is equal to

(A) $\frac{e^4}{2} - e$ (B) $\frac{e^4}{2} - 1$ (C) $\frac{e^4}{2} - \frac{1}{2}$

(D) $2e^4 - e$ (E) $2e^4 - 2$

---

24. If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x ?

(A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

25. A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

(A) is independent of m.
(B) increases as m increases.
(C) decreases as m increases.
(D) decreases as m increases when $m < \frac{1}{2}$; increases as m increases when $m > \frac{1}{2}$.
(E) increases as m increases when $m < \frac{1}{2}$; decreases as m increases when $m > \frac{1}{2}$.

---

26. $\int_0^1 \sqrt{x^2 - 2x + 1}\, dx$ is

(A) -1
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 1
(E) none of the above

---

27. If $\frac{dy}{dx} = \tan x$, then $y =$

(A) $\frac{1}{2}\tan^2 x + C$ (B) $\sec^2 x + C$ (C) $\ln|\sec x| + C$
(D) $\ln|\cos x| + C$ (E) $\sec x \tan x + C$

---

28. The function defined by $f(x) = \sqrt{3}\cos x + 3\sin x$ has an amplitude of

(A) $3 - \sqrt{3}$ (B) $\sqrt{3}$ (C) $2\sqrt{3}$ (D) $3 + \sqrt{3}$ (E) $3\sqrt{3}$

29. $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x}\, dx =$

(A) $\ln\sqrt{2}$ (B) $\ln\frac{\pi}{4}$ (C) $\ln\sqrt{3}$ (D) $\ln\frac{\sqrt{3}}{2}$ (E) $\ln e$

---

30. If a function f is continuous for all x and if f has a relative maximum at $(-1,4)$ and a relative minimum at $(3,-2)$, which of the following statements must be true?

(A) The graph of f has a point of inflection somewhere between $x=-1$ and $x=3$.
(B) $f'(-1)=0$
(C) The graph of f has a horizontal asymptote.
(D) The graph of f has a horizontal tangent line at $x=3$.
(E) The graph of f intersects both axes.

---

31. If $f'(x)=-f(x)$ and $f(1)=1$, then $f(x)=$

(A) $\frac{1}{2}e^{-2x+2}$ (B) e^{-x-1} (C) e^{1-x} (D) e^{-x} (E) $-e^{x}$

---

32. If $a,b,c,d,$ and e are real numbers and $a\neq 0$, then the polynomial equation $ax^7+bx^5+cx^3+dx+e=0$ has

(A) only one real root.
(B) at least one real root.
(C) an odd number of nonreal roots.
(D) no real roots.
(E) no positive real roots.

---

33. What is the average (mean) value of $3t^3-t^2$ over the interval $-1\le t\le 2$?

(A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

34. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = \frac{1}{x} + k$ (where k takes all real values)?

(A) $y = -x$ (B) $y = -x^2$ (C) $y = -\frac{1}{3}x^3$ (D) $y = \frac{1}{3}x^3$ (E) $y = \ln x$

---

35. At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

(A) 32 (B) 48 (C) 64 (D) 96 (E) 192

---

36. The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

---

37. Which is the best of the following polynomial approximations to $\cos 2x$ near $x = 0$?

(A) $1 + \frac{x}{2}$ (B) $1 + x$ (C) $1 - \frac{x^2}{2}$ (D) $1 - 2x^2$ (E) $1 - 2x + x^2$

---

38. $\int \frac{x^2}{e^{x^3}}\, dx =$

(A) $-\frac{1}{3}\ln e^{x^3} + C$ (B) $-\frac{e^{x^3}}{3} + C$ (C) $-\frac{1}{3e^{x^3}} + C$

(D) $\frac{1}{3}\ln e^{x^3} + C$ (E) $\frac{x^3}{3e^{x^3}} + C$

---

39. If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $\frac{2}{e}$ (E) $\sec^2 e$

40. If n is a non-negative integer, then $\int_0^1 x^n\,dx = \int_0^1 (1-x)^n\,dx$ for

(A) no n (B) n even, only (C) n odd, only
(D) nonzero n, only (E) all n

---

41. If $\begin{cases} f(x) = 8 - x^2 & \text{for } -2 \le x \le 2, \\ f(x) = x^2 & \text{elsewhere}, \end{cases}$ then $\int_{-1}^{3} f(x)\,dx$ is a number between

(A) 0 and 8 (B) 8 and 16 (C) 16 and 24 (D) 24 and 32 (E) 32 and 40

---

42. What are all values of k for which the graph of $y = x^3 - 3x^2 + k$ will have three distinct x-intercepts?

(A) All $k > 0$ (B) All $k < 4$ (C) $k = 0, 4$ (D) $0 < k < 4$ (E) All k

---

43. $\int \sin(2x+3)\,dx =$

(A) $\frac{1}{2}\cos(2x+3) + C$ (B) $\cos(2x+3) + C$ (C) $-\cos(2x+3) + C$

(D) $-\frac{1}{2}\cos(2x+3) + C$ (E) $-\frac{1}{5}\cos(2x+3) + C$

---

44. The fundamental period of the function defined by $f(x) = 3 - 2\cos^2 \frac{\pi x}{3}$ is

(A) 1 (B) 2 (C) 3 (D) 5 (E) 6

---

45. If $\frac{d}{dx}\left(f(x)\right) = g(x)$ and $\frac{d}{dx}\left(g(x)\right) = f(x^2)$, then $\frac{d^2}{dx^2}\left(f(x^3)\right) =$

(A) $f\left(x^6\right)$ (B) $g\left(x^3\right)$ (C) $3x^2 g\left(x^3\right)$

(D) $9x^4 f\left(x^6\right) + 6x\,g\left(x^3\right)$ (E) $f\left(x^6\right) + g\left(x^3\right)$

90 Minutes—No Calculator

Note: In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

1. The asymptotes of the graph of the parametric equations $x = \frac{1}{t},\ y = \frac{t}{t+1}$ are

(A) $x = 0,\ y = 0$ (B) $x = 0$ only (C) $x = -1,\ y = 0$
(D) $x = -1$ only (E) $x = 0,\ y = 1$

---

2. What are the coordinates of the inflection point on the graph of $y = (x+1)\arctan x$?

(A) $(-1,0)$ (B) $(0,0)$ (C) $(0,1)$ (D) $\left(1,\frac{\pi}{4}\right)$ (E) $\left(1,\frac{\pi}{2}\right)$

---

3. The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between $(0,0)$ and $(4,2)$. What are the coordinates of this point?

(A) $(2,1)$
(B) $(1,1)$
(C) $\left(2,\sqrt{2}\right)$
(D) $\left(\frac{1}{2},\frac{1}{\sqrt{2}}\right)$
(E) None of the above

---

4. $\int_0^8 \frac{dx}{\sqrt{1+x}} =$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

---

5. If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

(A) –2 (B) 0 (C) 2 (D) 4 (E) not defined

6. What is $\lim_{h\to 0} \dfrac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) The limit does not exist.

(E) It cannot be determined from the information given.

---

7. For what value of k will $x+\dfrac{k}{x}$ have a relative maximum at $x=-2$?

(A) –4 (B) –2 (C) 2 (D) 4 (E) None of these

---

8. If $h(x) = f^2(x) - g^2(x)$, $f'(x) = -g(x)$, and $g'(x) = f(x)$, then $h'(x) =$

(A) 0 (B) 1 (C) $-4f(x)g(x)$

(D) $\left(-g(x)\right)^2 - \left(f(x)\right)^2$ (E) $-2\left(-g(x)+f(x)\right)$

---

9. The area of the closed region bounded by the polar graph of $r=\sqrt{3+\cos\theta}$ is given by the integral

(A) $\int_0^{2\pi} \sqrt{3+\cos\theta}\, d\theta$ (B) $\int_0^{\pi} \sqrt{3+\cos\theta}\, d\theta$ (C) $2\int_0^{\pi/2} \left(3+\cos\theta\right) d\theta$

(D) $\int_0^{\pi} \left(3+\cos\theta\right) d\theta$ (E) $2\int_0^{\pi/2} \sqrt{3+\cos\theta}\, d\theta$

---

10. $\int_0^1 \dfrac{x^2}{x^2+1}\, dx =$

(A) $\dfrac{4-\pi}{4}$ (B) $\ln 2$ (C) 0 (D) $\dfrac{1}{2}\ln 2$ (E) $\dfrac{4+\pi}{4}$

11. The point on the curve $x^2 + 2y = 0$ that is nearest the point $\left(0, -\frac{1}{2}\right)$ occurs where y is

(A) $\frac{1}{2}$

(B) 0

(C) $-\frac{1}{2}$

(D) -1

(E) none of the above

---

12. If $F(x) = \int_0^x e^{-t^2}\,dt$, then $F'(x) =$

(A) $2xe^{-x^2}$ (B) $-2xe^{-x^2}$ (C) $\dfrac{e^{-x^2+1}}{-x^2+1} - e$

(D) $e^{-x^2} - 1$ (E) e^{-x^2}

---

13. The region bounded by the x-axis and the part of the graph of $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \frac{\pi}{2}$, then $k =$

(A) $\arcsin\left(\frac{1}{4}\right)$ (B) $\arcsin\left(\frac{1}{3}\right)$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{4}$ (E) $\frac{\pi}{3}$

---

14. If $y = x^2 + 2$ and $u = 2x - 1$, then $\frac{dy}{du} =$

(A) $\dfrac{2x^2 - 2x + 4}{(2x-1)^2}$ (B) $6x^2 - 2x + 4$ (C) x^2

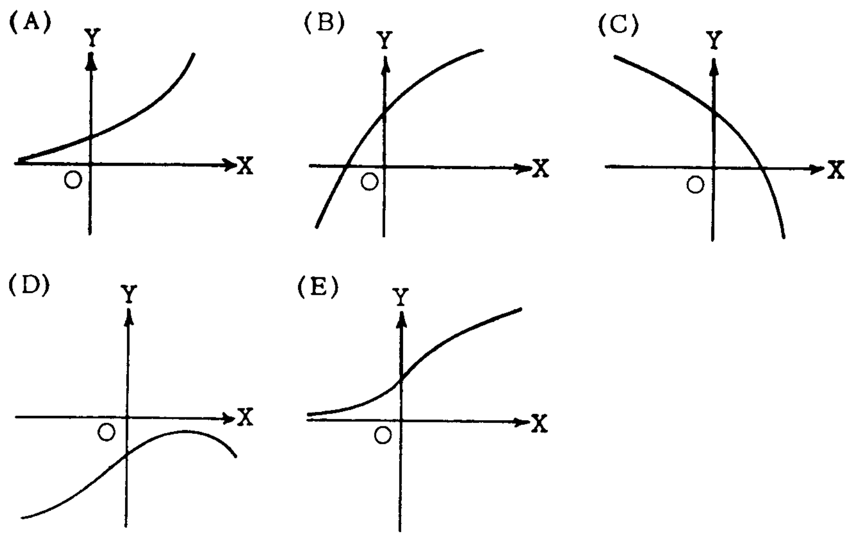
(D) x (E) $\dfrac{1}{x}$

15. If $f'(x)$ and $g'(x)$ exist and $f'(x) > g'(x)$ for all real x, then the graph of $y = f(x)$ and the graph of $y = g(x)$

(A) intersect exactly once.
(B) intersect no more than once.
(C) do not intersect.
(D) could intersect more than once.
(E) have a common tangent at each point of intersection.

---

16. If y is a function x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



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17. The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) $(0,0)$ only
(B) $(3,162)$ only
(C) $(4,256)$ only
(D) $(0,0)$ and $(3,162)$
(E) $(0,0)$ and $(4,256)$

---

18. If $f(x) = 2 + |x - 3|$ for all x, then the value of the derivative $f'(x)$ at $x = 3$ is

(A) -1
(B) 0
(C) 1
(D) 2
(E) nonexistent

19. A point moves on the x-axis in such a way that its velocity at time t $(t > 0)$ is given by $v = \frac{\ln t}{t}$. At what value of t does v attain its maximum?

(A) 1 (B) $e^{\frac{1}{2}}$ (C) e (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for v.

---

20. An equation for a tangent to the graph of $y = \arcsin \frac{x}{2}$ at the origin is

(A) $x - 2y = 0$ (B) $x - y = 0$ (C) $x = 0$

(D) $y = 0$ (E) $\pi x - 2y = 0$

---

21. At $x = 0$, which of the following is true of the function f defined by $f(x) = x^2 + e^{-2x}$?

(A) f is increasing.
(B) f is decreasing.
(C) f is discontinuous.
(D) f has a relative minimum.
(E) f has a relative maximum.

---

22. If $f(x) = \int_0^x \frac{1}{\sqrt{t^3 + 2}}\,dt$, which of the following is FALSE?

(A) $f(0) = 0$

(B) f is continuous at x for all $x \geq 0$.

(C) $f(1) > 0$

(D) $f'(1) = \frac{1}{\sqrt{3}}$

(E) $f(-1) > 0$

23. If the graph of $y = f(x)$ contains the point $(0, 2)$, $\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$ and $f(x) > 0$ for all x, then $f(x) =$

(A) $3+e^{-x^2}$
(B) $\sqrt{3} + e^{-x}$
(C) $1+e^{-x}$
(D) $\sqrt{3+e^{-x^2}}$
(E) $\sqrt{3+e^{x^2}}$

---

24. If $\sin x = e^y$, $0 < x < \pi$, what is $\frac{dy}{dx}$ in terms of x ?

(A) $-\tan x$
(B) $-\cot x$
(C) $\cot x$
(D) $\tan x$
(E) $\csc x$

---

25. A region in the plane is bounded by the graph of $y = \frac{1}{x}$, the x-axis, the line $x = m$, and the line $x = 2m$, $m > 0$. The area of this region

(A) is independent of m .
(B) increases as m increases.
(C) decreases as m increases.
(D) decreases as m increases when $m < \frac{1}{2}$; increases as m increases when $m > \frac{1}{2}$.
(E) increases as m increases when $m < \frac{1}{2}$; decreases as m increases when $m > \frac{1}{2}$.

---

26. $\int_0^1 \sqrt{x^2 - 2x + 1}\, dx$ is

(A) -1
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 1
(E) none of the above

27. If $\frac{dy}{dx} = \tan x$, then $y =$

(A) $\frac{1}{2}\tan^2 x + C$ (B) $\sec^2 x + C$ (C) $\ln|\sec x| + C$
(D) $\ln|\cos x| + C$ (E) $\sec x \tan x + C$

---

28. What is $\lim_{x\to 0} \frac{e^{2x}-1}{\tan x}$?

(A) -1 (B) 0 (C) 1 (D) 2 (E) The limit does not exist.

---

29. $\int_0^1 \left(4 - x^2\right)^{-\frac{3}{2}} dx =$

(A) $\frac{2-\sqrt{3}}{3}$ (B) $\frac{2\sqrt{3}-3}{4}$ (C) $\frac{\sqrt{3}}{12}$ (D) $\frac{\sqrt{3}}{3}$ (E) $\frac{\sqrt{3}}{2}$

---

30. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ is the Taylor series about zero for which of the following functions?

(A) $\sin x$ (B) $\cos x$ (C) e^x (D) e^{-x} (E) $\ln(1+x)$

---

31. If $f'(x) = -f(x)$ and $f(1) = 1$, then $f(x) =$

(A) $\frac{1}{2}e^{-2x+2}$ (B) e^{-x-1} (C) e^{1-x} (D) e^{-x} (E) $-e^x$

---

32. For what values of x does the series $1 + 2^x + 3^x + 4^x + \cdots + n^x + \cdots$ converge?

(A) No values of x (B) $x < -1$ (C) $x \geq -1$ (D) $x > -1$ (E) All values of x

---

33. What is the average (mean) value of $3t^3 - t^2$ over the interval $-1 \leq t \leq 2$?

(A) $\frac{11}{4}$ (B) $\frac{7}{2}$ (C) 8 (D) $\frac{33}{4}$ (E) 16

34. Which of the following is an equation of a curve that intersects at right angles every curve of the family $y = \frac{1}{x} + k$ (where k takes all real values)?

(A) $y = -x$ (B) $y = -x^2$ (C) $y = -\frac{1}{3}x^3$ (D) $y = \frac{1}{3}x^3$ (E) $y = \ln x$

---

35. At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

(A) 32 (B) 48 (C) 64 (D) 96 (E) 192

---

36. The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is

(A) 2.00 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24

---

37. Of the following choices of δ, which is the largest that could be used successfully with an arbitrary ε in an epsilon-delta proof of $\lim_{x \to 2} (1 - 3x) = -5$?

(A) $\delta = 3\varepsilon$ (B) $\delta = \varepsilon$ (C) $\delta = \frac{\varepsilon}{2}$ (D) $\delta = \frac{\varepsilon}{4}$ (E) $\delta = \frac{\varepsilon}{5}$

---

38. If $f(x) = (x^2 + 1)^{(2-3x)}$, then $f'(1) =$

(A) $-\frac{1}{2}\ln(8e)$ (B) $-\ln(8e)$ (C) $-\frac{3}{2}\ln(2)$ (D) $-\frac{1}{2}$ (E) $\frac{1}{8}$

---

39. If $y = \tan u$, $u = v - \frac{1}{v}$, and $v = \ln x$, what is the value of $\frac{dy}{dx}$ at $x = e$?

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) $\frac{2}{e}$ (E) $\sec^2 e$

40. If n is a non-negative integer, then $\int_0^1 x^n\,dx = \int_0^1 (1-x)^n\,dx$ for

(A) no n
(B) n even, only
(C) n odd, only
(D) nonzero n, only
(E) all n

---

41. If $\begin{cases} f(x) = 8 - x^2 & \text{for } -2 \le x \le 2, \\ f(x) = x^2 & \text{elsewhere}, \end{cases}$ then $\int_{-1}^{3} f(x)\,dx$ is a number between

(A) 0 and 8
(B) 8 and 16
(C) 16 and 24
(D) 24 and 32
(E) 32 and 40

---

42. If $\int x^2 \cos x\,dx = f(x) - \int 2x \sin x\,dx$, then $f(x) =$

(A) $2\sin x + 2x\cos x + C$

(B) $x^2 \sin x + C$

(C) $2x\cos x - x^2 \sin x + C$

(D) $4\cos x - 2x\sin x + C$

(E) $\left(2 - x^2\right)\cos x - 4\sin x + C$

---

43. Which of the following integrals gives the length of the graph of $y = \tan x$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$?

(A) $\int_a^b \sqrt{x^2 + \tan^2 x}\,dx$

(B) $\int_a^b \sqrt{x + \tan x}\,dx$

(C) $\int_a^b \sqrt{1 + \sec^2 x}\,dx$

(D) $\int_a^b \sqrt{1 + \tan^2 x}\,dx$

(E) $\int_a^b \sqrt{1 + \sec^4 x}\,dx$

44. If $f''(x) - f'(x) - 2f(x) = 0$, $f'(0) = -2$, and $f(0) = 2$, then $f(1) =$

(A) $e^2 + e^{-1}$ (B) 1 C) 0 (D) e^2 (E) $2e^{-1}$

---

45. The complete interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x+1)^k}{k^2}$ is

(A) $0 < x < 2$ (B) $0 \le x \le 2$ (C) $-2 < x \le 0$

(D) $-2 \le x < 0$ (E) $-2 \le x \le 0$

90 Minutes—No Calculator

Note: In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

1. $\int \left(x^3 - 3x\right) dx =$

(A) $3x^2 - 3 + C$ (B) $4x^4 - 6x^2 + C$ (C) $\frac{x^4}{3} - 3x^2 + C$

(D) $\frac{x^4}{4} - 3x + C$ (E) $\frac{x^4}{4} - \frac{3x^2}{2} + C$

---

2. If $f(x) = x^3 + 3x^2 + 4x + 5$ and $g(x) = 5$, then $g\left(f(x)\right) =$

(A) $5x^2 + 15x + 25$ (B) $5x^3 + 15x^2 + 20x + 25$ (C) 1125

(D) 225 (E) 5

---

3. The slope of the line tangent to the graph of $y = \ln\left(x^2\right)$ at $x = e^2$ is

(A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$

---

4. If $f(x) = x + \sin x$, then $f'(x) =$

(A) $1 + \cos x$ (B) $1 - \cos x$ (C) $\cos x$

(D) $\sin x - x\cos x$ (E) $\sin x + x\cos x$

---

5. If $f(x) = e^x$, which of the following lines is an asymptote to the graph of f?

(A) $y = 0$ (B) $x = 0$ (C) $y = x$ (D) $y = -x$ (E) $y = 1$

---

6. If $f(x) = \frac{x-1}{x+1}$ for all $x \neq -1$, then $f'(1) =$

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

7. Which of the following equations has a graph that is symmetric with respect to the origin?

(A) $y = \frac{x+1}{x}$ (B) $y = -x^5 + 3x$ (C) $y = x^4 - 2x^2 + 6$

(D) $y = (x-1)^3 + 1$ (E) $y = \left(x^2 + 1\right)^2 - 1$

---

8. A particle moves in a straight line with velocity $v(t) = t^2$. How far does the particle move between times $t = 1$ and $t = 2$?

(A) $\frac{1}{3}$ (B) $\frac{7}{3}$ (C) 3 (D) 7 (E) 8

---

9. If $y = \cos^2 3x$, then $\frac{dy}{dx} =$

(A) $-6\sin 3x \cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$

(D) $6\cos 3x$ (E) $2\sin 3x \cos 3x$

---

10. The *derivative* of $f(x) = \frac{x^4}{3} - \frac{x^5}{5}$ attains its maximum value at $x =$

(A) –1 (B) 0 (C) 1 (D) $\frac{4}{3}$ (E) $\frac{5}{3}$

---

11. If the line $3x - 4y = 0$ is tangent in the first quadrant to the curve $y = x^3 + k$, then k is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 0 (D) $-\frac{1}{8}$ (E) $-\frac{1}{2}$

---

12. If $f(x) = 2x^3 + Ax^2 + Bx - 5$ and if $f(2) = 3$ and $f(-2) = -37$, what is the value of $A + B$?

(A) –6 (B) –3 (C) –1 (D) 2

(E) It cannot be determined from the information given.

13. The acceleration α of a body moving in a straight line is given in terms of time t by $\alpha = 8 - 6t$. If the velocity of the body is 25 at $t = 1$ and if $s(t)$ is the distance of the body from the origin at time t, what is $s(4) - s(2)$?

(A) 20 (B) 24 (C) 28 (D) 32 (E) 42

---

14. If $f(x) = x^{\frac{1}{3}}(x-2)^{\frac{2}{3}}$ for all x, then the domain of f' is

(A) $\{x \mid x \neq 0\}$
(B) $\{x \mid x > 0\}$
(C) $\{x \mid 0 \leq x \leq 2\}$
(D) $\{x \mid x \neq 0 \text{ and } x \neq 2\}$
(E) $\{x \mid x \text{ is a real number}\}$

---

15. The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{\frac{x}{2}}$ is

(A) $\frac{e-1}{2}$ (B) $e - 1$ (C) $2(e-1)$ (D) $2e - 1$ (E) $2e$

---

16. The number of bacteria in a culture is growing at a rate of $3000e^{\frac{2t}{5}}$ per unit of time t. At $t = 0$, the number of bacteria present was 7,500. Find the number present at $t = 5$.

(A) $1,200e^2$ (B) $3,000e^2$ (C) $7,500e^2$ (D) $7,500e^5$ (E) $\frac{15,000}{7}e^7$

---

17. What is the area of the region completely bounded by the curve $y = -x^2 + x + 6$ and the line $y = 4$?

(A) $\frac{3}{2}$ (B) $\frac{7}{3}$ (C) $\frac{9}{2}$ (D) $\frac{31}{6}$ (E) $\frac{33}{2}$

---

18. $\frac{d}{dx}(\arcsin 2x) =$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$
(B) $\frac{-2}{\sqrt{4x^2-1}}$
(C) $\frac{1}{2\sqrt{1-4x^2}}$
(D) $\frac{2}{\sqrt{1-4x^2}}$
(E) $\frac{2}{\sqrt{4x^2-1}}$

19. Suppose that f is a function that is defined for all real numbers. Which of the following conditions assures that f has an inverse function?

(A) The function f is periodic.
(B) The graph of f is symmetric with respect to the y-axis.
(C) The graph of f is concave up.
(D) The function f is a strictly increasing function.
(E) The function f is continuous.

---

20. If F and f are continuous functions such that $F'(x) = f(x)$ for all x, then $\int_a^b f(x)\,dx$ is

(A) $F'(a) - F'(b)$
(B) $F'(b) - F'(a)$
(C) $F(a) - F(b)$
(D) $F(b) - F(a)$
(E) none of the above

---

21. $\int_0^1 (x+1)e^{x^2+2x}\,dx =$

(A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

---

22. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

(A) $x > 0$
(B) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$
(C) $-2 < x < 0$ or $x > 2$
(D) $x > \sqrt{2}$
(E) $-2 < x < 2$

23. $\lim_{h\to 0} \frac{1}{h}\ln\left(\frac{2+h}{2}\right)$ is

(A) e^2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) nonexistent

---

24. Let $f(x)=\cos(\arctan x)$. What is the range of f?

(A) $\left\{x \middle| -\frac{\pi}{2}<x<\frac{\pi}{2}\right\}$ (B) $\{x \mid 0<x\le 1\}$ (C) $\{x \mid 0\le x\le 1\}$

(D) $\{x \mid -1<x<1\}$ (E) $\{x \mid -1\le x\le 1\}$

---

25. $\int_0^{\pi/4} \tan^2 x\,dx=$

(A) $\frac{\pi}{4}-1$ (B) $1-\frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2}-1$ (E) $\frac{\pi}{4}+1$

---

26. The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S=4\pi r^2 \text{ and } V=\frac{4}{3}\pi r^3\right)$

(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

---

27. $\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}}\,dx=$

(A) $1-\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}\ln\frac{3}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{6}-1$ (E) $2-\sqrt{3}$

---

28. A point moves in a straight line so that its distance at time t from a fixed point of the line is $8t-3t^2$. What is the *total* distance covered by the point between $t=1$ and $t=2$?

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) 2 (E) 5

29. Let $f(x)=\left|\sin x-\frac{1}{2}\right|$. The maximum value attained by f is

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) $\frac{\pi}{2}$ (E) $\frac{3\pi}{2}$

---

30. $\int_1^2 \frac{x-4}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\ln 2-2$ (C) $\ln 2$ (D) 2 (E) $\ln 2+2$

---

31. If $\log_a\left(2^a\right)=\frac{a}{4}$, then $a =$

(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

---

32. $\int \frac{5}{1+x^2}\,dx =$

(A) $\frac{-10x}{\left(1+x^2\right)^2}+C$ (B) $\frac{5}{2x}\ln\left(1+x^2\right)+C$ (C) $5x-\frac{5}{x}+C$

(D) $5\arctan x+C$ (E) $5\ln\left(1+x^2\right)+C$

---

33. Suppose that f is an odd function; i.e., $f(-x)=-f(x)$ for all x. Suppose that $f'\left(x_0\right)$ exists. Which of the following must necessarily be equal to $f'\left(-x_0\right)$?

(A) $f'\left(x_0\right)$

(B) $-f'\left(x_0\right)$

(C) $\frac{1}{f'\left(x_0\right)}$

(D) $\frac{-1}{f'\left(x_0\right)}$

(E) None of the above

34. The average value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

(A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$

---

35. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\frac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\frac{8\pi}{3}$

---

36. If $y = e^{nx}$, then $\frac{d^n y}{dx^n} =$

(A) $n^n e^{nx}$ (B) $n!e^{nx}$ (C) $n e^{nx}$ (D) $n^n e^x$ (E) $n!e^x$

---

37. If $\frac{dy}{dx} = 4y$ and if $y = 4$ when $x = 0$, then $y =$

(A) $4e^{4x}$ (B) e^{4x} (C) $3 + e^{4x}$ (D) $4 + e^{4x}$ (E) $2x^2 + 4$

---

38. If $\int_1^2 f(x - c)\,dx = 5$ where c is a constant, then $\int_{1-c}^{2-c} f(x)\,dx =$

(A) $5 + c$ (B) 5 (C) $5 - c$ (D) $c - 5$ (E) –5

---

39. The point on the curve $2y = x^2$ nearest to $(4,1)$ is

(A) $(0,0)$ (B) $(2,2)$ (C) $(\sqrt{2},1)$ (D) $(2\sqrt{2},4)$ (E) $(4,8)$

---

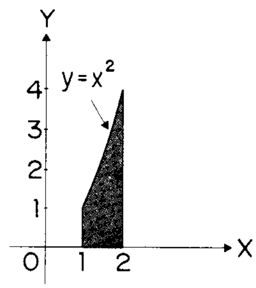
40. If $\tan(xy) = x$, then $\frac{dy}{dx} =$

(A) $\frac{1 - y\tan(xy)\sec(xy)}{x\tan(xy)\sec(xy)}$ (B) $\frac{\sec^2(xy) - y}{x}$ (C) $\cos^2(xy)$

(D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy) - y}{x}$

41. Given $f(x) = \begin{cases} x+1 & \text{for } x < 0, \\ \cos \pi x & \text{for } x \geq 0, \end{cases}$ $\int_{-1}^{1} f(x)\,dx =$

(A) $\frac{1}{2} + \frac{1}{\pi}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2} - \frac{1}{\pi}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{2} + \pi$



42. Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x = \frac{4}{3}$ and $x = \frac{5}{3}$.

(A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$

43. If the solutions of $f(x) = 0$ are -1 and 2, then the solutions of $f\left(\frac{x}{2}\right) = 0$ are

(A) -1 and 2 (B) $-\frac{1}{2}$ and $\frac{5}{2}$ (C) $-\frac{3}{2}$ and $\frac{3}{2}$

(D) $-\frac{1}{2}$ and 1 (E) -2 and 4

44. For small values of h, the function $\sqrt[4]{16+h}$ is best approximated by which of the following?

(A) $4 + \frac{h}{32}$ (B) $2 + \frac{h}{32}$ (C) $\frac{h}{32}$

(D) $4 - \frac{h}{32}$ (E) $2 - \frac{h}{32}$

45. If f is a continuous function on $[a,b]$, which of the following is necessarily true?

(A) f' exists on (a,b).

(B) If $f(x_0)$ is a maximum of f, then $f'(x_0) = 0$.

(C) $\lim_{x \to x_0} f(x) = f\left(\lim_{x \to x_0} x\right)$ for $x_0 \in (a,b)$

(D) $f'(x) = 0$ for some $x \in [a,b]$

(E) The graph of f' is a straight line.

90 Minutes—No Calculator

Note: In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

1. If $f(x) = e^{1/x}$, then $f'(x) =$

(A) $-\dfrac{e^{1/x}}{x^2}$ (B) $-e^{1/x}$ (C) $\dfrac{e^{1/x}}{x}$ (D) $\dfrac{e^{1/x}}{x^2}$ (E) $\dfrac{1}{x}e^{(1/x)-1}$

---

2. $\int_0^3 (x+1)^{1/2}\,dx =$

(A) $\dfrac{21}{2}$ (B) 7 (C) $\dfrac{16}{3}$ (D) $\dfrac{14}{3}$ (E) $-\dfrac{1}{4}$

---

3. If $f(x) = x + \dfrac{1}{x}$, then the set of values for which f increases is

(A) $(-\infty, -1] \cup [1, \infty)$ (B) $[-1, 1]$ (C) $(-\infty, \infty)$

(D) $(0, \infty)$ (E) $(-\infty, 0) \cup (0, \infty)$

---

4. For what non-negative value of b is the line given by $y = -\dfrac{1}{3}x + b$ normal to the curve $y = x^3$?

(A) 0 (B) 1 (C) $\dfrac{4}{3}$ (D) $\dfrac{10}{3}$ (E) $\dfrac{10\sqrt{3}}{3}$

---

5. $\int_{-1}^{2} \dfrac{|x|}{x}\,dx$ is

(A) –3 (B) 1 (C) 2 (D) 3 (E) nonexistent

---

6. If $f(x) = \dfrac{x-1}{x+1}$ for all $x \neq -1$, then $f'(1) =$

(A) –1 (B) $-\dfrac{1}{2}$ (C) 0 (D) $\dfrac{1}{2}$ (E) 1

7. If $y = \ln\left(x^2 + y^2\right)$, then the value of $\frac{dy}{dx}$ at the point $(1,0)$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) undefined

---

8. If $y = \sin x$ and $y^{(n)}$ means "the nth derivative of y with respect to x," then the smallest positive integer n for which $y^{(n)} = y$ is

(A) 2 (B) 4 (C) 5 (D) 6 (E) 8

---

9. If $y = \cos^2 3x$, then $\frac{dy}{dx} =$

(A) $-6\sin 3x\cos 3x$ (B) $-2\cos 3x$ (C) $2\cos 3x$
(D) $6\cos 3x$ (E) $2\sin 3x\cos 3x$

---

10. The length of the curve $y = \ln\sec x$ from $x = 0$ to $x = b$, where $0 < b < \frac{\pi}{2}$, may be expressed by which of the following integrals?

(A) $\int_0^b \sec x\,dx$

(B) $\int_0^b \sec^2 x\,dx$

(C) $\int_0^b (\sec x\tan x)\,dx$

(D) $\int_0^b \sqrt{1+\left(\ln\sec x\right)^2}\,dx$

(E) $\int_0^b \sqrt{1+\left(\sec^2 x\tan^2 x\right)}\,dx$

---

11. Let $y = x\sqrt{1+x^2}$. When $x = 0$ and $dx = 2$, the value of dy is

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

12. If n is a known positive integer, for what value of k is $\int_1^k x^{n-1}\,dx = \frac{1}{n}$?

(A) 0 (B) $\left(\frac{2}{n}\right)^{1/n}$ (C) $\left(\frac{2n-1}{n}\right)^{1/n}$

(D) $2^{1/n}$ (E) 2^n

---

13. The acceleration α of a body moving in a straight line is given in terms of time t by $\alpha = 8 - 6t$. If the velocity of the body is 25 at $t = 1$ and if $s(t)$ is the distance of the body from the origin at time t, what is $s(4) - s(2)$?

(A) 20 (B) 24 (C) 28 (D) 32 (E) 42

---

14. If $x = t^2 - 1$ and $y = 2e^t$, then $\frac{dy}{dx} =$

(A) $\frac{e^t}{t}$ (B) $\frac{2e^t}{t}$ (C) $\frac{e^{|t|}}{t^2}$ (D) $\frac{4e^t}{2t-1}$ (E) e^t

---

15. The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{x/2}$ is

(A) $\frac{e-1}{2}$ (B) $e-1$ (C) $2(e-1)$ (D) $2e-1$ (E) $2e$

---

16. A series expansion of $\frac{\sin t}{t}$ is

(A) $1 - \frac{t^2}{3!} + \frac{t^4}{5!} - \frac{t^6}{7!} + \cdots$

(B) $\frac{1}{t} - \frac{t}{2!} + \frac{t^3}{4!} - \frac{t^5}{6!} + \cdots$

(C) $1 + \frac{t^2}{3!} + \frac{t^4}{5!} + \frac{t^6}{7!} + \cdots$

(D) $\frac{1}{t} + \frac{t}{2!} + \frac{t^3}{4!} + \frac{t^5}{6!} + \cdots$

(E) $t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \cdots$

17. The number of bacteria in a culture is growing at a rate of $3,000e^{2t/5}$ per unit of time t. At $t=0$, the number of bacteria present was 7,500. Find the number present at $t=5$.

(A) $1,200e^2$ (B) $3,000e^2$ (C) $7,500e^2$ (D) $7,500e^5$ (E) $\frac{15,000}{7}e^7$

---

18. Let g be a continuous function on the closed interval $[0,1]$. Let $g(0)=1$ and $g(1)=0$. Which of the following is NOT necessarily true?

(A) There exists a number h in $[0,1]$ such that $g(h) \geq g(x)$ for all x in $[0,1]$.

(B) For all a and b in $[0,1]$, if $a=b$, then $g(a)=g(b)$.

(C) There exists a number h in $[0,1]$ such that $g(h)=\frac{1}{2}$.

(D) There exists a number h in $[0,1]$ such that $g(h)=\frac{3}{2}$.

(E) For all h in the open interval $(0,1)$, $\lim_{x\to h} g(x)=g(h)$.

---

19. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ II. $\sum_{n=1}^{\infty} \frac{1}{n}$ III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(A) I only (B) III only (C) I and II only (D) I and III only (E) I, II, and III

---

20. $\int x\sqrt{4-x^2}\,dx =$

(A) $\frac{\left(4-x^2\right)^{3/2}}{3}+C$ (B) $-\left(4-x^2\right)^{3/2}+C$ (C) $\frac{x^2\left(4-x^2\right)^{3/2}}{3}+C$

(D) $-\frac{x^2\left(4-x^2\right)^{3/2}}{3}+C$ (E) $-\frac{\left(4-x^2\right)^{3/2}}{3}+C$

---

21. $\int_0^1 (x+1)e^{x^2+2x}\,dx =$

(A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e

22. A particle moves on the curve $y = \ln x$ so that the x-component has velocity $x'(t) = t + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point $(1,0)$. At time $t = 1$, the particle is at the point

(A) $(2, \ln 2)$ (B) $(e^2, 2)$ (C) $\left(\frac{5}{2}, \ln\frac{5}{2}\right)$
(D) $(3, \ln 3)$ (E) $\left(\frac{3}{2}, \ln\frac{3}{2}\right)$

---

23. $\lim_{h\to 0} \frac{1}{h}\ln\left(\frac{2+h}{2}\right)$ is

(A) e^2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) nonexistent

---

24. Let $f(x) = 3x + 1$ for all real x and let $\varepsilon > 0$. For which of the following choices of δ is $|f(x) - 7| < \varepsilon$ whenever $|x - 2| < \delta$?

(A) $\frac{\varepsilon}{4}$ (B) $\frac{\varepsilon}{2}$ (C) $\frac{\varepsilon}{\varepsilon+1}$ (D) $\frac{\varepsilon+1}{\varepsilon}$ (E) 3ε

---

25. $\int_0^{\pi/4} \tan^2 x\, dx =$

(A) $\frac{\pi}{4} - 1$ (B) $1 - \frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2} - 1$ (E) $\frac{\pi}{4} + 1$

---

26. Which of the following is true about the graph of $y = \ln\left|x^2 - 1\right|$ in the interval $(-1,1)$?

(A) It is increasing.
(B) It attains a relative minimum at $(0,0)$.
(C) It has a range of all real numbers.
(D) It is concave down.
(E) It has an asymptote of $x = 0$.

---

27. If $f(x) = \frac{1}{3}x^3 - 4x^2 + 12x - 5$ and the domain is the set of all x such that $0 \leq x \leq 9$, then the absolute maximum value of the function f occurs when x is

(A) 0 (B) 2 (C) 4 (D) 6 (E) 9

28. If the substitution $\sqrt{x} = \sin y$ is made in the integrand of $\int_0^{1/2} \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$, the resulting integral is

(A) $\int_0^{1/2} \sin^2 y\,dy$ (B) $2\int_0^{1/2} \frac{\sin^2 y}{\cos y}\,dy$ (C) $2\int_0^{\pi/4} \sin^2 y\,dy$

(D) $\int_0^{\pi/4} \sin^2 y\,dy$ (E) $2\int_0^{\pi/6} \sin^2 y\,dy$

---

29. If $y'' = 2y'$ and if $y = y' = e$ when $x = 0$, then when $x = 1$, $y =$

(A) $\frac{e}{2}\left(e^2+1\right)$ (B) e (C) $\frac{e^3}{2}$ (D) $\frac{e}{2}$ (E) $\frac{\left(e^3-e\right)}{2}$

---

30. $\int_1^2 \frac{x-4}{x^2}\,dx$

(A) $-\frac{1}{2}$ (B) $\ln 2 - 2$ (C) $\ln 2$ (D) 2 (E) $\ln 2 + 2$

---

31. If $f(x) = \ln(\ln x)$, then $f'(x) =$

(A) $\frac{1}{x}$ (B) $\frac{1}{\ln x}$ (C) $\frac{\ln x}{x}$ (D) x (E) $\frac{1}{x \ln x}$

---

32. If $y = x^{\ln x}$, then y' is

(A) $\frac{x^{\ln x} \ln x}{x^2}$

(B) $x^{1/x} \ln x$

(C) $\frac{2x^{\ln x} \ln x}{x}$

(D) $\frac{x^{\ln x} \ln x}{x}$

(E) None of the above

33. Suppose that f is an odd function; i.e., $f(-x) = -f(x)$ for all x. Suppose that $f'(x_0)$ exists. Which of the following must necessarily be equal to $f'(-x_0)$?

(A) $f'(x_0)$

(B) $-f'(x_0)$

(C) $\dfrac{1}{f'(x_0)}$

(D) $-\dfrac{1}{f'(x_0)}$

(E) None of the above

---

34. The average (mean) value of $\sqrt{x}$ over the interval $0 \le x \le 2$ is

(A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$

---

35. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \dfrac{\pi}{4}$, and the axes is rotated about the x-axis. What is the volume of the solid generated?

(A) $\dfrac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\dfrac{8\pi}{3}$

---

36. $\displaystyle\int_0^1 \frac{x+1}{x^2+2x-3}\,dx$ is

(A) $-\ln\sqrt{3}$ (B) $-\dfrac{\ln\sqrt{3}}{2}$ (C) $\dfrac{1-\ln\sqrt{3}}{2}$ (D) $\ln\sqrt{3}$ (E) divergent

---

37. $\displaystyle\lim_{x\to 0} \frac{1-\cos^2(2x)}{x^2} =$

(A) –2 (B) 0 (C) 1 (D) 2 (E) 4

---

38. If $\displaystyle\int_1^2 f(x-c)\,dx = 5$ where c is a constant, then $\displaystyle\int_{1-c}^{2-c} f(x)\,dx =$

(A) $5+c$ (B) 5 (C) $5-c$ (D) $c-5$ (E) –5

39. Let f and g be differentiable functions such that

$f(1)=2$, $f'(1)=3$, $f'(2)=-4$,

$g(1)=2$, $g'(1)=-3$, $g'(2)=5$.

If $h(x)=f\left(g(x)\right)$, then $h'(1)=$

(A) -9 (B) -4 (C) 0 (D) 12 (E) 15

---

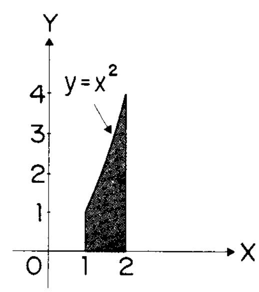
40. The area of the region enclosed by the polar curve $r=1-\cos\theta$ is

(A) $\frac{3}{4}\pi$ (B) π (C) $\frac{3}{2}\pi$ (D) 2π (E) 3π

---

41. Given $f(x)=\begin{cases} x+1 & \text{for } x<0, \\ \cos\pi x & \text{for } x\geq 0, \end{cases}$ $\int_{-1}^{1} f(x)\,dx=$

(A) $\frac{1}{2}+\frac{1}{\pi}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}-\frac{1}{\pi}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{2}+\pi$



42. Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x=\frac{4}{3}$ and $x=\frac{5}{3}$.

(A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$

43. $\int \arcsin x \, dx =$

(A) $\sin x - \int \frac{x \, dx}{\sqrt{1-x^2}}$

(B) $\frac{(\arcsin x)^2}{2} + C$

(C) $\arcsin x + \int \frac{dx}{\sqrt{1-x^2}}$

(D) $x \arccos x - \int \frac{x \, dx}{\sqrt{1-x^2}}$

(E) $x \arcsin x - \int \frac{x \, dx}{\sqrt{1-x^2}}$

---

44. If f is the solution of $x f'(x) - f(x) = x$ such that $f(-1) = 1$, then $f\left(e^{-1}\right) =$

(A) $-2e^{-1}$ (B) 0 C) e^{-1} (D) $-e^{-1}$ (E) $2e^{-2}$

---

45. Suppose $g'(x) < 0$ for all $x \geq 0$ and $F(x) = \int_0^x t\, g'(t)\, dt$ for all $x \geq 0$. Which of the following statements is FALSE?

(A) F takes on negative values.

(B) F is continuous for all $x > 0$.

(C) $F(x) = x\, g(x) - \int_0^x g(t)\, dt$

(D) $F'(x)$ exists for all $x > 0$.

(E) F is an increasing function.

90 Minutes—No Calculator

Notes: (1) In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_1^2 x^{-3}\,dx =$

(A) $-\frac{7}{8}$ (B) $-\frac{3}{4}$ (C) $\frac{15}{64}$ (D) $\frac{3}{8}$ (E) $\frac{15}{16}$

---

2. If $f(x) = (2x+1)^4$, then the 4th derivative of $f(x)$ at $x = 0$ is

(A) 0 (B) 24 (C) 48 (D) 240 (E) 384

---

3. If $y = \frac{3}{4+x^2}$, then $\frac{dy}{dx} =$

(A) $\frac{-6x}{\left(4+x^2\right)^2}$ (B) $\frac{3x}{\left(4+x^2\right)^2}$ (C) $\frac{6x}{\left(4+x^2\right)^2}$ (D) $\frac{-3}{\left(4+x^2\right)^2}$ (E) $\frac{3}{2x}$

---

4. If $\frac{dy}{dx} = \cos(2x)$, then $y =$

(A) $-\frac{1}{2}\cos(2x)+C$ (B) $-\frac{1}{2}\cos^2(2x)+C$ (C) $\frac{1}{2}\sin(2x)+C$

(D) $\frac{1}{2}\sin^2(2x)+C$ (E) $-\frac{1}{2}\sin(2x)+C$

---

5. $\lim_{n\to\infty} \frac{4n^2}{n^2+10{,}000n}$ is

(A) 0 (B) $\frac{1}{2{,}500}$ (C) 1 (D) 4 (E) nonexistent

6. If $f(x) = x$, then $f'(5) =$

(A) 0 (B) $\frac{1}{5}$ (C) 1 (D) 5 (E) $\frac{25}{2}$

---

7. Which of the following is equal to $\ln 4$?

(A) $\ln 3 + \ln 1$ (B) $\frac{\ln 8}{\ln 2}$ (C) $\int_1^4 e^t\,dt$ (D) $\int_1^4 \ln x\,dx$ (E) $\int_1^4 \frac{1}{t}\,dt$

---

8. The slope of the line tangent to the graph of $y = \ln\left(\frac{x}{2}\right)$ at $x = 4$ is

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1 (E) 4

---

9. If $\int_{-1}^{1} e^{-x^2}\,dx = k$, then $\int_{-1}^{0} e^{-x^2}\,dx =$

(A) $-2k$ (B) $-k$ (C) $-\frac{k}{2}$ (D) $\frac{k}{2}$ (E) $2k$

---

10. If $y = 10^{\left(x^2-1\right)}$, then $\frac{dy}{dx} =$

(A) $(\ln 10)10^{\left(x^2-1\right)}$ (B) $(2x)10^{\left(x^2-1\right)}$ (C) $\left(x^2-1\right)10^{\left(x^2-2\right)}$

(D) $2x(\ln 10)10^{\left(x^2-1\right)}$ (E) $x^2(\ln 10)10^{\left(x^2-1\right)}$

---

11. The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

(A) 0 (B) 2 (C) 4 (D) 8 (E) 12

---

12. If $f\left(g(x)\right) = \ln\left(x^2 + 4\right)$, $f(x) = \ln\left(x^2\right)$, and $g(x) > 0$ for all real x, then $g(x) =$

(A) $\frac{1}{\sqrt{x^2+4}}$ (B) $\frac{1}{x^2+4}$ (C) $\sqrt{x^2+4}$ (D) $x^2 + 4$ (E) $x + 2$

13. If $x^2 + xy + y^3 = 0$, then, in terms of x and y, $\frac{dy}{dx} =$

(A) $-\frac{2x+y}{x+3y^2}$ (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$

---

14. The velocity of a particle moving on a line at time t is $v = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ meters per second. How many meters did the particle travel from $t = 0$ to $t = 4$?

(A) 32 (B) 40 (C) 64 (D) 80 (E) 184

---

15. The domain of the function defined by $f(x) = \ln\left(x^2 - 4\right)$ is the set of all real numbers x such that

(A) $|x| < 2$ (B) $|x| \le 2$ (C) $|x| > 2$ (D) $|x| \ge 2$ (E) x is a real number

---

16. The function defined by $f(x) = x^3 - 3x^2$ for all real numbers x has a relative maximum at $x =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 4

---

17. $\int_0^1 xe^{-x}dx =$

(A) $1-2e$ (B) -1 (C) $1-2e^{-1}$ (D) 1 (E) $2e-1$

---

18. If $y = \cos^2 x - \sin^2 x$, then $y' =$

(A) -1 (B) 0 (C) $-2\sin(2x)$ (D) $-2(\cos x + \sin x)$ (E) $2(\cos x - \sin x)$

---

19. If $f(x_1) + f(x_2) = f(x_1 + x_2)$ for all real numbers x_1 and x_2, which of the following could define f?

(A) $f(x) = x+1$ (B) $f(x) = 2x$ (C) $f(x) = \frac{1}{x}$ (D) $f(x) = e^x$ (E) $f(x) = x^2$

20. If $y = \arctan(\cos x)$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{-\sin x}{1+\cos^2 x}$ (B) $-(\operatorname{arcsec}(\cos x))^2 \sin x$ (C) $(\operatorname{arcsec}(\cos x))^2$

(D) $\dfrac{1}{(\arccos x)^2 + 1}$ (E) $\dfrac{1}{1+\cos^2 x}$

---

21. If the domain of the function f given by $f(x) = \dfrac{1}{1-x^2}$ is $\{x : |x| > 1\}$, what is the range of f?

(A) $\{x : -\infty < x < -1\}$ (B) $\{x : -\infty < x < 0\}$ (C) $\{x : -\infty < x < 1\}$

(D) $\{x : -1 < x < \infty\}$ (E) $\{x : 0 < x < \infty\}$

---

22. $\displaystyle\int_1^2 \frac{x^2-1}{x+1}\,dx =$

(A) $\dfrac{1}{2}$ (B) 1 (C) 2 (D) $\dfrac{5}{2}$ (E) $\ln 3$

---

23. $\dfrac{d}{dx}\left(\dfrac{1}{x^3} - \dfrac{1}{x} + x^2\right)$ at $x = -1$ is

(A) -6 (B) -4 (C) 0 (D) 2 (E) 6

---

24. If $\displaystyle\int_{-2}^{2} (x^7 + k)\,dx = 16$, then $k =$

(A) -12 (B) -4 (C) 0 (D) 4 (E) 12

---

25. If $f(x) = e^x$, which of the following is equal to $f'(e)$?

(A) $\displaystyle\lim_{h\to 0} \frac{e^{x+h}}{h}$ (B) $\displaystyle\lim_{h\to 0} \frac{e^{x+h} - e^e}{h}$ (C) $\displaystyle\lim_{h\to 0} \frac{e^{e+h} - e}{h}$

(D) $\displaystyle\lim_{h\to 0} \frac{e^{x+h} - 1}{h}$ (E) $\displaystyle\lim_{h\to 0} \frac{e^{e+h} - e^e}{h}$

26. The graph of $y^2 = x^2 + 9$ is symmetric to which of the following?

I. The x-axis
II. The y-axis
III. The origin

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

---

27. $\int_0^3 |x-1|\,dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) 6

---

28. If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0 \le t \le 3$ is

(A) -45 (B) -30 (C) -15 (D) -10 (E) -5

---

29. Which of the following functions are continuous for all real numbers x ?

I. $y = x^{\frac{2}{3}}$
II. $y = e^x$
III. $y = \tan x$

(A) None (B) I only (C) II only (D) I and II (E) I and III

---

30. $\int \tan(2x)\,dx =$

(A) $-2\ln|\cos(2x)| + C$ (B) $-\frac{1}{2}\ln|\cos(2x)| + C$ (C) $\frac{1}{2}\ln|\cos(2x)| + C$

(D) $2\ln|\cos(2x)| + C$ (E) $\frac{1}{2}\sec(2x)\tan(2x) + C$

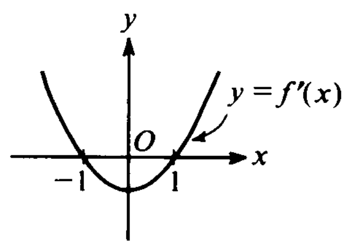
31. The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

(A) $\frac{1}{2}\pi$ (B) 10π (C) 24π (D) 54π (E) 108π

---

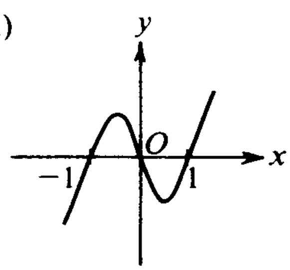
32. $\int_0^{\frac{\pi}{3}} \sin(3x)\,dx =$

(A) -2 (B) $-\frac{2}{3}$ (C) 0 (D) $\frac{2}{3}$ (E) 2

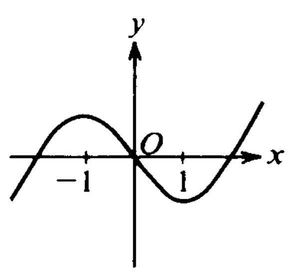


33. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

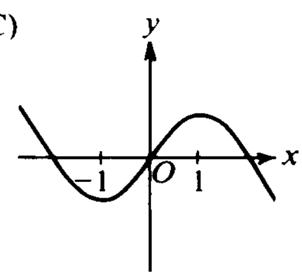
(A)



(B)



(C)



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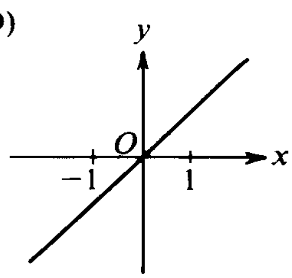
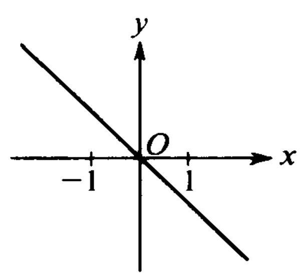
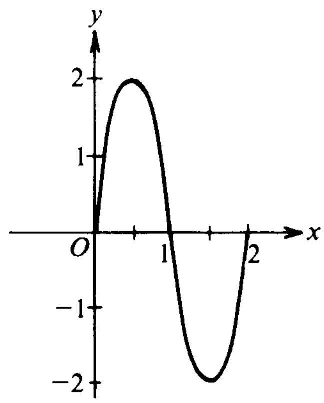


(E)



34. The area of the region in the <u>first quadrant</u> that is enclosed by the graphs of $y = x^3 + 8$ and $y = x + 8$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) $\frac{65}{4}$



35. The figure above shows the graph of a sine function for one complete period. Which of the following is an equation for the graph?

(A) $y = 2\sin\left(\frac{\pi}{2}x\right)$ (B) $y = \sin(\pi x)$ (C) $y = 2\sin(2x)$

(D) $y = 2\sin(\pi x)$ (E) $y = \sin(2x)$

---

36. If f is a continuous function defined for all real numbers x and if the maximum value of $f(x)$ is 5 and the minimum value of $f(x)$ is -7, then which of the following must be true?

I. The maximum value of $f(|x|)$ is 5.
II. The maximum value of $|f(x)|$ is 7.
III. The minimum value of $f(|x|)$ is 0.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

---

37. $\lim_{x\to 0} (x \csc x)$ is

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

38. Let f and g have continuous first and second derivatives everywhere. If $f(x) \leq g(x)$ for all real x, which of the following must be true?

I. $f'(x) \leq g'(x)$ for all real x

II. $f''(x) \leq g''(x)$ for all real x

III. $\int_0^1 f(x)\,dx \leq \int_0^1 g(x)\,dx$

(A) None (B) I only (C) III only (D) I and II only (E) I, II, and III

---

39. If $f(x) = \frac{\ln x}{x}$, for all $x > 0$, which of the following is true?

(A) f is increasing for all x greater than 0.
(B) f is increasing for all x greater than 1.
(C) f is decreasing for all x between 0 and 1.
(D) f is decreasing for all x between 1 and e.
(E) f is decreasing for all x greater than e.

---

40. Let f be a continuous function on the closed interval $[0,2]$. If $2 \leq f(x) \leq 4$, then the greatest possible value of $\int_0^2 f(x)\,dx$ is

(A) 0 (B) 2 (C) 4 (D) 8 (E) 16

---

41. If $\lim_{x \to a} f(x) = L$, where L is a real number, which of the following must be true?

(A) $f'(a)$ exists.
(B) $f(x)$ is continuous at $x = a$.
(C) $f(x)$ is defined at $x = a$.
(D) $f(a) = L$
(E) None of the above

42. $\frac{d}{dx}\int_2^x \sqrt{1+t^2}\,dt =$

(A) $\frac{x}{\sqrt{1+x^2}}$ (B) $\sqrt{1+x^2}-5$ (C) $\sqrt{1+x^2}$

(D) $\frac{x}{\sqrt{1+x^2}}-\frac{1}{\sqrt{5}}$ (E) $\frac{1}{2\sqrt{1+x^2}}-\frac{1}{2\sqrt{5}}$

---

43. An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$ (B) $y = -3x + 1$ (C) $y = 2x + 10$

(D) $y = 3x - 1$ (E) $y = 4x + 1$

---

44. The average value of $f(x) = x^2\sqrt{x^3+1}$ on the closed interval $[0,2]$ is

(A) $\frac{26}{9}$ (B) $\frac{13}{3}$ (C) $\frac{26}{3}$ (D) 13 (E) 26

---

45. The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x-axis is revolved about the y-axis. The volume of the solid generated is

(A) 8π (B) $\frac{32}{5}\pi$ (C) $\frac{16}{3}\pi$ (D) 4π (E) $\frac{8}{3}\pi$

90 Minutes—No Calculator

Notes: (1) In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. The area of the region between the graph of $y = 4x^3 + 2$ and the x-axis from $x = 1$ to $x = 2$ is

(A) 36 (B) 23 (C) 20 (D) 17 (E) 9

---

2. At what values of x does $f(x) = 3x^5 - 5x^3 + 15$ have a relative maximum?

(A) –1 only (B) 0 only (C) 1 only (D) –1 and 1 only (E) –1, 0 and 1

---

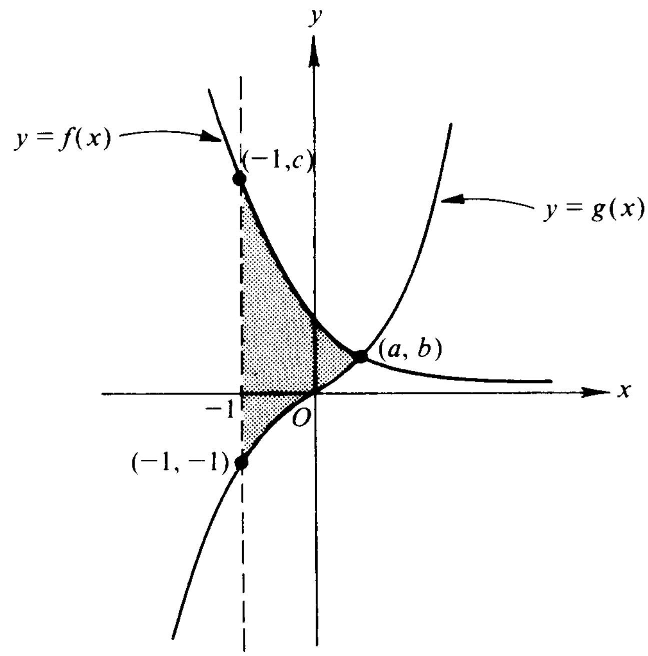
3. $\int_1^2 \frac{x+1}{x^2+2x}\,dx =$

(A) $\ln 8 - \ln 3$ (B) $\frac{\ln 8 - \ln 3}{2}$ (C) $\ln 8$ (D) $\frac{3\ln 2}{2}$ (E) $\frac{3\ln 2 + 2}{2}$

---

4. A particle moves in the xy-plane so that at any time t its coordinates are $x = t^2 - 1$ and $y = t^4 - 2t^3$. At $t = 1$, its acceleration vector is

(A) $(0, -1)$ (B) $(0, 12)$ (C) $(2, -2)$ (D) $(2, 0)$ (E) $(2, 8)$



5. The curves $y = f(x)$ and $y = g(x)$ shown in the figure above intersect at the point (a,b). The area of the shaded region enclosed by these curves and the line $x = -1$ is given by

(A) $\int_0^a (f(x) - g(x))\,dx + \int_{-1}^0 (f(x) + g(x))\,dx$

(B) $\int_{-1}^b g(x)\,dx + \int_b^c f(x)\,dx$

(C) $\int_{-1}^c (f(x) - g(x))\,dx$

(D) $\int_{-1}^a (f(x) - g(x))\,dx$

(E) $\int_{-1}^a (|f(x)| - |g(x)|)\,dx$

---

6. If $f(x) = \dfrac{x}{\tan x}$, then $f'\left(\dfrac{\pi}{4}\right) =$

(A) 2 (B) $\dfrac{1}{2}$ (C) $1 + \dfrac{\pi}{2}$ (D) $\dfrac{\pi}{2} - 1$ (E) $1 - \dfrac{\pi}{2}$

7. Which of the following is equal to $\int \frac{1}{\sqrt{25-x^2}}\,dx$?

(A) $\arcsin\frac{x}{5}+C$ (B) $\arcsin x+C$ (C) $\frac{1}{5}\arcsin\frac{x}{5}+C$

(D) $\sqrt{25-x^2}+C$ (E) $2\sqrt{25-x^2}+C$

---

8. If f is a function such that $\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=0$, which of the following must be true?

(A) The limit of $f(x)$ as x approaches 2 does not exist.

(B) f is not defined at $x=2$.

(C) The derivative of f at $x=2$ is 0.

(D) f is continuous at $x=0$.

(E) $f(2)=0$

---

9. If $xy^2+2xy=8$, then, at the point $(1, 2)$, y' is

(A) $-\frac{5}{2}$ (B) $-\frac{4}{3}$ (C) -1 (D) $-\frac{1}{2}$ (E) 0

---

10. For $-1<x<1$ if $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^{2n-1}}{2n-1}$, then $f'(x)=$

(A) $\sum_{n=1}^{\infty}(-1)^{n+1}x^{2n-2}$

(B) $\sum_{n=1}^{\infty}(-1)^{n}x^{2n-2}$

(C) $\sum_{n=1}^{\infty}(-1)^{2n}x^{2n}$

(D) $\sum_{n=1}^{\infty}(-1)^{n}x^{2n}$

(E) $\sum_{n=1}^{\infty}(-1)^{n+1}x^{2n}$

11. $\frac{d}{dx}\ln\left(\frac{1}{1-x}\right) =$

(A) $\frac{1}{1-x}$ (B) $\frac{1}{x-1}$ (C) $1-x$ (D) $x-1$ (E) $(1-x)^2$

---

12. $\int \frac{dx}{(x-1)(x+2)} =$

(A) $\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C$ (B) $\frac{1}{3}\ln\left|\frac{x+2}{x-1}\right| + C$ (C) $\frac{1}{3}\ln\left|(x-1)(x+2)\right| + C$

(D) $\left(\ln|x-1|\right)\left(\ln|x+2|\right) + C$ (E) $\ln\left|(x-1)(x+2)^2\right| + C$

---

13. Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0,3]$?

(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

---

14. Which of the following series are convergent?

I. $1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots + \frac{1}{n^2} + \ldots$

II. $1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} + \ldots$

III. $1 - \frac{1}{3} + \frac{1}{3^2} - \ldots + \frac{(-1)^{n+1}}{3^{n-1}} + \ldots$

(A) I only (B) III only (C) I and III only (D) II and III only (E) I, II, and III

---

15. If the velocity of a particle moving along the x-axis is $v(t) = 2t - 4$ and if at $t = 0$ its position is 4, then at any time t its position $x(t)$ is

(A) $t^2 - 4t$ (B) $t^2 - 4t - 4$ (C) $t^2 - 4t + 4$ (D) $2t^2 - 4t$ (E) $2t^2 - 4t + 4$

16. Which of the following functions shows that the statement "If a function is continuous at $x = 0$, then it is differentiable at $x = 0$" is false?

(A) $f(x) = x^{-\frac{4}{3}}$ (B) $f(x) = x^{-\frac{1}{3}}$ (C) $f(x) = x^{\frac{1}{3}}$ (D) $f(x) = x^{\frac{4}{3}}$ (E) $f(x) = x^3$

---

17. If $f(x) = x\ln\left(x^2\right)$, then $f'(x) =$

(A) $\ln\left(x^2\right)+1$ (B) $\ln\left(x^2\right)+2$ (C) $\ln\left(x^2\right)+\frac{1}{x}$ (D) $\frac{1}{x^2}$ (E) $\frac{1}{x}$

---

18. $\int \sin(2x+3)\,dx =$

(A) $-2\cos(2x+3)+C$ (B) $-\cos(2x+3)+C$ (C) $-\frac{1}{2}\cos(2x+3)+C$

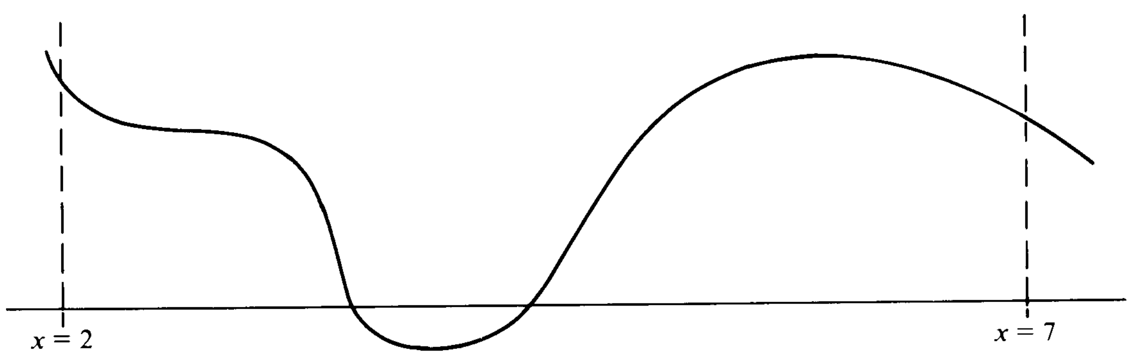
(D) $\frac{1}{2}\cos(2x+3)+C$ (E) $\cos(2x+3)+C$

---

19. If f and g are twice differentiable functions such that $g(x) = e^{f(x)}$ and $g''(x) = h(x)e^{f(x)}$, then $h(x) =$

(A) $f'(x)+f''(x)$ (B) $f'(x)+\left(f''(x)\right)^2$ (C) $\left(f'(x)+f''(x)\right)^2$

(D) $\left(f'(x)\right)^2+f''(x)$ (E) $2f'(x)+f''(x)$



20. The graph of $y = f(x)$ on the closed interval $[2,7]$ is shown above. How many points of inflection does this graph have on this interval?

(A) One (B) Two (C) Three (D) Four (E) Five

21. If $\int f(x)\sin x\,dx = -f(x)\cos x + \int 3x^2 \cos x\,dx$, then $f(x)$ could be

(A) $3x^2$ (B) x^3 (C) $-x^3$ (D) $\sin x$ (E) $\cos x$

---

22. The area of a circular region is increasing at a rate of 96π square meters per second. When the area of the region is 64π square meters, how fast, in meters per second, is the radius of the region increasing?

(A) 6 (B) 8 (C) 16 (D) $4\sqrt{3}$ (E) $12\sqrt{3}$

---

23. $\lim\limits_{h\to 0} \dfrac{\int_1^{1+h} \sqrt{x^5+8}\,dx}{h}$ is

(A) 0 (B) 1 (C) 3 (D) $2\sqrt{2}$ (E) nonexistent

---

24. The area of the region enclosed by the polar curve $r = \sin(2\theta)$ for $0 \le \theta \le \frac{\pi}{2}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{\pi}{8}$ (E) $\frac{\pi}{4}$

---

25. A particle moves along the x-axis so that at any time t its position is given by $x(t) = te^{-2t}$. For what values of t is the particle at rest?

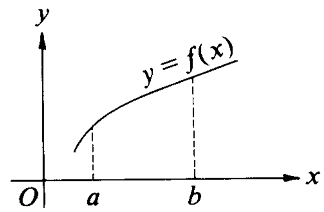
(A) No values (B) 0 only (C) $\frac{1}{2}$ only (D) 1 only (E) 0 and $\frac{1}{2}$

---

26. For $0 < x < \frac{\pi}{2}$, if $y = (\sin x)^x$, then $\frac{dy}{dx}$ is

(A) $x\ln(\sin x)$ (B) $(\sin x)^x \cot x$ (C) $x(\sin x)^{x-1}(\cos x)$

(D) $(\sin x)^x(x\cos x + \sin x)$ (E) $(\sin x)^x(x\cot x + \ln(\sin x))$



27. If f is the continuous, strictly increasing function on the interval $a \le x \le b$ as shown above, which of the following must be true?

I. $\int_a^b f(x)\,dx < f(b)(b-a)$

II. $\int_a^b f(x)\,dx > f(a)(b-a)$

III. $\int_a^b f(x)\,dx = f(c)(b-a)$ for some number c such that $a < c < b$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

---

28. An antiderivative of $f(x) = e^{x+e^x}$ is

(A) $\dfrac{e^{x+e^x}}{1+e^x}$ (B) $\left(1+e^x\right)e^{x+e^x}$ (C) e^{1+e^x} (D) e^{x+e^x} (E) e^{e^x}

---

29. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sin\left(x - \frac{\pi}{4}\right)}{x - \frac{\pi}{4}}$ is

(A) 0 (B) $\dfrac{1}{\sqrt{2}}$ (C) $\dfrac{\pi}{4}$ (D) 1 (E) nonexistent

---

30. If $x = t^3 - t$ and $y = \sqrt{3t+1}$, then $\dfrac{dy}{dx}$ at $t = 1$ is

(A) $\dfrac{1}{8}$ (B) $\dfrac{3}{8}$ (C) $\dfrac{3}{4}$ (D) $\dfrac{8}{3}$ (E) 8

---

31. What are all values of x for which the series $\sum\limits_{n=1}^{\infty} \dfrac{(x-1)^n}{n}$ converges?

(A) $-1 \le x < 1$ (B) $-1 \le x \le 1$ (C) $0 < x < 2$ (D) $0 \le x < 2$ (E) $0 \le x \le 2$

32. An equation of the line <u>normal</u> to the graph of $y = x^3 + 3x^2 + 7x - 1$ at the point where $x = -1$ is

(A) $4x + y = -10$ (B) $x - 4y = 23$ (C) $4x - y = 2$ (D) $x + 4y = 25$ (E) $x + 4y = -25$

---

33. If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

(A) $-\frac{\ln 2}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{\ln 2}{2}$ (D) $\frac{\sqrt{2}}{2}$ (E) $\ln 2$

---

34. Which of the following gives the area of the surface generated by revolving about the y-axis the arc of $x = y^3$ from $y = 0$ to $y = 1$?

(A) $2\pi\int_0^1 y^3\sqrt{1+9y^4}\,dy$

(B) $2\pi\int_0^1 y^3\sqrt{1+y^6}\,dy$

(C) $2\pi\int_0^1 y^3\sqrt{1+3y^2}\,dy$

(D) $2\pi\int_0^1 y\sqrt{1+9y^4}\,dy$

(E) $2\pi\int_0^1 y\sqrt{1+y^6}\,dy$

---

35. The region in the first quadrant between the x-axis and the graph of $y = 6x - x^2$ is rotated around the y-axis. The volume of the resulting solid of revolution is given by

(A) $\int_0^6 \pi\left(6x - x^2\right)^2 dx$

(B) $\int_0^6 2\pi x\left(6x - x^2\right) dx$

(C) $\int_0^6 \pi x\left(6x - x^2\right)^2 dx$

(D) $\int_0^6 \pi\left(3 + \sqrt{9 - y}\right)^2 dy$

(E) $\int_0^9 \pi\left(3 + \sqrt{9 - y}\right)^2 dy$

36. $\int_{-1}^{1} \frac{3}{x^2}\,dx$ is

(A) –6 (B) –3 (C) 0 (D) 6 (E) nonexistent

---

37. The general solution for the equation $\frac{dy}{dx} + y = xe^{-x}$ is

(A) $y = \frac{x^2}{2}e^{-x} + Ce^{-x}$

(B) $y = \frac{x^2}{2}e^{-x} + e^{-x} + C$

(C) $y = -e^{-x} + \frac{C}{1+x}$

(D) $y = xe^{-x} + Ce^{-x}$

(E) $y = C_1e^{x} + C_2xe^{-x}$

---

38. $\lim_{x\to\infty}\left(1+5e^{x}\right)^{\frac{1}{x}}$ is

(A) 0 (B) 1 (C) e (D) e^5 (E) nonexistent

---

39. The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is

(A) $\frac{\left(1-e^{-6}\right)}{2}$ (B) $\frac{1}{2}e^{-6}$ (C) e^{-6} (D) e^{-3} (E) $1-e^{-3}$

---

40. If the substitution $u = \frac{x}{2}$ is made, the integral $\int_{2}^{4} \frac{1-\left(\frac{x}{2}\right)^2}{x}\,dx =$

(A) $\int_{1}^{2} \frac{1-u^2}{u}\,du$

(B) $\int_{2}^{4} \frac{1-u^2}{u}\,du$

(C) $\int_{1}^{2} \frac{1-u^2}{2u}\,du$

(D) $\int_{1}^{2} \frac{1-u^2}{4u}\,du$

(E) $\int_{2}^{4} \frac{1-u^2}{2u}\,du$

41. What is the length of the arc of $y = \frac{2}{3}x^{\frac{3}{2}}$ from $x = 0$ to $x = 3$?

(A) $\frac{8}{3}$ (B) 4 (C) $\frac{14}{3}$ (D) $\frac{16}{3}$ (E) 7

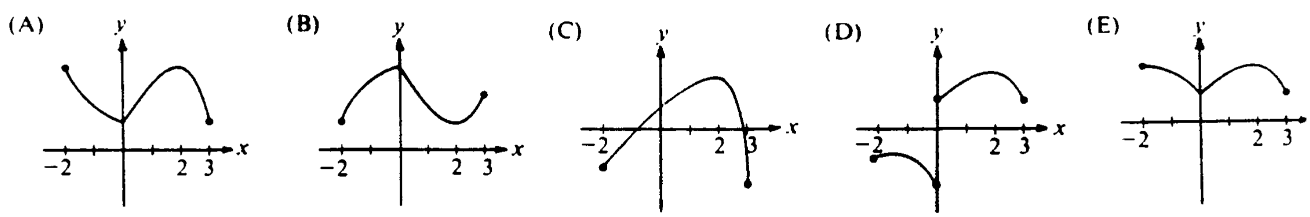
---

42. The coefficient of x^3 in the Taylor series for e^{3x} about $x = 0$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{2}$ (E) $\frac{9}{2}$

---

43. Let f be a function that is continuous on the closed interval $[-2,3]$ such that $f'(0)$ does not exist, $f'(2) = 0$, and $f''(x) < 0$ for all x except $x = 0$. Which of the following could be the graph of f?



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44. At each point (x, y) on a certain curve, the slope of the curve is $3x^2y$. If the curve contains the point $(0,8)$, then its equation is

(A) $y = 8e^{x^3}$ (B) $y = x^3 + 8$ (C) $y = e^{x^3} + 7$

(D) $y = \ln(x+1) + 8$ (E) $y^2 = x^3 + 8$

---

45. If n is a positive integer, then $\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + \ldots + \left(\frac{3n}{n}\right)^2\right]$ can be expressed as

(A) $\int_0^1 \frac{1}{x^2}\,dx$ (B) $3\int_0^1 \left(\frac{1}{x}\right)^2 dx$ (C) $\int_0^3 \left(\frac{1}{x}\right)^2 dx$

(D) $\int_0^3 x^2\,dx$ (E) $3\int_0^3 x^2\,dx$

90 Minutes—No Calculator

Notes: (1) In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $y = x^2 e^x$, then $\frac{dy}{dx} =$

(A) $2xe^x$ (B) $x\left(x + 2e^x\right)$ (C) $xe^x\left(x + 2\right)$

(D) $2x + e^x$ (E) $2x + e$

---

2. What is the domain of the function f given by $f(x) = \frac{\sqrt{x^2 - 4}}{x - 3}$?

(A) $\{x : x \neq 3\}$ (B) $\{x : |x| \leq 2\}$ (C) $\{x : |x| \geq 2\}$

(D) $\{x : |x| \geq 2 \text{ and } x \neq 3\}$ (E) $\{x : x \geq 2 \text{ and } x \neq 3\}$

---

3. A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

(A) $e^2 - 1$ (B) $e - 1$ (C) $2e$ (D) e^2 (E) $\frac{e^3}{3}$

---

4. The graph of $y = \frac{-5}{x - 2}$ is concave downward for all values of x such that

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x > 2$

---

5. $\int \sec^2 x \, dx =$

(A) $\tan x + C$ (B) $\csc^2 x + C$ (C) $\cos^2 x + C$

(D) $\frac{\sec^3 x}{3} + C$ (E) $2\sec^2 x \tan x + C$

6. If $y = \dfrac{\ln x}{x}$, then $\dfrac{dy}{dx} =$

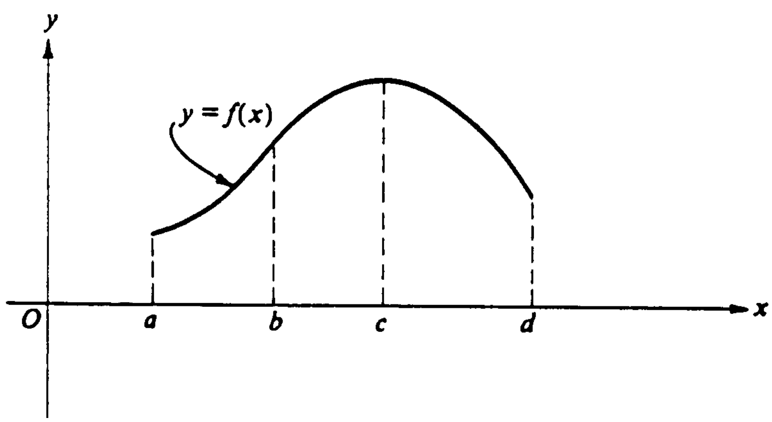
(A) $\dfrac{1}{x}$ (B) $\dfrac{1}{x^2}$ (C) $\dfrac{\ln x - 1}{x^2}$ (D) $\dfrac{1 - \ln x}{x^2}$ (E) $\dfrac{1 + \ln x}{x^2}$

---

7. $\displaystyle\int \frac{x\,dx}{\sqrt{3x^2+5}} =$

(A) $\frac{1}{9}\left(3x^2+5\right)^{\frac{3}{2}} + C$ (B) $\frac{1}{4}\left(3x^2+5\right)^{\frac{3}{2}} + C$ (C) $\frac{1}{12}\left(3x^2+5\right)^{\frac{1}{2}} + C$

(D) $\frac{1}{3}\left(3x^2+5\right)^{\frac{1}{2}} + C$ (E) $\frac{3}{2}\left(3x^2+5\right)^{\frac{1}{2}} + C$



8. The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\dfrac{dy}{dx} > 0$ and $\dfrac{d^2y}{dx^2} < 0$?

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

9. If $x + 2xy - y^2 = 2$, then at the point $(1,1)$, $\frac{dy}{dx}$ is

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{3}{2}$ (E) nonexistent

---

10. If $\int_0^k \left(2kx - x^2\right) dx = 18$, then $k =$

(A) –9 (B) –3 (C) 3 (D) 9 (E) 18

---

11. An equation of the line tangent to the graph of $f(x) = x(1-2x)^3$ at the point $(1,-1)$ is

(A) $y = -7x + 6$ (B) $y = -6x + 5$ (C) $y = -2x + 1$

(D) $y = 2x - 3$ (E) $y = 7x - 8$

---

12. If $f(x) = \sin x$, then $f'\left(\frac{\pi}{3}\right) =$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

---

13. If the function f has a continuous derivative on $[0,c]$, then $\int_0^c f'(x)\,dx =$

(A) $f(c) - f(0)$ (B) $\left|f(c) - f(0)\right|$ (C) $f(c)$ (D) $f(x) + c$ (E) $f''(c) - f''(0)$

---

14. $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta =$

(A) $-2\left(\sqrt{2} - 1\right)$ (B) $-2\sqrt{2}$ (C) $2\sqrt{2}$

(D) $2\left(\sqrt{2} - 1\right)$ (E) $2\left(\sqrt{2} + 1\right)$

15. If $f(x)=\sqrt{2x}$, then $f'(2)=$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) 1 (E) $\sqrt{2}$

---

16. A particle moves along the x-axis so that at any time $t \ge 0$ its position is given by $x(t)=t^3-3t^2-9t+1$. For what values of t is the particle at rest?

(A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

---

17. $\int_0^1 (3x-2)^2\, dx =$

(A) $-\frac{7}{3}$ (B) $-\frac{7}{9}$ (C) $\frac{1}{9}$ (D) 1 (E) 3

---

18. If $y=2\cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2}=$

(A) $-8\cos\left(\frac{x}{2}\right)$ (B) $-2\cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$ (D) $-\cos\left(\frac{x}{2}\right)$ (E) $-\frac{1}{2}\cos\left(\frac{x}{2}\right)$

---

19. $\int_2^3 \frac{x}{x^2+1}\, dx =$

(A) $\frac{1}{2}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln 2$ (C) $\ln 2$ (D) $2\ln 2$ (E) $\frac{1}{2}\ln 5$

---

20. Let f be a polynomial function with degree greater than 2. If $a \ne b$ and $f(a)=f(b)=1$, which of the following must be true for at least one value of x <u>between a and b</u>?

I. $f(x)=0$
II. $f'(x)=0$
III. $f''(x)=0$

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

21. The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) 2 (E) $\frac{14}{3}$

---

22. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

---

23. If $f'(x) = \cos x$ and $g'(x) = 1$ for all x, and if $f(0) = g(0) = 0$, then $\lim_{x \to 0} \frac{f(x)}{g(x)}$ is

(A) $\frac{\pi}{2}$ (B) 1 (C) 0 (D) -1 (E) nonexistent

---

24. $\frac{d}{dx}\left(x^{\ln x}\right) =$

(A) $x^{\ln x}$ (B) $(\ln x)^x$ (C) $\frac{2}{x}(\ln x)\left(x^{\ln x}\right)$ (D) $(\ln x)\left(x^{\ln x - 1}\right)$ (E) $2(\ln x)\left(x^{\ln x}\right)$

---

25. For all $x > 1$, if $f(x) = \int_1^x \frac{1}{t}\,dt$, then $f'(x) =$

(A) 1 (B) $\frac{1}{x}$ (C) $\ln x - 1$ (D) $\ln x$ (E) e^x

---

26. $\int_0^{\frac{\pi}{2}} x \cos x \, dx =$

(A) $-\frac{\pi}{2}$ (B) -1 (C) $1 - \frac{\pi}{2}$ (D) 1 (E) $\frac{\pi}{2} - 1$

27. At $x = 3$, the function given by $f(x) = \begin{cases} x^2, & x < 3 \\ 6x - 9, & x \geq 3 \end{cases}$ is

(A) undefined.
(B) continuous but not differentiable.
(C) differentiable but not continuous.
(D) neither continuous nor differentiable.
(E) both continuous and differentiable.

---

28. $\int_1^4 |x - 3|\, dx =$

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$ (E) 5

---

29. The $\lim_{h \to 0} \frac{\tan 3(x+h) - \tan 3x}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

---

30. A region in the first quadrant is enclosed by the graphs of $y = e^{2x}$, $x = 1$, and the coordinate axes. If the region is rotated about the <u>y-axis</u>, the volume of the solid that is generated is represented by which of the following integrals?

(A) $2\pi \int_0^1 x e^{2x}\, dx$

(B) $2\pi \int_0^1 e^{2x}\, dx$

(C) $\pi \int_0^1 e^{4x}\, dx$

(D) $\pi \int_0^e y \ln y\, dy$

(E) $\frac{\pi}{4} \int_0^e \ln^2 y\, dy$

31. If $f(x)=\dfrac{x}{x+1}$, then the inverse function, f^{-1}, is given by $f^{-1}(x)=$

(A) $\dfrac{x-1}{x}$ (B) $\dfrac{x+1}{x}$ (C) $\dfrac{x}{1-x}$ (D) $\dfrac{x}{x+1}$ (E) x

---

32. Which of the following does NOT have a period of π ?

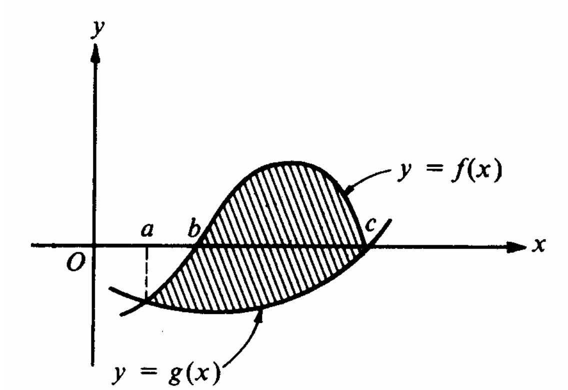
(A) $f(x)=\sin\left(\frac{1}{2}x\right)$ (B) $f(x)=\left|\sin x\right|$ (C) $f(x)=\sin^2 x$

(D) $f(x)=\tan x$ (E) $f(x)=\tan^2 x$

---

33. The absolute maximum value of $f(x)=x^3-3x^2+12$ on the closed interval $[-2,4]$ occurs at $x=$

(A) 4 (B) 2 (C) 1 (D) 0 (E) –2



34. The area of the shaded region in the figure above is represented by which of the following integrals?

(A) $\int_a^c \left(|f(x)|-|g(x)|\right)dx$

(B) $\int_b^c f(x)\,dx-\int_a^c g(x)\,dx$

(C) $\int_a^c \left(g(x)-f(x)\right)dx$

(D) $\int_a^c \left(f(x)-g(x)\right)dx$

(E) $\int_a^b \left(g(x)-f(x)\right)dx+\int_b^c \left(f(x)-g(x)\right)dx$

35. $4\cos\left(x+\frac{\pi}{3}\right)=$

(A) $2\sqrt{3}\cos x-2\sin x$ (B) $2\cos x-2\sqrt{3}\sin x$ (C) $2\cos x+2\sqrt{3}\sin x$

(D) $2\sqrt{3}\cos x+2\sin x$ (E) $4\cos x+2$

---

36. What is the average value of y for the part of the curve $y=3x-x^2$ which is in the <u>first quadrant</u>?

(A) –6 (B) –2 (C) $\frac{3}{2}$ (D) $\frac{9}{4}$ (E) $\frac{9}{2}$

---

37. If $f(x)=e^x\sin x$, then the number of zeros of f on the closed interval $[0,2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

---

38. For $x>0$, $\int\left(\frac{1}{x}\int_1^x\frac{du}{u}\right)dx=$

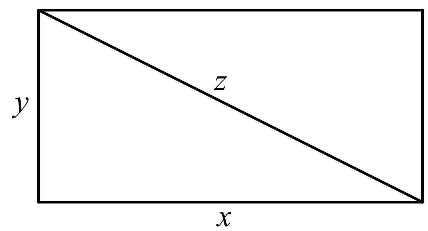
(A) $\frac{1}{x^3}+C$ (B) $\frac{8}{x^4}-\frac{2}{x^2}+C$ (C) $\ln(\ln x)+C$

(D) $\frac{\ln\left(x^2\right)}{2}+C$ (E) $\frac{(\ln x)^2}{2}+C$

---

39. If $\int_1^{10}f(x)\,dx=4$ and $\int_{10}^{3}f(x)\,dx=7$, then $\int_1^{3}f(x)\,dx=$

(A) –3 (B) 0 (C) 3 (D) 10 (E) 11



40. The sides of the rectangle above increase in such a way that $\frac{dz}{dt}=1$ and $\frac{dx}{dt}=3\frac{dy}{dt}$. At the instant when $x=4$ and $y=3$, what is the value of $\frac{dx}{dt}$?

(A) $\frac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5

---

41. If $\lim_{x\to 3} f(x)=7$, which of the following must be true?

I. f is continuous at $x=3$.
II. f is differentiable at $x=3$.
III. $f(3)=7$

(A) None (B) II only (C) III only
(D) I and III only (E) I, II, and III

---

42. The graph of which of the following equations has $y=1$ as an asymptote?

(A) $y=\ln x$ (B) $y=\sin x$ (C) $y=\frac{x}{x+1}$ (D) $y=\frac{x^2}{x-1}$ (E) $y=e^{-x}$

---

43. The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2+9y^2=9$ about the x-axis is

(A) 2π (B) 4π (C) 6π (D) 9π (E) 12π

44. Let f and g be odd functions. If p, r, and s are nonzero functions defined as follows, which must be odd?

I. $p(x) = f\left(g(x)\right)$
II. $r(x) = f(x) + g(x)$
III. $s(x) = f(x)g(x)$

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

---

45. The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

(A) $2\sqrt[3]{2}$
(B) $2\sqrt{2}$
(C) $2\sqrt[3]{4}$
(D) 4
(E) 8

1988 AP Calculus BC: Section I

90 Minutes—No Calculator

Notes: (1) In this examination, ln x denotes the natural logarithm of x (that is, logarithm to the base e).

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. The area of the region in the first quadrant enclosed by the graph of $y = x(1-x)$ and the x-axis is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$ (E) 1

---

2. $\int_0^1 x\left(x^2+2\right)^2\,dx =$

(A) $\frac{19}{2}$ (B) $\frac{19}{3}$ (C) $\frac{9}{2}$ (D) $\frac{19}{6}$ (E) $\frac{1}{6}$

---

3. If $f(x) = \ln\left(\sqrt{x}\right)$, then $f''(x) =$

(A) $-\frac{2}{x^2}$ (B) $-\frac{1}{2x^2}$ (C) $-\frac{1}{2x}$ (D) $-\frac{1}{2x^{\frac{3}{2}}}$ (E) $\frac{2}{x^2}$

---

4. If u, v, and w are nonzero differentiable functions, then the derivative of $\frac{uv}{w}$ is

(A) $\frac{uv'+u'v}{w'}$ (B) $\frac{u'v'w-uvw'}{w^2}$ (C) $\frac{uvw'-uv'w-u'vw}{w^2}$

(D) $\frac{u'vw+uv'w+uvw'}{w^2}$ (E) $\frac{uv'w+u'vw-uvw'}{w^2}$

5. Let f be the function defined by the following.

$$f(x)=\begin{cases}\sin x, & x<0\\ x^2, & 0\le x<1\\ 2-x, & 1\le x<2\\ x-3, & x\ge 2\end{cases}$$

For what values of x is f NOT continuous?

(A) 0 only (B) 1 only (C) 2 only (D) 0 and 2 only (E) 0, 1, and 2

---

6. If $y^2-2xy=16$, then $\dfrac{dy}{dx}=$

(A) $\dfrac{x}{y-x}$ (B) $\dfrac{y}{x-y}$ (C) $\dfrac{y}{y-x}$ (D) $\dfrac{y}{2y-x}$ (E) $\dfrac{2y}{x-y}$

---

7. $\int_2^{+\infty}\dfrac{dx}{x^2}$ is

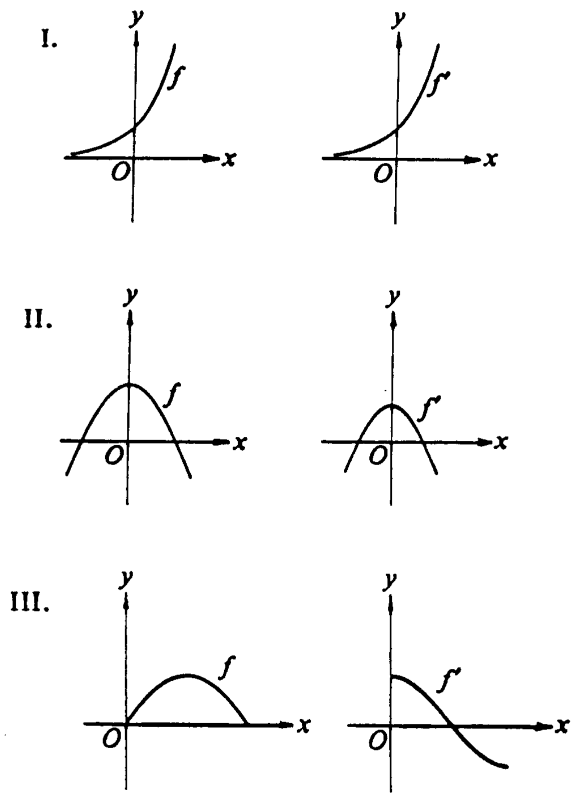
(A) $\dfrac{1}{2}$ (B) $\ln 2$ (C) 1 (D) 2 (E) nonexistent

---

8. If $f(x)=e^x$, then $\ln\left(f'(2)\right)=$

(A) 2 (B) 0 (C) $\dfrac{1}{e^2}$ (D) $2e$ (E) e^2

9. Which of the following pairs of graphs could represent the graph of a function and the graph of its derivative?



(A) I only (B) II only (C) III only (D) I and III (E) II and III

10. $\lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h}$ is

(A) 0 (B) 1 (C) $\sin x$ (D) $\cos x$ (E) nonexistent

11. If $x+7y=29$ is an equation of the line <u>normal</u> to the graph of f at the point $(1,4)$, then $f'(1)=$

(A) 7 (B) $\frac{1}{7}$ (C) $-\frac{1}{7}$ (D) $-\frac{7}{29}$ (E) -7

12. A particle travels in a straight line with a constant acceleration of 3 meters per second per second. If the velocity of the particle is 10 meters per second at time 2 seconds, how far does the particle travel during the time interval when its velocity increases from 4 meters per second to 10 meters per second?

(A) 20 m (B) 14 m (C) 7 m (D) 6 m (E) 3 m

13. $\sin(2x) =$

(A) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^{n-1} x^{2n-1}}{(2n-1)!} + \ldots$

(B) $2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \ldots + \frac{(-1)^{n-1}(2x)^{2n-1}}{(2n-1)!} + \ldots$

(C) $-\frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \ldots + \frac{(-1)^n (2x)^{2n}}{(2n)!} + \ldots$

(D) $\frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots + \frac{x^{2n}}{(2n)!} + \ldots$

(E) $2x + \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} + \ldots + \frac{(2x)^{2n-1}}{(2n-1)!} + \ldots$

---

14. If $F(x) = \int_1^{x^2} \sqrt{1+t^3}\, dt$, then $F'(x) =$

(A) $2x\sqrt{1+x^6}$

(B) $2x\sqrt{1+x^3}$

(C) $\sqrt{1+x^6}$

(D) $\sqrt{1+x^3}$

(E) $\int_1^{x^2} \frac{3t^2}{2\sqrt{1+t^3}}\, dt$

---

15. For any time $t \geq 0$, if the position of a particle in the xy-plane is given by $x = t^2 + 1$ and $y = \ln(2t+3)$, then the acceleration vector is

(A) $\left(2t, \frac{2}{(2t+3)}\right)$

(B) $\left(2t, \frac{-4}{(2t+3)^2}\right)$

(C) $\left(2, \frac{4}{(2t+3)^2}\right)$

(D) $\left(2, \frac{2}{(2t+3)^2}\right)$

(E) $\left(2, \frac{-4}{(2t+3)^2}\right)$

16. $\int xe^{2x}dx =$

(A) $\frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + C$ (B) $\frac{xe^{2x}}{2} - \frac{e^{2x}}{2} + C$ (C) $\frac{xe^{2x}}{2} + \frac{e^{2x}}{4} + C$

(D) $\frac{xe^{2x}}{2} + \frac{e^{2x}}{2} + C$ (E) $\frac{x^2e^{2x}}{4} + C$

---

17. $\int_2^3 \frac{3}{(x-1)(x+2)}dx =$

(A) $-\frac{33}{20}$ (B) $-\frac{9}{20}$ (C) $\ln\left(\frac{5}{2}\right)$ (D) $\ln\left(\frac{8}{5}\right)$ (E) $\ln\left(\frac{2}{5}\right)$

---

18. If three equal subdivisions of $[-4, 2]$ are used, what is the trapezoidal approximation of $\int_{-4}^{2} \frac{e^{-x}}{2}dx$?

(A) $e^2 + e^0 + e^{-2}$ (B) $e^4 + e^2 + e^0$ (C) $e^4 + 2e^2 + 2e^0 + e^{-2}$

(D) $\frac{1}{2}\left(e^4 + e^2 + e^0 + e^{-2}\right)$ (E) $\frac{1}{2}\left(e^4 + 2e^2 + 2e^0 + e^{-2}\right)$

---

19. A polynomial $p(x)$ has a relative maximum at $(-2,4)$, a relative minimum at $(1,1)$, a relative maximum at $(5,7)$ and no other critical points. How many zeros does $p(x)$ have?

(A) One (B) Two (C) Three (D) Four (E) Five

---

20. The statement "$\lim_{x\to a} f(x) = L$" means that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that

(A) if $0 < |x-a| < \varepsilon$, then $|f(x)-L| < \delta$

(B) if $0 < |f(x)-L| < \varepsilon$, then $|x-a| < \delta$

(C) if $|f(x)-L| < \delta$, then $0 < |x-a| < \varepsilon$

(D) $0 < |x-a| < \delta$ and $|f(x)-L| < \varepsilon$

(E) if $0 < |x-a| < \delta$, then $|f(x)-L| < \varepsilon$

21. The average value of $\frac{1}{x}$ on the closed interval $[1,3]$ is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{\ln 2}{2}$ (D) $\frac{\ln 3}{2}$ (E) $\ln 3$

---

22. If $f(x)=\left(x^2+1\right)^x$, then $f'(x)=$

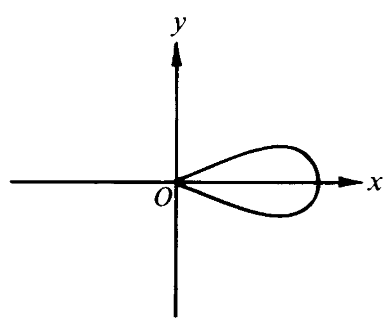
(A) $x\left(x^2+1\right)^{x-1}$

(B) $2x^2\left(x^2+1\right)^{x-1}$

(C) $x\ln\left(x^2+1\right)$

(D) $\ln\left(x^2+1\right)+\frac{2x^2}{x^2+1}$

(E) $\left(x^2+1\right)^x\left[\ln\left(x^2+1\right)+\frac{2x^2}{x^2+1}\right]$



23. Which of the following gives the area of the region enclosed by the loop of the graph of the polar curve $r=4\cos(3\theta)$ shown in the figure above?

(A) $16\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\cos(3\theta)\,d\theta$ (B) $8\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos(3\theta)\,d\theta$ (C) $8\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}\cos^2(3\theta)\,d\theta$

(D) $16\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos^2(3\theta)\,d\theta$ (E) $8\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos^2(3\theta)\,d\theta$

24. If c is the number that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^3 - 2x^2$ on the interval $0 \le x \le 2$, then $c =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) $\frac{4}{3}$ (E) 2

---

25. The base of a solid is the region in the first quadrant enclosed by the parabola $y = 4x^2$, the line $x = 1$, and the x-axis. Each plane section of the solid perpendicular to the x-axis is a square. The volume of the solid is

(A) $\frac{4\pi}{3}$ (B) $\frac{16\pi}{5}$ (C) $\frac{4}{3}$ (D) $\frac{16}{5}$ (E) $\frac{64}{5}$

---

26. If f is a function such that $f'(x)$ exists for all x and $f(x) > 0$ for all x, which of the following is NOT necessarily true?

(A) $\int_{-1}^{1} f(x)\,dx > 0$

(B) $\int_{-1}^{1} 2f(x)\,dx = 2\int_{-1}^{1} f(x)\,dx$

(C) $\int_{-1}^{1} f(x)\,dx = 2\int_{0}^{1} f(x)\,dx$

(D) $\int_{-1}^{1} f(x)\,dx = -\int_{1}^{-1} f(x)\,dx$

(E) $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$

---

27. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b?

(A) –3 (B) 0 (C) 1 (D) 3

(E) It cannot be determined from the information given.

28. $\dfrac{d}{dx}\ln\left|\cos\left(\dfrac{\pi}{x}\right)\right|$ is

(A) $\dfrac{-\pi}{x^2\cos\left(\dfrac{\pi}{x}\right)}$ (B) $-\tan\left(\dfrac{\pi}{x}\right)$ (C) $\dfrac{1}{\cos\left(\dfrac{\pi}{x}\right)}$

(D) $\dfrac{\pi}{x}\tan\left(\dfrac{\pi}{x}\right)$ (E) $\dfrac{\pi}{x^2}\tan\left(\dfrac{\pi}{x}\right)$

---

29. The region R in the first quadrant is enclosed by the lines $x = 0$ and $y = 5$ and the graph of $y = x^2 + 1$. The volume of the solid generated when R is revolved about the <u>y-axis</u> is

(A) 6π (B) 8π (C) $\dfrac{34\pi}{3}$ (D) 16π (E) $\dfrac{544\pi}{15}$

---

30. $\displaystyle\sum_{i=n}^{\infty}\left(\frac{1}{3}\right)^i =$

(A) $\dfrac{3}{2}-\left(\dfrac{1}{3}\right)^n$ (B) $\dfrac{3}{2}\left[1-\left(\dfrac{1}{3}\right)^n\right]$ (C) $\dfrac{3}{2}\left(\dfrac{1}{3}\right)^n$

(D) $\dfrac{2}{3}\left(\dfrac{1}{3}\right)^n$ (E) $\dfrac{2}{3}\left(\dfrac{1}{3}\right)^{n+1}$

---

31. $\displaystyle\int_0^2 \sqrt{4-x^2}\,dx =$

(A) $\dfrac{8}{3}$ (B) $\dfrac{16}{3}$ (C) π (D) 2π (E) 4π

---

32. The general solution of the differential equation $y' = y + x^2$ is $y =$

(A) Ce^x (B) $Ce^x + x^2$ (C) $-x^2 - 2x - 2 + C$

(D) $e^x - x^2 - 2x - 2 + C$ (E) $Ce^x - x^2 - 2x - 2$

33. The length of the curve $y = x^3$ from $x = 0$ to $x = 2$ is given by

(A) $\int_0^2 \sqrt{1+x^6}\,dx$ (B) $\int_0^2 \sqrt{1+3x^2}\,dx$ (C) $\pi\int_0^2 \sqrt{1+9x^4}\,dx$

(D) $2\pi\int_0^2 \sqrt{1+9x^4}\,dx$ (E) $\int_0^2 \sqrt{1+9x^4}\,dx$

---

34. A curve in the plane is defined parametrically by the equations $x = t^3 + t$ and $y = t^4 + 2t^2$. An equation of the line tangent to the curve at $t = 1$ is

(A) $y = 2x$ (B) $y = 8x$ (C) $y = 2x - 1$

(D) $y = 4x - 5$ (E) $y = 8x + 13$

---

35. If k is a positive integer, then $\lim_{x\to+\infty} \frac{x^k}{e^x}$ is

(A) 0 (B) 1 (C) e (D) $k!$ (E) nonexistent

---

36. Let R be the region between the graphs of $y = 1$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$. The volume of the solid obtained by revolving R about the x-axis is given by

(A) $2\pi\int_0^{\frac{\pi}{2}} x\sin x\,dx$ (B) $2\pi\int_0^{\frac{\pi}{2}} x\cos x\,dx$ (C) $\pi\int_0^{\frac{\pi}{2}} (1-\sin x)^2\,dx$

(D) $\pi\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$ (E) $\pi\int_0^{\frac{\pi}{2}} \left(1-\sin^2 x\right)dx$

---

37. A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of $\frac{4}{9}$ meter per second, at what rate, in meters per second, is the person walking?

(A) $\frac{4}{27}$ (B) $\frac{4}{9}$ (C) $\frac{3}{4}$ (D) $\frac{4}{3}$ (E) $\frac{16}{9}$

38. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converges?

(A) $-1 \le x \le 1$ (B) $-1 < x \le 1$ (C) $-1 \le x < 1$

(D) $-1 < x < 1$ (E) All real x

---

39. If $\frac{dy}{dx} = y \sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

(A) $e^{\tan x} + 4$ (B) $e^{\tan x} + 5$ (C) $5e^{\tan x}$

(D) $\tan x + 5$ (E) $\tan x + 5e^x$

---

40. Let f and g be functions that are differentiable everywhere. If g is the inverse function of f and if $g(-2) = 5$ and $f'(5) = -\frac{1}{2}$, then $g'(-2) =$

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $-\frac{1}{5}$ (E) -2

---

41. $\lim_{n\to\infty} \frac{1}{n}\left[\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \ldots + \sqrt{\frac{n}{n}}\right] =$

(A) $\frac{1}{2}\int_0^1 \frac{1}{\sqrt{x}}\,dx$ (B) $\int_0^1 \sqrt{x}\,dx$ (C) $\int_0^1 x\,dx$

(D) $\int_1^2 x\,dx$ (E) $2\int_1^2 x\sqrt{x}\,dx$

---

42. If $\int_1^4 f(x)\,dx = 6$, what is the value of $\int_1^4 f(5-x)\,dx$?

(A) 6 (B) 3 (C) 0 (D) -1 (E) -6

43. Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

(A) $\dfrac{3\ln 3}{\ln 2}$ (B) $\dfrac{2\ln 3}{\ln 2}$ (C) $\dfrac{\ln 3}{\ln 2}$ (D) $\ln\left(\dfrac{27}{2}\right)$ (E) $\ln\left(\dfrac{9}{2}\right)$

---

44. Which of the following series converge?

I. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$

II. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{3}{2}\right)^{n}$

III. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n}$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

---

45. What is the area of the largest rectangle that can be inscribed in the ellipse $4x^2 + 9y^2 = 36$?

(A) $6\sqrt{2}$ (B) 12 (C) 24 (D) $24\sqrt{2}$ (E) 36

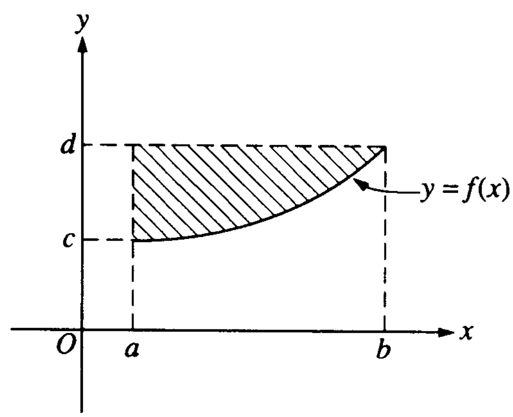
90 Minutes—Scientific Calculator

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $f(x) = x^{\frac{3}{2}}$, then $f'(4) =$

(A) –6 (B) –3 (C) 3 (D) 6 (E) 8



2. Which of the following represents the area of the shaded region in the figure above?

(A) $\int_c^d f(y)dy$ (B) $\int_a^b \left(d - f(x)\right)dx$ (C) $f'(b) - f'(a)$

(D) $(b-a)\left[f(b) - f(a)\right]$ (E) $(d-c)\left[f(b) - f(a)\right]$

---

3. $\lim\limits_{n\to\infty} \dfrac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ is

(A) –5 (B) –2 (C) 1 (D) 3 (E) nonexistent

4. If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y, $\frac{dy}{dx} =$

(A) $-\frac{x^2+y}{x+2y^2}$

(B) $-\frac{x^2+y}{x+y^2}$

(C) $-\frac{x^2+y}{x+2y}$

(D) $-\frac{x^2+y}{2y^2}$

(E) $\frac{-x^2}{1+2y^2}$

---

5. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2-4}{x+2}$ when $x \neq -2$, then $f(-2) =$

(A) –4 (B) –2 (C) –1 (D) 0 (E) 2

---

6. The area of the region enclosed by the curve $y = \frac{1}{x-1}$, the x-axis, and the lines $x = 3$ and $x = 4$ is

(A) $\frac{5}{36}$ (B) $\ln\frac{2}{3}$ (C) $\ln\frac{4}{3}$ (D) $\ln\frac{3}{2}$ (E) $\ln 6$

---

7. An equation of the line tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point $(1,5)$ is

(A) $13x - y = 8$ (B) $13x + y = 18$ (C) $x - 13y = 64$

(D) $x + 13y = 66$ (E) $-2x + 3y = 13$

8. If $y = \tan x - \cot x$, then $\dfrac{dy}{dx} =$

(A) $\sec x \csc x$ (B) $\sec x - \csc x$ (C) $\sec x + \csc x$ (D) $\sec^2 x - \csc^2 x$ (E) $\sec^2 x + \csc^2 x$

---

9. If h is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$

(A) $3x^3 - |x|$ (B) $|3x^2 - 1|$ (C) $3x^2|x| - 1$ (D) $3|x| - 1$ (E) $3x^2 - 1$

---

10. If $f(x) = (x-1)^2 \sin x$, then $f'(0) =$

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

---

11. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 6t - 2$. If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

(A) $9t^2 + 1$

(B) $3t^2 - 2t + 4$

(C) $t^3 - t^2 + 4t + 6$

(D) $t^3 - t^2 + 9t - 20$

(E) $36t^3 - 4t^2 - 77t + 55$

---

12. If f and g are continuous functions, and if $f(x) \geq 0$ for all real numbers x, which of the following must be true?

I. $\displaystyle\int_a^b f(x)g(x)dx = \left(\int_a^b f(x)dx\right)\left(\int_a^b g(x)dx\right)$

II. $\displaystyle\int_a^b \left(f(x) + g(x)\right)dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

III. $\displaystyle\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)dx}$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

13. The fundamental period of $2\cos(3x)$ is

(A) $\frac{2\pi}{3}$ (B) 2π (C) 6π (D) 2 (E) 3

---

14. $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx =$

(A) $2\sqrt{x^3+1}+C$

(B) $\frac{3}{2}\sqrt{x^3+1}+C$

(C) $\sqrt{x^3+1}+C$

(D) $\ln\sqrt{x^3+1}+C$

(E) $\ln(x^3+1)+C$

---

15. For what value of x does the function $f(x)=(x-2)(x-3)^2$ have a relative maximum?

(A) -3 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$

---

16. The slope of the line <u>normal</u> to the graph of $y=2\ln(\sec x)$ at $x=\frac{\pi}{4}$ is

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

(E) nonexistent

17. $\int (x^2+1)^2\,dx =$

(A) $\dfrac{(x^2+1)^3}{3}+C$

(B) $\dfrac{(x^2+1)^3}{6x}+C$

(C) $\left(\dfrac{x^3}{3}+x\right)^2+C$

(D) $\dfrac{2x(x^2+1)^3}{3}+C$

(E) $\dfrac{x^5}{5}+\dfrac{2x^3}{3}+x+C$

---

18. If $f(x)=\sin\left(\dfrac{x}{2}\right)$, then there exists a number c in the interval $\dfrac{\pi}{2}<x<\dfrac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c ?

(A) $\dfrac{2\pi}{3}$ (B) $\dfrac{3\pi}{4}$ (C) $\dfrac{5\pi}{6}$ (D) π (E) $\dfrac{3\pi}{2}$

---

19. Let f be the function defined by $f(x)=\begin{cases} x^3 & \text{for } x\le 0, \\ x & \text{for } x>0. \end{cases}$ Which of the following statements about f is true?

(A) f is an odd function.

(B) f is discontinuous at $x=0$.

(C) f has a relative maximum.

(D) $f'(0)=0$

(E) $f'(x)>0$ for $x\neq 0$

20. Let R be the region in the first quadrant enclosed by the graph of $y = (x+1)^{\frac{1}{3}}$, the line $x = 7$, the x-axis, and the y-axis. The volume of the solid generated when R is revolved about the y-axis is given by

(A) $\pi\int_0^7 (x+1)^{\frac{2}{3}}\,dx$ (B) $2\pi\int_0^7 x(x+1)^{\frac{1}{3}}\,dx$ (C) $\pi\int_0^2 (x+1)^{\frac{2}{3}}\,dx$

(D) $2\pi\int_0^2 x(x+1)^{\frac{1}{3}}\,dx$ (E) $\pi\int_0^7 (y^3-1)^2\,dy$

---

21. At what value of x does the graph of $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection?

(A) 0 (B) 1 (C) 2 (D) 3 (E) At no value of x

---

22. An antiderivative for $\frac{1}{x^2 - 2x + 2}$ is

(A) $-(x^2 - 2x + 2)^{-2}$

(B) $\ln(x^2 - 2x + 2)$

(C) $\ln\left|\frac{x-2}{x+1}\right|$

(D) $\operatorname{arcsec}(x-1)$

(E) $\arctan(x-1)$

---

23. How many critical points does the function $f(x) = (x+2)^5(x-3)^4$ have?

(A) One (B) Two (C) Three (D) Five (E) Nine

---

24. If $f(x) = (x^2 - 2x - 1)^{\frac{2}{3}}$, then $f'(0)$ is

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

25. $\frac{d}{dx}\left(2^x\right) =$

(A) 2^{x-1} (B) $(2^{x-1})x$ (C) $(2^x)\ln 2$ (D) $(2^{x-1})\ln 2$ (E) $\frac{2x}{\ln 2}$

---

26. A particle moves along a line so that at time t, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

(A) –5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13

---

27. The function f given by $f(x) = x^3 + 12x - 24$ is

(A) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
(B) decreasing for $x < 0$, increasing for $x > 0$
(C) increasing for all x
(D) decreasing for all x
(E) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

---

28. $\int_1^{500}\left(13^x - 11^x\right)dx + \int_2^{500}\left(11^x - 13^x\right)dx =$

(A) 0.000 (B) 14.946 (C) 34.415 (D) 46.000 (E) 136.364

---

29. $\lim_{\theta \to 0} \frac{1 - \cos\theta}{2\sin^2\theta}$ is

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) 1 (E) nonexistent

---

30. The region enclosed by the x-axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

(A) 3π (B) $2\sqrt{3}\,\pi$ (C) $\frac{9}{2}\pi$ (D) 9π (E) $\frac{36\sqrt{3}}{5}\pi$

31. If $f(x)=e^{3\ln(x^2)}$, then $f'(x)=$

(A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2}e^{3\ln(x^2)}$ (C) $6(\ln x)e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$

---

32. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} =$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{1}{2}\ln 2$ (E) $-\ln 2$

---

33. If $\frac{dy}{dx}=2y^2$ and if $y=-1$ when $x=1$, then when $x=2$, $y=$

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

---

34. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

(A) $-\frac{7}{8}$ feet per minute

(B) $-\frac{7}{24}$ feet per minute

(C) $\frac{7}{24}$ feet per minute

(D) $\frac{7}{8}$ feet per minute

(E) $\frac{21}{25}$ feet per minute

---

35. If the graph of $y=\frac{ax+b}{x+c}$ has a horizontal asymptote $y=2$ and a vertical asymptote $x=-3$, then $a+c=$

(A) –5 (B) –1 (C) 0 (D) 1 (E) 5

36. If the definite integral $\int_0^2 e^{x^2}\,dx$ is first approximated by using two inscribed rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, the difference between the two approximations is

(A) 53.60 (B) 30.51 (C) 27.80 (D) 26.80 (E) 12.78

---

37. If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$

II. $\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$

III. $\lim\limits_{x\to a}\dfrac{f(x+h)-f(x)}{h}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

---

38. If the second derivative of f is given by $f''(x) = 2x - \cos x$, which of the following could be $f(x)$?

(A) $\dfrac{x^3}{3} + \cos x - x + 1$

(B) $\dfrac{x^3}{3} - \cos x - x + 1$

(C) $x^3 + \cos x - x + 1$

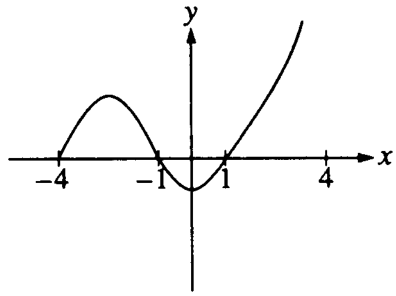
(D) $x^2 - \sin x + 1$

(E) $x^2 + \sin x + 1$

---

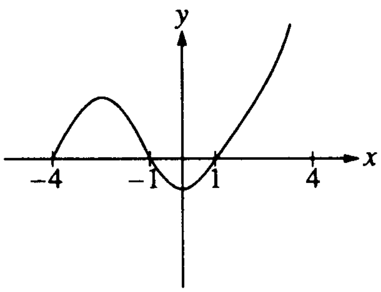
39. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

(A) $\dfrac{1}{\pi}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{2}{\pi}$ (D) 1 (E) 2

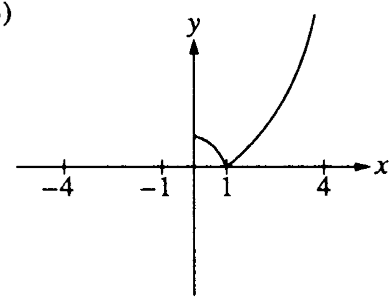


40. The graph of $y = f(x)$ is shown in the figure above. Which of the following could be the graph of $y = f(|x|)$?

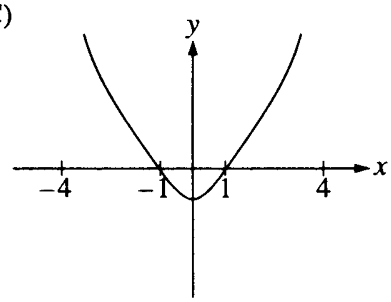
(A)



(B)



(C)



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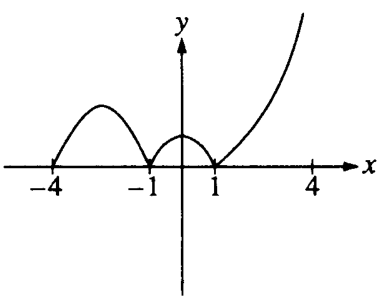
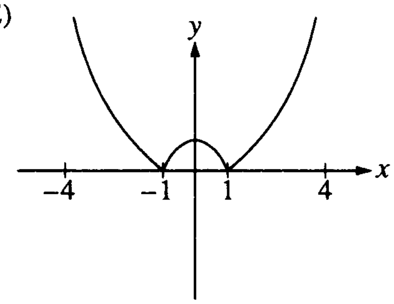


(E)



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41. $\dfrac{d}{dx}\displaystyle\int_0^x \cos(2\pi u)\,du$ is

(A) 0 (B) $\dfrac{1}{2\pi}\sin x$ (C) $\dfrac{1}{2\pi}\cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi\cos(2\pi x)$

---

42. A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

(A) 4.2 pounds (B) 4.6 pounds (C) 4.8 pounds (D) 5.6 pounds (E) 6.5 pounds

43. $\int x f(x)\,dx =$

(A) $x f(x) - \int x f'(x)\,dx$

(B) $\frac{x^2}{2} f(x) - \int \frac{x^2}{2} f'(x)dx$

(C) $x f(x) - \frac{x^2}{2} f(x) + C$

(D) $x f(x) - \int f'(x)\,dx$

(E) $\frac{x^2}{2} \int f(x)\,dx$

---

44. What is the minimum value of $f(x) = x \ln x$?

(A) $-e$ (B) -1 (C) $-\frac{1}{e}$ (D) 0 (E) $f(x)$ has no minimum value.

---

45. If Newton's method is used to approximate the real root of $x^3 + x - 1 = 0$, then a first approximation $x_1 = 1$ would lead to a <u>third</u> approximation of $x_3 =$

(A) 0.682 (B) 0.686 (C) 0.694 (D) 0.750 (E) 1.637

90 Minutes—Scientific Calculator

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. The area of the region enclosed by the graphs of $y = x^2$ and $y = x$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{5}{6}$ (E) 1

---

2. If $f(x) = 2x^2 + 1$, then $\lim_{x \to 0} \frac{f(x) - f(0)}{x^2}$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) nonexistent

---

3. If p is a polynomial of degree n, $n > 0$, what is the degree of the polynomial $Q(x) = \int_0^x p(t)dt$?

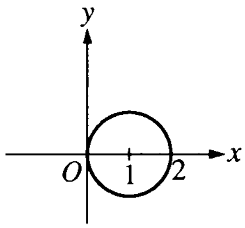
(A) 0 (B) 1 (C) $n-1$ (D) n (E) $n+1$

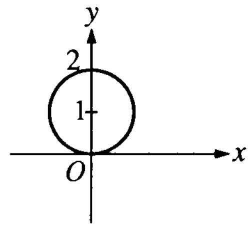
---

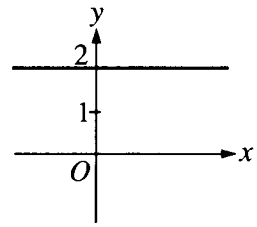
4. A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?

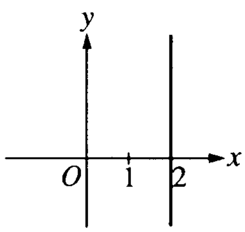
(A) $-\frac{5}{2}$ (B) $-\frac{6}{5}$ (C) 0 (D) $\frac{4}{5}$ (E) $\frac{6}{5}$

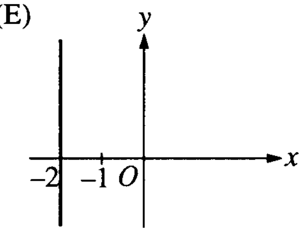
5. Which of the following represents the graph of the polar curve $r = 2\sec\theta$?

(A)


(B)


(C)


(D)


(E)


---

6. If $x = t^2 + 1$ and $y = t^3$, then $\dfrac{d^2y}{dx^2} =$

(A) $\dfrac{3}{4t}$ (B) $\dfrac{3}{2t}$ (C) $3t$ (D) $6t$ (E) $\dfrac{3}{2}$

---

7. $\int_0^1 x^3 e^{x^4}\,dx =$

(A) $\dfrac{1}{4}(e-1)$ (B) $\dfrac{1}{4}e$ (C) $e-1$ (D) e (E) $4(e-1)$

---

8. If $f(x) = \ln(e^{2x})$, then $f'(x) =$

(A) 1 (B) 2 (C) $2x$ (D) e^{-2x} (E) $2e^{-2x}$

9. If $f(x) = 1 + x^{\frac{2}{3}}$, which of the following is NOT true?

(A) f is continuous for all real numbers.

(B) f has a minimum at $x = 0$.

(C) f is increasing for $x > 0$.

(D) $f'(x)$ exists for all x.

(E) $f''(x)$ is negative for $x > 0$.

---

10. Which of the following functions are continuous at $x = 1$?

I. $\ln x$

II. e^x

III. $\ln(e^x - 1)$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

---

11. $\int_4^\infty \frac{-2x}{\sqrt[3]{9-x^2}}\,dx$ is

(A) $7^{\frac{2}{3}}$ (B) $\frac{3}{2}\left(7^{\frac{2}{3}}\right)$ (C) $9^{\frac{2}{3}} + 7^{\frac{2}{3}}$ (D) $\frac{3}{2}\left(9^{\frac{2}{3}} + 7^{\frac{2}{3}}\right)$ (E) nonexistent

---

12. The position of a particle moving along the x-axis is $x(t) = \sin(2t) - \cos(3t)$ for time $t \geq 0$. When $t = \pi$, the acceleration of the particle is

(A) 9 (B) $\frac{1}{9}$ (C) 0 (D) $-\frac{1}{9}$ (E) –9

---

13. If $\frac{dy}{dx} = x^2 y$, then y could be

(A) $3\ln\left(\frac{x}{3}\right)$ (B) $e^{\frac{x^3}{3}} + 7$ (C) $2e^{\frac{x^3}{3}}$ (D) $3e^{2x}$ (E) $\frac{x^3}{3} + 1$

14. The derivative of f is $x^4(x-2)(x+3)$. At how many points will the graph of f have a relative maximum?

(A) None (B) One (C) Two (D) Three (E) Four

---

15. If $f(x) = e^{\tan^2 x}$, then $f'(x) =$

(A) $e^{\tan^2 x}$

(B) $\sec^2 x\, e^{\tan^2 x}$

(C) $\tan^2 x\, e^{\tan^2 x - 1}$

(D) $2\tan x \sec^2 x\, e^{\tan^2 x}$

(E) $2\tan x\, e^{\tan^2 x}$

---

16. Which of the following series diverge?

I. $\sum_{k=3}^{\infty} \frac{2}{k^2+1}$

II. $\sum_{k=1}^{\infty} \left(\frac{6}{7}\right)^k$

III. $\sum_{k=2}^{\infty} \frac{(-1)^k}{k}$

(A) None (B) II only (C) III only (D) I and III (E) II and III

---

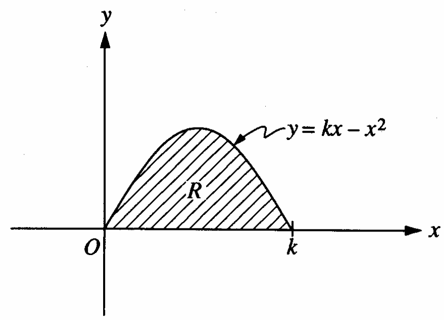
17. The slope of the line tangent to the graph of $\ln(xy) = x$ at the point where $x = 1$ is

(A) 0 (B) 1 (C) e (D) e^2 (E) $1-e$

---

18. If $e^{f(x)} = 1 + x^2$, then $f'(x) =$

(A) $\frac{1}{1+x^2}$ (B) $\frac{2x}{1+x^2}$ (C) $2x(1+x^2)$ (D) $2x\left(e^{1+x^2}\right)$ (E) $2x\ln(1+x^2)$



19. The shaded region R, shown in the figure above, is rotated about the y-axis to form a solid whose volume is 10 cubic units. Of the following, which best approximates k ?

(A) 1.51 (B) 2.09 (C) 2.49 (D) 4.18 (E) 4.77

---

20. A particle moves along the x-axis so that at any time $t \geq 0$ the acceleration of the particle is $a(t) = e^{-2t}$. If at $t = 0$ the velocity of the particle is $\frac{5}{2}$ and its position is $\frac{17}{4}$, then its position at any time $t > 0$ is $x(t) =$

(A) $-\frac{e^{-2t}}{2} + 3$

(B) $\frac{e^{-2t}}{4} + 4$

(C) $4e^{-2t} + \frac{9}{2}t + \frac{1}{4}$

(D) $\frac{e^{-2t}}{2} + 3t + \frac{15}{4}$

(E) $\frac{e^{-2t}}{4} + 3t + 4$

---

21. The value of the derivative of $y = \frac{\sqrt[3]{x^2 + 8}}{\sqrt[4]{2x+1}}$ at $x = 0$ is

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

22. If $f(x) = x^2 e^x$, then the graph of f is decreasing for all x such that

(A) $x < -2$ (B) $-2 < x < 0$ (C) $x > -2$ (D) $x < 0$ (E) $x > 0$

---

23. The length of the curve determined by the equations $x = t^2$ and $y = t$ from $t = 0$ to $t = 4$ is

(A) $\int_0^4 \sqrt{4t+1}\, dt$

(B) $2\int_0^4 \sqrt{t^2+1}\, dt$

(C) $\int_0^4 \sqrt{2t^2+1}\, dt$

(D) $\int_0^4 \sqrt{4t^2+1}\, dt$

(E) $2\pi\int_0^4 \sqrt{4t^2+1}\, dt$

---

24. Let f and g be functions that are differentiable for all real numbers, with $g(x) \neq 0$ for $x \neq 0$. If $\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = 0$ and $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$ exists, then $\lim_{x\to 0} \frac{f(x)}{g(x)}$ is

(A) 0

(B) $\frac{f'(x)}{g'(x)}$

(C) $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$

(D) $\frac{f'(x)g(x) - f(x)g'(x)}{\left(f(x)\right)^2}$

(E) nonexistent

---

25. Consider the curve in the xy-plane represented by $x = e^t$ and $y = te^{-t}$ for $t \geq 0$. The slope of the line tangent to the curve at the point where $x = 3$ is

(A) 20.086 (B) 0.342 (C) –0.005 (D) –0.011 (E) –0.033

26. If $y = \arctan(e^{2x})$, then $\frac{dy}{dx} =$

(A) $\frac{2e^{2x}}{\sqrt{1-e^{4x}}}$ (B) $\frac{2e^{2x}}{1+e^{4x}}$ (C) $\frac{e^{2x}}{1+e^{4x}}$ (D) $\frac{1}{\sqrt{1-e^{4x}}}$ (E) $\frac{1}{1+e^{4x}}$

---

27. The interval of convergence of $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is

(A) $-3 < x \le 3$
(B) $-3 \le x \le 3$
(C) $-2 < x < 4$
(D) $-2 \le x < 4$
(E) $0 \le x \le 2$

---

28. If a particle moves in the *xy*-plane so that at time $t > 0$ its position vector is $\left(\ln(t^2 + 2t), 2t^2\right)$, then at time $t = 2$, its velocity vector is

(A) $\left(\frac{3}{4}, 8\right)$ (B) $\left(\frac{3}{4}, 4\right)$ (C) $\left(\frac{1}{8}, 8\right)$ (D) $\left(\frac{1}{8}, 4\right)$ (E) $\left(-\frac{5}{16}, 4\right)$

---

29. $\int x \sec^2 x \, dx =$

(A) $x \tan x + C$
(B) $\frac{x^2}{2} \tan x + C$
(C) $\sec^2 x + 2\sec^2 x \tan x + C$
(D) $x \tan x - \ln|\cos x| + C$
(E) $x \tan x + \ln|\cos x| + C$

---

30. What is the volume of the solid generated by rotating about the *x*-axis the region enclosed by the curve $y = \sec x$ and the lines $x = 0$, $y = 0$, and $x = \frac{\pi}{3}$?

(A) $\frac{\pi}{\sqrt{3}}$

(B) π

(C) $\pi\sqrt{3}$

(D) $\frac{8\pi}{3}$

(E) $\pi \ln\left(\frac{1}{2} + \sqrt{3}\right)$

31. If $s_n = \left(\dfrac{(5+n)^{100}}{5^{n+1}}\right)\left(\dfrac{5^n}{(4+n)^{100}}\right)$, to what number does the sequence $\{s_n\}$ converge?

(A) $\dfrac{1}{5}$ (B) 1 (C) $\dfrac{5}{4}$ (D) $\left(\dfrac{5}{4}\right)^{100}$ (E) The sequence does not converge.

---

32. If $\int_a^b f(x)dx = 5$ and $\int_a^b g(x)dx = -1$, which of the following must be true?

I. $f(x) > g(x)$ for $a \le x \le b$

II. $\int_a^b \left(f(x) + g(x)\right) dx = 4$

III. $\int_a^b \left(f(x)g(x)\right) dx = -5$

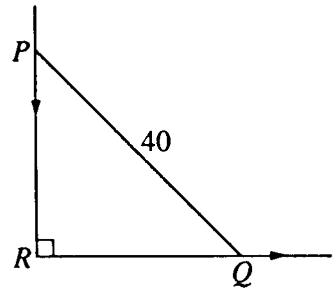
(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

---

33. Which of the following is equal to $\int_0^{\pi} \sin x \, dx$?

(A) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx$ (B) $\int_0^{\pi} \cos x \, dx$ (C) $\int_{-\pi}^{0} \sin x \, dx$

(D) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x \, dx$ (E) $\int_{\pi}^{2\pi} \sin x \, dx$



34. In the figure above, PQ represents a 40-foot ladder with end P against a vertical wall and end Q on level ground. If the ladder is slipping down the wall, what is the distance RQ at the instant when Q is moving along the ground $\frac{3}{4}$ as fast as P is moving down the wall?

(A) $\frac{6}{5}\sqrt{10}$ (B) $\frac{8}{5}\sqrt{10}$ (C) $\frac{80}{\sqrt{7}}$ (D) 24 (E) 32

---

35. If F and f are differentiable functions such that $F(x) = \int_0^x f(t)dt$, and if $F(a) = -2$ and $F(b) = -2$ where $a < b$, which of the following must be true?

(A) $f(x) = 0$ for some x such that $a < x < b$.

(B) $f(x) > 0$ for all x such that $a < x < b$.

(C) $f(x) < 0$ for all x such that $a < x < b$.

(D) $F(x) \le 0$ for all x such that $a < x < b$.

(E) $F(x) = 0$ for some x such that $a < x < b$.

---

36. Consider all right circular cylinders for which the sum of the height and circumference is 30 centimeters. What is the radius of the one with maximum volume?

(A) 3 cm (B) 10 cm (C) 20 cm (D) $\frac{30}{\pi^2}$ cm (E) $\frac{10}{\pi}$ cm

37. If $f(x)=\begin{cases} x & \text{for } x \le 1 \\ \dfrac{1}{x} & \text{for } x > 1, \end{cases}$ then $\int_0^e f(x)dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) e (E) $e+\frac{1}{2}$

---

38. During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

(A) 343 (B) 1,343 (C) 1,367 (D) 1,400 (E) 2,057

---

39. If $\frac{dy}{dx}=\frac{1}{x}$, then the average rate of change of y with respect to x on the closed interval $[1,4]$ is

(A) $-\frac{1}{4}$ (B) $\frac{1}{2}\ln 2$ (C) $\frac{2}{3}\ln 2$ (D) $\frac{2}{5}$ (E) 2

---

40. Let R be the region in the first quadrant enclosed by the x-axis and the graph of $y=\ln(1+2x-x^2)$. If Simpson's Rule with 2 subintervals is used to approximate the area of R, the approximation is

(A) 0.462 (B) 0.693 (C) 0.924 (D) 0.986 (E) 1.850

---

41. Let $f(x)=\int_{-2}^{x^2-3x} e^{t^2}\,dt$. At what value of x is $f(x)$ a minimum?

(A) For no value of x (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) 2 (E) 3

---

42. $\lim_{x\to 0}(1+2x)^{\csc x} =$

(A) 0 (B) 1 (C) 2 (D) e (E) e^2

43. The coefficient of x^6 in the Taylor series expansion about $x = 0$ for $f(x) = \sin\left(x^2\right)$ is

(A) $-\frac{1}{6}$ (B) 0 (C) $\frac{1}{120}$ (D) $\frac{1}{6}$ (E) 1

---

44. If f is continuous on the interval $[a,b]$, then there exists c such that $a < c < b$ and $\int_a^b f(x)dx =$

(A) $\frac{f(c)}{b-a}$ (B) $\frac{f(b)-f(a)}{b-a}$ (C) $f(b)-f(a)$ (D) $f'(c)(b-a)$ (E) $f(c)(b-a)$

---

45. If $f(x) = \sum_{k=1}^{\infty}\left(\sin^2 x\right)^k$, then $f(1)$ is

(A) 0.369 (B) 0.585 (C) 2.400 (D) 2.426 (E) 3.426

1997 AP Calculus AB: Section I, Part A

50 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_1^2 (4x^3 - 6x)\,dx =$

   (A) 2
   (B) 4
   (C) 6
   (D) 36
   (E) 42

---

2. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$

   (A) $\dfrac{3x-3}{\sqrt{2x-3}}$

   (B) $\dfrac{x}{\sqrt{2x-3}}$

   (C) $\dfrac{1}{\sqrt{2x-3}}$

   (D) $\dfrac{-x+3}{\sqrt{2x-3}}$

   (E) $\dfrac{5x-6}{2\sqrt{2x-3}}$

---

3. If $\int_a^b f(x)\,dx = a + 2b$, then $\int_a^b (f(x)+5)\,dx =$

   (A) $a+2b+5$ (B) $5b-5a$ (C) $7b-4a$ (D) $7b-5a$ (E) $7b-6a$

---

4. If $f(x) = -x^3 + x + \dfrac{1}{x}$, then $f'(-1) =$

   (A) 3 (B) 1 (C) –1 (D) –3 (E) –5

5. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$

---

6. $\frac{1}{2}\int e^{\frac{t}{2}}\,dt =$

(A) $e^{-t} + C$ (B) $e^{-\frac{t}{2}} + C$ (C) $e^{\frac{t}{2}} + C$ (D) $2e^{\frac{t}{2}} + C$ (E) $e^{t} + C$

---

7. $\frac{d}{dx}\cos^2(x^3) =$

(A) $6x^2 \sin(x^3)\cos(x^3)$

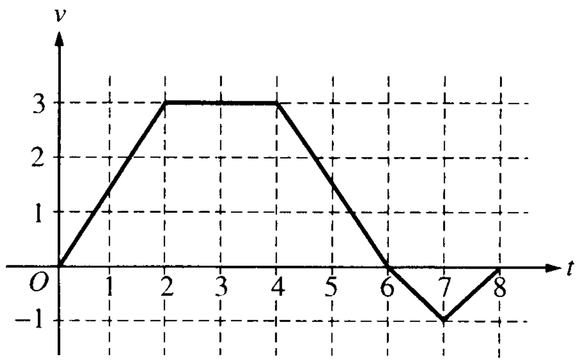
(B) $6x^2 \cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2 \sin(x^3)\cos(x^3)$

(E) $-2\sin(x^3)\cos(x^3)$

Questions 8-9 refer to the following situation.



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

8. At what value of t does the bug change direction?

(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

---

9. What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

---

10. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

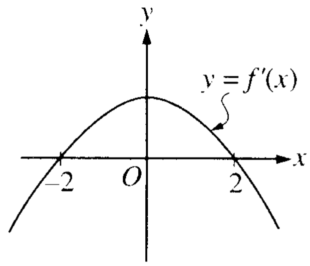
(A) $y - 1 = -\left(x - \frac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \frac{\pi}{4}\right)$

(C) $y = 2\left(x - \frac{\pi}{4}\right)$

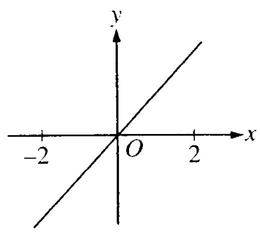
(D) $y = -\left(x - \frac{\pi}{4}\right)$

(E) $y = -2\left(x - \frac{\pi}{4}\right)$

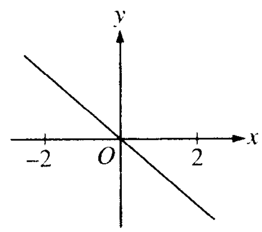


11. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

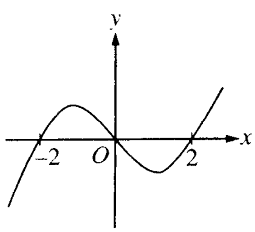
(A)



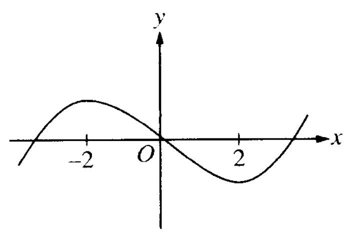
(B)



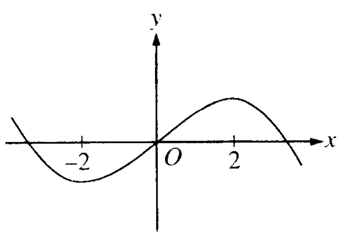
(C)



(D)



(E)



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12. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$?

(A) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{8}\right)$ (C) $\left(1, -\frac{1}{4}\right)$ (D) $\left(1, \frac{1}{2}\right)$ (E) $(2, 2)$

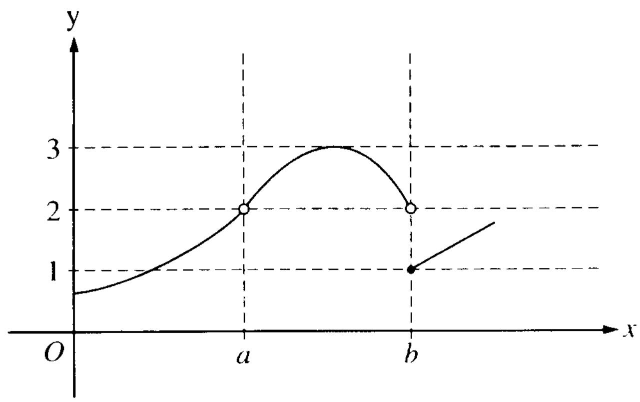
13. Let f be a function defined for all real numbers x. If $f'(x) = \frac{\left|4 - x^2\right|}{x - 2}$, then f is decreasing on the interval

(A) $(-\infty, 2)$ (B) $(-\infty, \infty)$ (C) $(-2, 4)$ (D) $(-2, \infty)$ (E) $(2, \infty)$

---

14. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5



15. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $\lim_{x \to a} f(x) = \lim_{x \to b} f(x)$

(B) $\lim_{x \to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 2$

(D) $\lim_{x \to b} f(x) = 1$

(E) $\lim_{x \to a} f(x)$ does not exist.

16. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8π

---

17. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4,3)$?

(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

---

18. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x} dx$ is

(A) 0 (B) 1 (C) $e - 1$ (D) e (E) $e + 1$

---

19. If $f(x) = \ln\left|x^2 - 1\right|$, then $f'(x) =$

(A) $\left|\frac{2x}{x^2 - 1}\right|$

(B) $\frac{2x}{\left|x^2 - 1\right|}$

(C) $\frac{2|x|}{x^2 - 1}$

(D) $\frac{2x}{x^2 - 1}$

(E) $\frac{1}{x^2 - 1}$

20. The average value of $\cos x$ on the interval $[-3,5]$ is

(A) $\dfrac{\sin 5 - \sin 3}{8}$

(B) $\dfrac{\sin 5 - \sin 3}{2}$

(C) $\dfrac{\sin 3 - \sin 5}{2}$

(D) $\dfrac{\sin 3 + \sin 5}{2}$

(E) $\dfrac{\sin 3 + \sin 5}{8}$

---

21. $\lim\limits_{x \to 1} \dfrac{x}{\ln x}$ is

(A) 0 (B) $\dfrac{1}{e}$ (C) 1 (D) e (E) nonexistent

---

22. What are all values of x for which the function f defined by $f(x) = (x^2 - 3)e^{-x}$ is increasing?

(A) There are no such values of x.

(B) $x < -1$ and $x > 3$

(C) $-3 < x < 1$

(D) $-1 < x < 3$

(E) All values of x

---

23. If the region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt{x}$ is revolved about the y-axis, the volume of the solid generated is

(A) $\dfrac{32\pi}{5}$ (B) $\dfrac{16\pi}{3}$ (C) $\dfrac{16\pi}{5}$ (D) $\dfrac{8\pi}{3}$ (E) π

24. The expression $\frac{1}{50}\left(\sqrt{\frac{1}{50}}+\sqrt{\frac{2}{50}}+\sqrt{\frac{3}{50}}+\cdots+\sqrt{\frac{50}{50}}\right)$ is a Riemann sum approximation for

(A) $\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(B) $\int_0^1 \sqrt{x}\,dx$

(C) $\frac{1}{50}\int_0^1 \sqrt{\frac{x}{50}}\,dx$

(D) $\frac{1}{50}\int_0^1 \sqrt{x}\,dx$

(E) $\frac{1}{50}\int_0^{50} \sqrt{x}\,dx$

---

25. $\int x\sin(2x)\,dx =$

(A) $-\frac{x}{2}\cos(2x)+\frac{1}{4}\sin(2x)+C$

(B) $-\frac{x}{2}\cos(2x)-\frac{1}{4}\sin(2x)+C$

(C) $\frac{x}{2}\cos(2x)-\frac{1}{4}\sin(2x)+C$

(D) $-2x\cos(2x)+\sin(2x)+C$

(E) $-2x\cos(2x)-4\sin(2x)+C$

40 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

76. If $f(x) = \dfrac{e^{2x}}{2x}$, then $f'(x) =$

(A) 1

(B) $\dfrac{e^{2x}(1-2x)}{2x^2}$

(C) e^{2x}

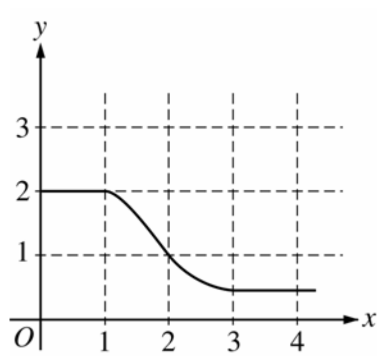
(D) $\dfrac{e^{2x}(2x+1)}{x^2}$

(E) $\dfrac{e^{2x}(2x-1)}{2x^2}$

---

77. The graph of the function $y = x^3 + 6x^2 + 7x - 2\cos x$ changes concavity at $x =$

(A) –1.58 (B) –1.63 (C) –1.67 (D) –1.89 (E) –2.33



78. The graph of f is shown in the figure above. If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3

79. Let f be a function such that $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = 5$. Which of the following must be true?

I. f is continuous at $x = 2$.

II. f is differentiable at $x = 2$.

III. The derivative of f is continuous at $x = 2$.

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

---

80. Let f be the function given by $f(x) = 2e^{4x^2}$. For what value of x is the slope of the line tangent to the graph of f at $(x, f(x))$ equal to 3?

(A) 0.168 (B) 0.276 (C) 0.318 (D) 0.342 (E) 0.551

---

81. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?

(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40

---

82. If $y = 2x - 8$, what is the minimum value of the product xy?

(A) –16 (B) –8 (C) –4 (D) 0 (E) 2

---

83. What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$, and the y-axis?

(A) 0.127 (B) 0.385 (C) 0.400 (D) 0.600 (E) 0.947

---

84. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln x}$, the line $x = e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) $\frac{1}{3}(e^3 - 1)$

85. If the derivative of f is given by $f'(x) = e^x - 3x^2$, at which of the following values of x does f have a relative maximum value?

(A) –0.46 (B) 0.20 (C) 0.91 (D) 0.95 (E) 3.73

---

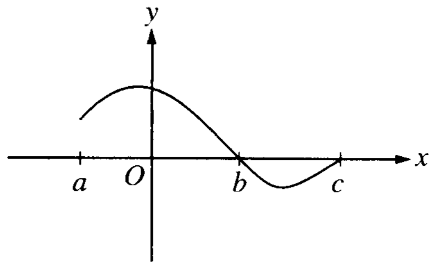
86. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

(A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

---

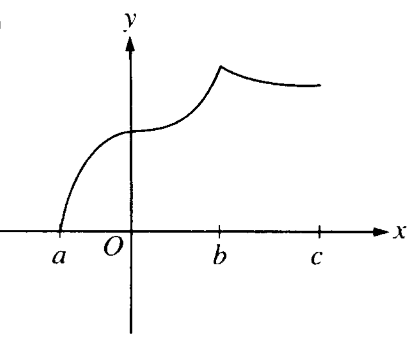
87. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is $a(t) = t + \sin t$. At $t = 0$, the velocity of the particle is –2. For what value t will the velocity of the particle be zero?

(A) 1.02 (B) 1.48 (C) 1.85 (D) 2.81 (E) 3.14



88. Let $f(x)=\int_a^x h(t)\,dt$, where h has the graph shown above. Which of the following could be the graph of f?

(A)



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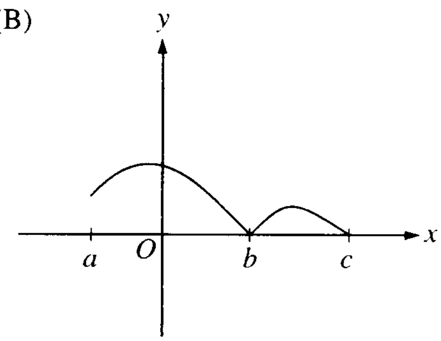

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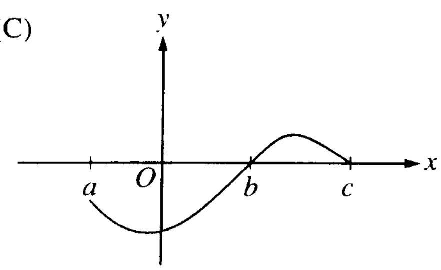

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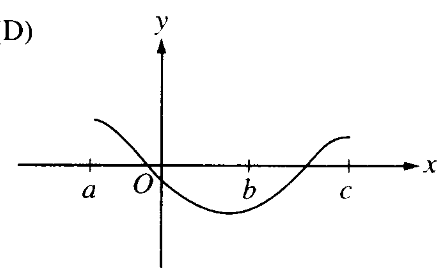
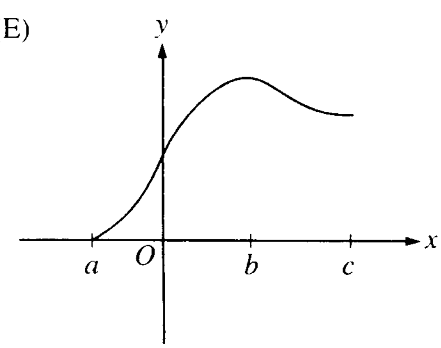


(E)



x	0	0.5	1.0	1.5	2.0
$f(x)$	3	3	5	8	13

89. A table of values for a continuous function f is shown above. If four equal subintervals of $[0,2]$ are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

---

90. Which of the following are antiderivatives of $f(x) = \sin x \cos x$?

I. $F(x) = \dfrac{\sin^2 x}{2}$

II. $F(x) = \dfrac{\cos^2 x}{2}$

III. $F(x) = \dfrac{-\cos(2x)}{4}$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

1997 AP Calculus BC: Section I, Part A

50 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_0^1 \sqrt{x}\,(x+1)\,dx =$

(A) 0 (B) 1 (C) $\frac{16}{15}$ (D) $\frac{7}{5}$ (E) 2

---

2. If $x = e^{2t}$ and $y = \sin(2t)$, then $\frac{dy}{dx} =$

(A) $4e^{2t}\cos(2t)$ (B) $\frac{e^{2t}}{\cos(2t)}$ (C) $\frac{\sin(2t)}{2e^{2t}}$ (D) $\frac{\cos(2t)}{2e^{2t}}$ (E) $\frac{\cos(2t)}{e^{2t}}$

---

3. The function f given by $f(x) = 3x^5 - 4x^3 - 3x$ has a relative maximum at $x =$

(A) -1 (B) $-\frac{\sqrt{5}}{5}$ (C) 0 (D) $\frac{\sqrt{5}}{5}$ (E) 1

---

4. $\frac{d}{dx}\left(xe^{\ln x^2}\right) =$

(A) $1+2x$ (B) $x+x^2$ (C) $3x^2$ (D) x^3 (E) x^2+x^3

---

5. If $f(x) = (x-1)^{\frac{3}{2}} + \frac{e^{x-2}}{2}$, then $f'(2) =$

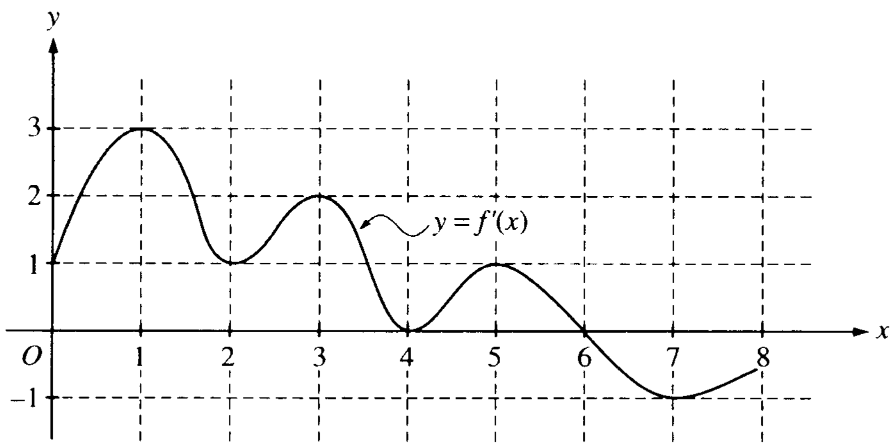
(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{7}{2}$ (E) $\frac{3+e}{2}$

---

6. The line normal to the curve $y = \sqrt{16-x}$ at the point $(0,4)$ has slope

(A) 8 (B) 4 (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$ (E) -8

Questions 7-9 refer to the graph and the information below.



The function f is defined on the closed interval $[0,8]$. The graph of its derivative f' is shown above.

7. The point $(3,5)$ is on the graph of $y = f(x)$. An equation of the line tangent to the graph of f at $(3,5)$ is

(A) $y = 2$

(B) $y = 5$

(C) $y - 5 = 2(x-3)$

(D) $y + 5 = 2(x-3)$

(E) $y + 5 = 2(x+3)$

---

8. How many points of inflection does the graph of f have?

(A) Two
(B) Three
(C) Four
(D) Five
(E) Six

9. At what value of x does the absolute minimum of f occur?

(A) 0
(B) 2
(C) 4
(D) 6
(E) 8

---

10. If $y = xy + x^2 + 1$, then when $x = -1$, $\frac{dy}{dx}$ is

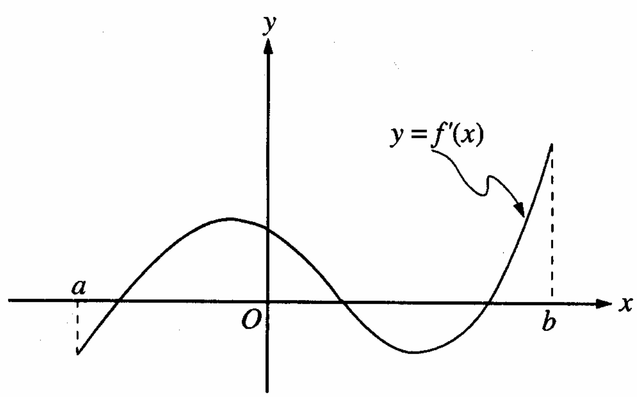
(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) -1 (D) -2 (E) nonexistent

---

11. $\int_1^{\infty} \frac{x}{(1+x^2)^2}\,dx$ is

(A) $-\frac{1}{2}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) divergent

---



12. The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a, b)?

(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima

13. A particle moves along the x-axis so that its acceleration at any time t is $a(t) = 2t - 7$. If the initial velocity of the particle is 6, at what time t during the interval $0 \le t \le 4$ is the particle farthest to the right?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

---

14. The sum of the infinite geometric series $\frac{3}{2} + \frac{9}{16} + \frac{27}{128} + \frac{81}{1,024} + \ldots$ is

(A) 1.60 (B) 2.35 (C) 2.40 (D) 2.45 (E) 2.50

---

15. The length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$, for $0 \le t \le \frac{\pi}{2}$, is given by

(A) $\int_0^{\frac{\pi}{2}} \sqrt{3\cos^2 t + 3\sin^2 t}\, dt$

(B) $\int_0^{\frac{\pi}{2}} \sqrt{-3\cos^2 t \sin t + 3\sin^2 t \cos t}\, dt$

(C) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t + 9\sin^4 t}\, dt$

(D) $\int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}\, dt$

(E) $\int_0^{\frac{\pi}{2}} \sqrt{\cos^6 t + \sin^6 t}\, dt$

---

16. $\lim_{h \to 0} \frac{e^h - 1}{2h}$ is

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) e (E) nonexistent

17. Let f be the function given by $f(x) = \ln(3 - x)$. The third-degree Taylor polynomial for f about $x = 2$ is

(A) $-(x-2)+\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(B) $-(x-2)-\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(C) $(x-2)+(x-2)^2+(x-2)^3$

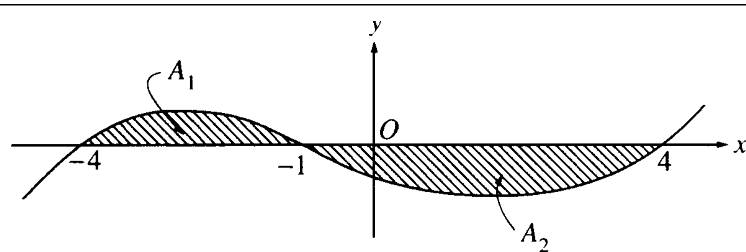
(D) $(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

(E) $(x-2)-\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

---

18. For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

(A) 0 only
(B) 1 only
(C) 0 and $\frac{2}{3}$ only
(D) 0, $\frac{2}{3}$, and 1
(E) No value



19. The graph of $y = f(x)$ is shown in the figure above. If A_1 and A_2 are positive numbers that represent the areas of the shaded regions, then in terms of A_1 and A_2,

$\int_{-4}^{4} f(x)\,dx - 2\int_{-1}^{4} f(x)\,dx =$

(A) A_1 (B) $A_1 - A_2$ (C) $2A_1 - A_2$ (D) $A_1 + A_2$ (E) $A_1 + 2A_2$

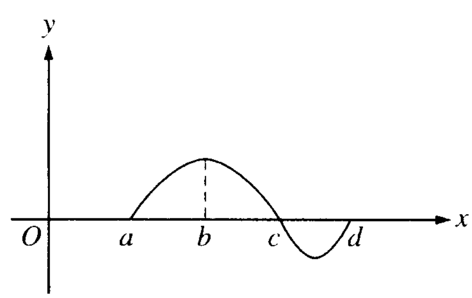
20. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$ converges?

(A) $-3 \le x \le 3$
(B) $-3 < x < 3$
(C) $-1 < x \le 5$
(D) $-1 \le x \le 5$
(E) $-1 \le x < 5$

---

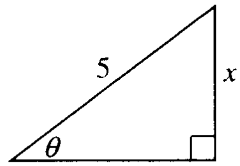
21. Which of the following is equal to the area of the region inside the polar curve $r = 2\cos\theta$ and outside the polar curve $r = \cos\theta$?

(A) $3\int_0^{\frac{\pi}{2}} \cos^2\theta \, d\theta$ (B) $3\int_0^{\pi} \cos^2\theta \, d\theta$ (C) $\frac{3}{2}\int_0^{\frac{\pi}{2}} \cos^2\theta \, d\theta$ (D) $3\int_0^{\frac{\pi}{2}} \cos\theta \, d\theta$ (E) $3\int_0^{\pi} \cos\theta \, d\theta$



22. The graph of f is shown in the figure above. If $g(x) = \int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a maximum?

(A) a
(B) b
(C) c
(D) d
(E) It cannot be determined from the information given.



23. In the triangle shown above, if θ increases at a constant rate of 3 radians per minute, at what rate is x increasing in units per minute when x equals 3 units?

(A) 3 (B) $\frac{15}{4}$ (C) 4 (D) 9 (E) 12

---

24. The Taylor series for $\sin x$ about $x = 0$ is $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$. If f is a function such that $f'(x) = \sin\left(x^2\right)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x = 0$ is

(A) $\frac{1}{7!}$ (B) $\frac{1}{7}$ (C) 0 (D) $-\frac{1}{42}$ (E) $-\frac{1}{7!}$

---

25. The closed interval $[a,b]$ is partitioned into n equal subintervals, each of width Δx, by the numbers $x_0, x_1, \dots, x_n$ where $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$. What is $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i}\,\Delta x$?

(A) $\frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

(B) $b^{\frac{3}{2}} - a^{\frac{3}{2}}$

(C) $\frac{3}{2}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

(D) $b^{\frac{1}{2}} - a^{\frac{1}{2}}$

(E) $2\left(b^{\frac{1}{2}} - a^{\frac{1}{2}}\right)$

40 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

76. Which of the following sequences converge?

I. $\left\{\dfrac{5n}{2n-1}\right\}$

II. $\left\{\dfrac{e^n}{n}\right\}$

III. $\left\{\dfrac{e^n}{1+e^n}\right\}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

---

77. When the region enclosed by the graphs of $y = x$ and $y = 4x - x^2$ is revolved about the y-axis, the volume of the solid generated is given by

(A) $\pi\int_0^3 \left(x^3 - 3x^2\right) dx$

(B) $\pi\int_0^3 \left(x^2 - \left(4x - x^2\right)^2\right) dx$

(C) $\pi\int_0^3 \left(3x - x^2\right)^2 dx$

(D) $2\pi\int_0^3 \left(x^3 - 3x^2\right) dx$

(E) $2\pi\int_0^3 \left(3x^2 - x^3\right) dx$

78. $\lim_{h\to 0} \frac{\ln(e+h)-1}{h}$ is

(A) $f'(e)$, where $f(x) = \ln x$

(B) $f'(e)$, where $f(x) = \frac{\ln x}{x}$

(C) $f'(1)$, where $f(x) = \ln x$

(D) $f'(1)$, where $f(x) = \ln(x+e)$

(E) $f'(0)$, where $f(x) = \ln x$

---

79. The position of an object attached to a spring is given by $y(t) = \frac{1}{6}\cos(5t) - \frac{1}{4}\sin(5t)$, where t is time in seconds. In the first 4 seconds, how many times is the velocity of the object equal to 0?

(A) Zero
(B) Three
(C) Five
(D) Six
(E) Seven

---

80. Let f be the function given by $f(x) = \cos(2x) + \ln(3x)$. What is the least value of x at which the graph of f changes concavity?

(A) 0.56 (B) 0.93 (C) 1.18 (D) 2.38 (E) 2.44

---

81. Let f be a continuous function on the closed interval $[-3,6]$. If $f(-3) = -1$ and $f(6) = 3$, then the Intermediate Value Theorem guarantees that

(A) $f(0) = 0$

(B) $f'(c) = \frac{4}{9}$ for at least one c between –3 and 6

(C) $-1 \le f(x) \le 3$ for all x between –3 and 6

(D) $f(c) = 1$ for at least one c between –3 and 6

(E) $f(c) = 0$ for at least one c between –1 and 3

82. If $0 \le x \le 4$, of the following, which is the greatest value of x such that $\int_0^x (t^2 - 2t)\, dt \ge \int_2^x t\, dt$?

(A) 1.35 (B) 1.38 (C) 1.41 (D) 1.48 (E) 1.59

---

83. If $\dfrac{dy}{dx} = (1 + \ln x)\, y$ and if $y = 1$ when $x = 1$, then $y =$

(A) $e^{\frac{x^2-1}{x^2}}$
(B) $1 + \ln x$
(C) $\ln x$
(D) $e^{2x + x\ln x - 2}$
(E) $e^{x \ln x}$

---

84. $\int x^2 \sin x\, dx =$

(A) $-x^2 \cos x - 2x \sin x - 2\cos x + C$
(B) $-x^2 \cos x + 2x \sin x - 2\cos x + C$
(C) $-x^2 \cos x + 2x \sin x + 2\cos x + C$
(D) $-\dfrac{x^3}{3} \cos x + C$
(E) $2x \cos x + C$

---

85. Let f be a twice differentiable function such that $f(1) = 2$ and $f(3) = 7$. Which of the following must be true for the function f on the interval $1 \le x \le 3$?

I. The average rate of change of f is $\dfrac{5}{2}$.

II. The average value of f is $\dfrac{9}{2}$.

III. The average value of f' is $\dfrac{5}{2}$.

(A) None
(B) I only
(C) III only
(D) I and III only
(E) II and III only

86. $\int \frac{dx}{(x-1)(x+3)} =$

(A) $\frac{1}{4}\ln\left|\frac{x-1}{x+3}\right| + C$

(B) $\frac{1}{4}\ln\left|\frac{x+3}{x-1}\right| + C$

(C) $\frac{1}{2}\ln\left|(x-1)(x+3)\right| + C$

(D) $\frac{1}{2}\ln\left|\frac{2x+2}{(x-1)(x+3)}\right| + C$

(E) $\ln\left|(x-1)(x+3)\right| + C$

---

87. The base of a solid is the region in the first quadrant enclosed by the graph of $y = 2 - x^2$ and the coordinate axes. If every cross section of the solid perpendicular to the y-axis is a square, the volume of the solid is given by

(A) $\pi\int_0^2 (2-y)^2\, dy$

(B) $\int_0^2 (2-y)\, dy$

(C) $\pi\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$

(D) $\int_0^{\sqrt{2}} \left(2-x^2\right)^2 dx$

(E) $\int_0^{\sqrt{2}} \left(2-x^2\right) dx$

88. Let $f(x) = \int_0^{x^2} \sin t \, dt$. At how many points in the closed interval $\left[0, \sqrt{\pi}\right]$ does the instantaneous rate of change of f equal the average rate of change of f on that interval?

(A) Zero
(B) One
(C) Two
(D) Three
(E) Four

---

89. If f is the antiderivative of $\frac{x^2}{1+x^5}$ such that $f(1) = 0$, then $f(4) =$

(A) -0.012 (B) 0 (C) 0.016 (D) 0.376 (E) 0.629

---

90. A force of 10 pounds is required to stretch a spring 4 inches beyond its natural length. Assuming Hooke's law applies, how much work is done in stretching the spring from its natural length to 6 inches beyond its natural length?

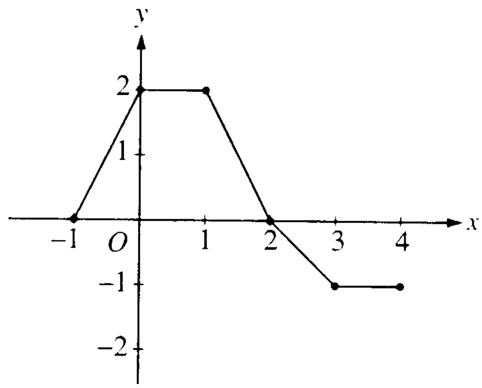
(A) 60.0 inch-pounds
(B) 45.0 inch-pounds
(C) 40.0 inch-pounds
(D) 15.0 inch-pounds
(E) 7.2 inch-pounds

55 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

(A) 5 (B) 0 (C) $-\frac{10}{3}$ (D) –5 (E) -10



2. The graph of a piecewise-linear function f, for $-1 \le x \le 4$, is shown above. What is the value of $\int_{-1}^{4} f(x)\,dx$?

(A) 1 (B) 2.5 (C) 4 (D) 5.5 (E) 8

---

3. $\int_{1}^{2} \frac{1}{x^2}\,dx =$

(A) $-\frac{1}{2}$ (B) $\frac{7}{24}$ (C) $\frac{1}{2}$ (D) 1 (E) $2\ln 2$

4. If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

(A) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.

(B) $f'(c) = 0$ for some c such that $a < c < b$.

(C) f has a minimum value on $a \le x \le b$.

(D) f has a maximum value on $a \le x \le b$.

(E) $\int_a^b f(x)dx$ exists.

---

5. $\int_0^x \sin t \, dt =$

(A) $\sin x$ (B) $-\cos x$ (C) $\cos x$ (D) $\cos x - 1$ (E) $1 - \cos x$

---

6. If $x^2 + xy = 10$, then when $x = 2$, $\dfrac{dy}{dx} =$

(A) $-\dfrac{7}{2}$ (B) -2 (C) $\dfrac{2}{7}$ (D) $\dfrac{3}{2}$ (E) $\dfrac{7}{2}$

---

7. $\int_1^e \left(\dfrac{x^2 - 1}{x} \right) dx =$

(A) $e - \dfrac{1}{e}$ (B) $e^2 - e$ (C) $\dfrac{e^2}{2} - e + \dfrac{1}{2}$ (D) $e^2 - 2$ (E) $\dfrac{e^2}{2} - \dfrac{3}{2}$

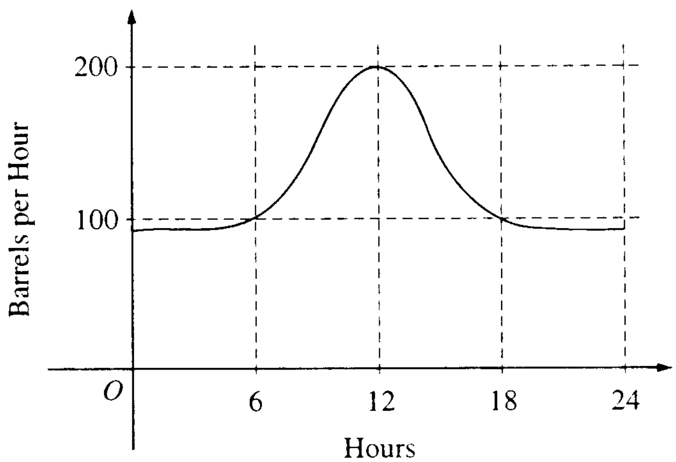
---

8. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x
(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

---

10. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \dfrac{x^2 - 2}{x - 1}$?

(A) -2 (B) $\dfrac{1}{6}$ (C) $\dfrac{1}{2}$ (D) 2 (E) 6

---

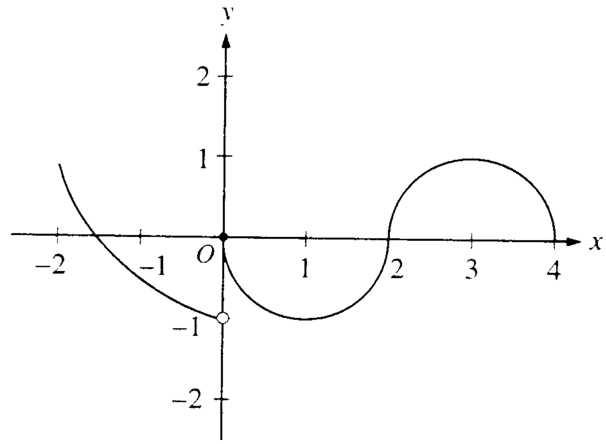
11. If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\dfrac{ab}{2}$ (D) $b - a$ (E) $\dfrac{b^2 - a^2}{2}$

---

12. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent



13. The graph of the function f shown in the figure above has a vertical tangent at the point $(2,0)$ and horizontal tangents at the points $(1,-1)$ and $(3,1)$. For what values of x, $-2 < x < 4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

---

14. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

---

15. If $F(x) = \int_0^x \sqrt{t^3 + 1}\, dt$, then $F'(2) =$

(A) -3 (B) -2 (C) 2 (D) 3 (E) 18

---

16. If $f(x) = \sin\left(e^{-x}\right)$, then $f'(x) =$

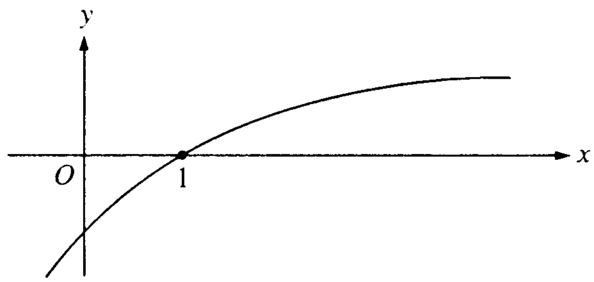
(A) $-\cos(e^{-x})$

(B) $\cos(e^{-x}) + e^{-x}$

(C) $\cos(e^{-x}) - e^{-x}$

(D) $e^{-x}\cos(e^{-x})$

(E) $-e^{-x}\cos(e^{-x})$



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

(D) $f''(1) < f(1) < f'(1)$

(E) $f''(1) < f'(1) < f(1)$

---

18. An equation of the line tangent to the graph of $y = x + \cos x$ at the point $(0,1)$ is

(A) $y = 2x + 1$ (B) $y = x + 1$ (C) $y = x$ (D) $y = x - 1$ (E) $y = 0$

---

19. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

(A) –1 only (B) 2 only (C) –1 and 0 only (D) –1 and 2 only (E) –1, 0, and 2 only

---

20. What are all values of k for which $\int_{-3}^{k} x^2 dx = 0$?

(A) –3 (B) 0 (C) 3 (D) –3 and 3 (E) –3, 0, and 3

---

21. If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

22. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

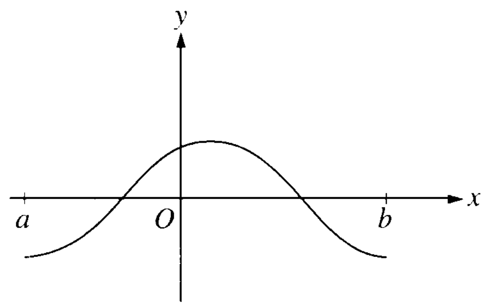
(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C) $(0, \infty)$

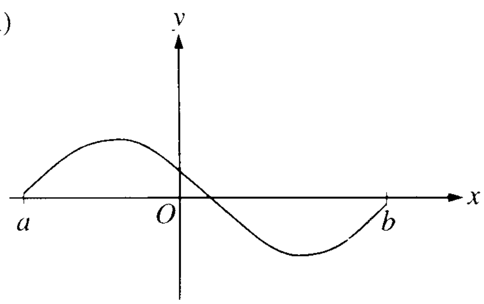
(D) $(-\infty, 0)$

(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

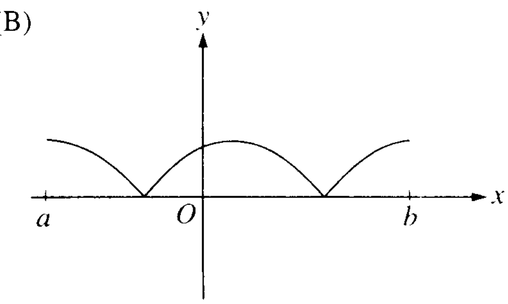


23. The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f?

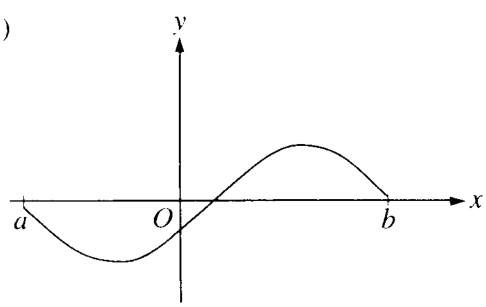
(A)



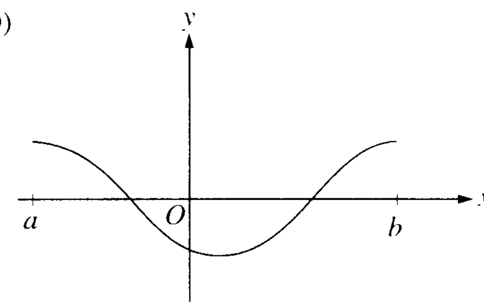
(B)



(C)



(D)



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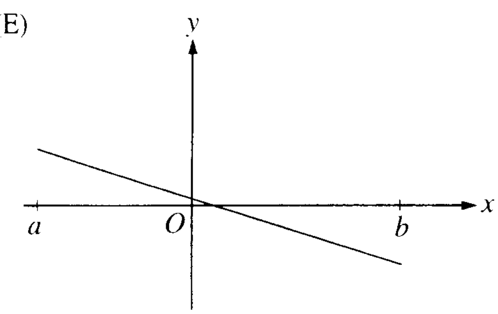


24. The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

---

25. What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$ (C) 4 (D) $\frac{14}{3}$ (E) $\frac{16}{3}$

---

x	0	1	2
$f(x)$	1	k	2

26. The function f is continuous on the closed interval $[0,2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0,2]$ if $k =$

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

---

27. What is the average value of $y = x^2\sqrt{x^3+1}$ on the interval $[0,2]$?

(A) $\frac{26}{9}$ (B) $\frac{52}{9}$ (C) $\frac{26}{3}$ (D) $\frac{52}{3}$ (E) 24

---

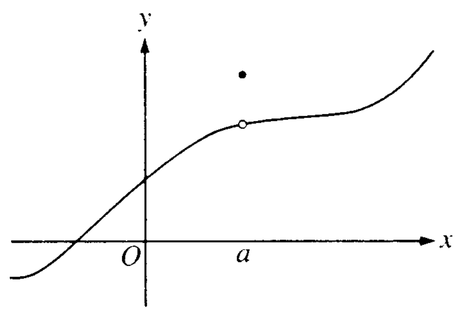
28. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

50 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.



76. The graph of a function f is shown above. Which of the following statements about f is false?

(A) f is continuous at $x = a$.

(B) f has a relative maximum at $x = a$.

(C) $x = a$ is in the domain of f.

(D) $\lim_{x\to a^+} f(x)$ is equal to $\lim_{x\to a^-} f(x)$.

(E) $\lim_{x\to a} f(x)$ exists.

---

77. Let f be the function given by $f(x) = 3e^{2x}$ and let g be the function given by $g(x) = 6x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) −0.701
(B) −0.567
(C) −0.391
(D) −0.302
(E) −0.258

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

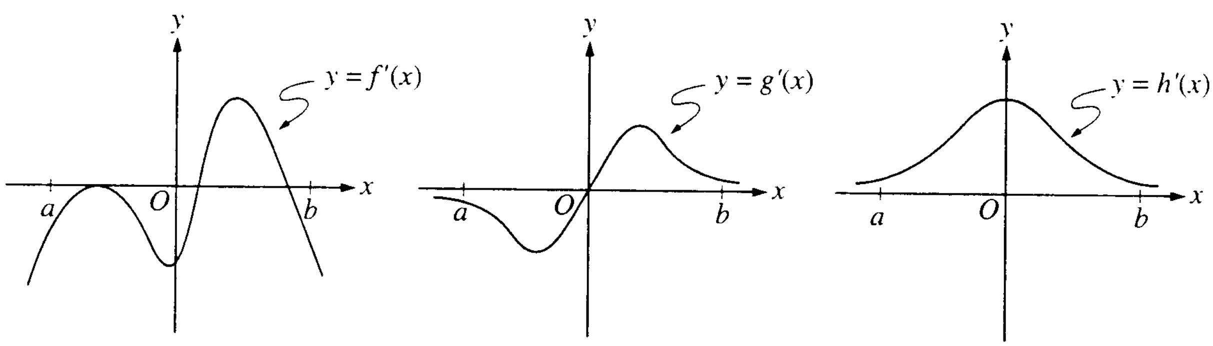
(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\frac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$



79. The graphs of the derivatives of the functions f, g, and h are shown above. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?

(A) f only
(B) g only
(C) h only
(D) f and g only
(E) f, g, and h

---

80. The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval $(0,10)$?

(A) One
(B) Three
(C) Four
(D) Five
(E) Seven

81. Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

I. f is continuous at $x = 0$.
II. f is differentiable at $x = 0$.
III. f has an absolute minimum at $x = 0$.

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

---

82. If f is a continuous function and if $F'(x) = f(x)$ for all real numbers x, then $\int_1^3 f(2x)\,dx =$

(A) $2F(3) - 2F(1)$

(B) $\frac{1}{2}F(3) - \frac{1}{2}F(1)$

(C) $2F(6) - 2F(2)$

(D) $F(6) - F(2)$

(E) $\frac{1}{2}F(6) - \frac{1}{2}F(2)$

---

83. If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

(A) $\frac{1}{a^2}$ (B) $\frac{1}{2a^2}$ (C) $\frac{1}{6a^2}$ (D) 0 (E) nonexistent

---

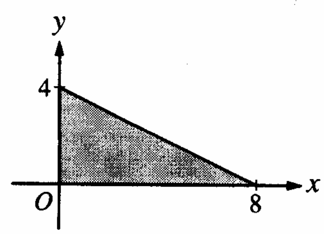
84. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) 0.069 (B) 0.200 (C) 0.301 (D) 3.322 (E) 5.000

x	2	5	7	8
$f(x)$	10	30	40	20

85. The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table above. Using the subintervals $[2,5]$, $[5,7]$, and $[7,8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210



86. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x+2y=8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

---

87. Which of the following is an equation of the line tangent to the graph of $f(x)=x^4+2x^2$ at the point where $f'(x)=1$?

(A) $y=8x-5$
(B) $y=x+7$
(C) $y=x+0.763$
(D) $y=x-0.122$
(E) $y=x-2.146$

---

88. Let $F(x)$ be an antiderivative of $\dfrac{(\ln x)^3}{x}$. If $F(1)=0$, then $F(9)=$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

89. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = \left(x^2 - 4\right) g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) f has relative minima at $x = -2$ and at $x = 2$.
(D) f has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if f has any relative extrema.

---

90. If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.

---

91. Let f be a function that is differentiable on the open interval $(1,10)$. If $f(2) = -5,\ f(5) = 5,$ and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some $c,\ 2 < c < 5,\ f(c) = 3$.

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II, and III

---

92. If $0 \le k < \frac{\pi}{2}$ and the area under the curve $y = \cos x$ from $x = k$ to $x = \frac{\pi}{2}$ is 0.1, then $k =$

(A) 1.471 (B) 1.414 (C) 1.277 (D) 1.120 (E) 0.436

55 Minutes—No Calculator

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

   (A) $-3 < x < 1$
   (B) $-1 < x < 1$
   (C) $x < -3$ or $x > 1$
   (D) $x < -1$ or $x > 3$
   (E) All real numbers

---

2. In the *xy*-plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$, for $-3 \le t \le 3$, is a line segment with slope

   (A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 3 (D) 5 (E) 13

---

3. The slope of the line tangent to the curve $y^2 + (xy+1)^3 = 0$ at $(2, -1)$ is

   (A) $-\frac{3}{2}$ (B) $-\frac{3}{4}$ (C) 0 (D) $\frac{3}{4}$ (E) $\frac{3}{2}$

---

4. $\int \frac{1}{x^2 - 6x + 8}\,dx =$

   (A) $\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C$

   (B) $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right| + C$

   (C) $\frac{1}{2}\ln\left|(x-2)(x-4)\right| + C$

   (D) $\frac{1}{2}\ln\left|(x-4)(x+2)\right| + C$

   (E) $\ln\left|(x-2)(x-4)\right| + C$

5. If f and g are twice differentiable and if $h(x) = f\left(g(x)\right)$, then $h''(x) =$

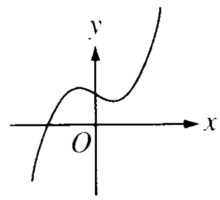
(A) $f''\left(g(x)\right)\left[g'(x)\right]^2 + f'\left(g(x)\right)g''(x)$

(B) $f''\left(g(x)\right)g'(x) + f'\left(g(x)\right)g''(x)$

(C) $f''\left(g(x)\right)\left[g'(x)\right]^2$

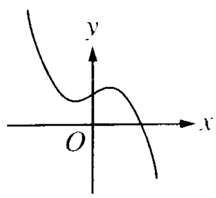
(D) $f''\left(g(x)\right)g''(x)$

(E) $f''\left(g(x)\right)$

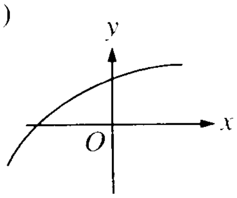


6. The graph of $y = h(x)$ is shown above. Which of the following could be the graph of $y = h'(x)$?

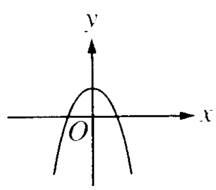
(A)



(B)



(C)



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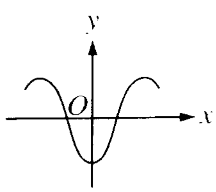
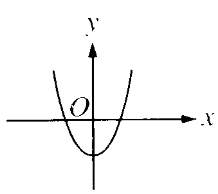


(E)



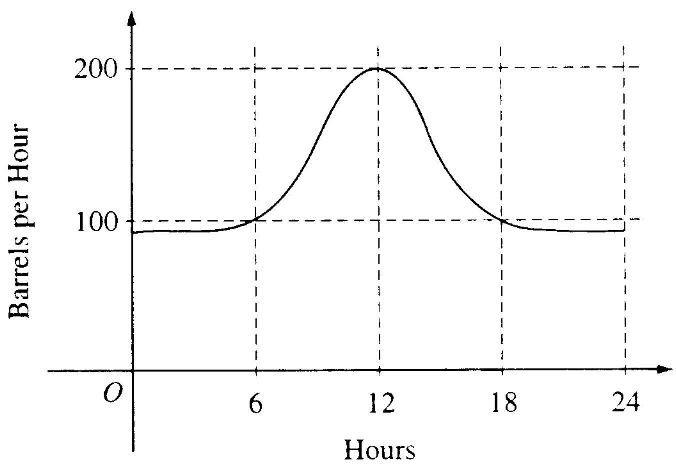
7. $\int_1^e \left(\frac{x^2-1}{x}\right) dx =$

(A) $e-\frac{1}{e}$ (B) e^2-e (C) $\frac{e^2}{2}-e+\frac{1}{2}$ (D) e^2-2 (E) $\frac{e^2}{2}-\frac{3}{2}$

---

8. If $\frac{dy}{dx}=\sin x\cos^2 x$ and if $y=0$ when $x=\frac{\pi}{2}$, what is the value of y when $x=0$?

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) 1



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

---

10. A particle moves on a plane curve so that at any time $t>0$ its x-coordinate is t^3-t and its y-coordinate is $(2t-1)^3$. The acceleration vector of the particle at $t=1$ is

(A) $(0,1)$ (B) $(2,3)$ (C) $(2,6)$ (D) $(6,12)$ (E) $(6,24)$

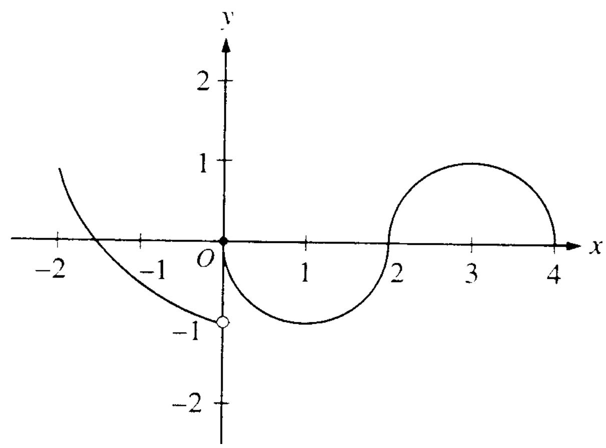
---

11. If f is a linear function and $0<a<b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b-a$ (E) $\frac{b^2-a^2}{2}$

12. If $f(x)=\begin{cases} \ln x & \text{for } 0<x\le 2 \\ x^2\ln 2 & \text{for } 2<x\le 4, \end{cases}$ then $\lim\limits_{x\to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent



13. The graph of the function f shown in the figure above has a vertical tangent at the point $(2,0)$ and horizontal tangents at the points $(1,-1)$ and $(3,1)$. For what values of x, $-2<x<4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

---

14. What is the approximation of the value of sin 1 obtained by using the fifth-degree Taylor polynomial about $x=0$ for $\sin x$?

(A) $1-\frac{1}{2}+\frac{1}{24}$

(B) $1-\frac{1}{2}+\frac{1}{4}$

(C) $1-\frac{1}{3}+\frac{1}{5}$

(D) $1-\frac{1}{4}+\frac{1}{8}$

(E) $1-\frac{1}{6}+\frac{1}{120}$

15. $\int x \cos x \, dx =$

(A) $x \sin x - \cos x + C$

(B) $x \sin x + \cos x + C$

(C) $-x \sin x + \cos x + C$

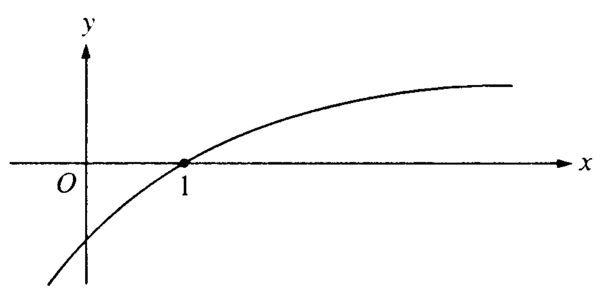
(D) $x \sin x + C$

(E) $\frac{1}{2} x^2 \sin x + C$

---

16. If f is the function defined by $f(x) = 3x^5 - 5x^4$, what are all the x-coordinates of points of inflection for the graph of f?

(A) -1 (B) 0 (C) 1 (D) 0 and 1 (E) -1, 0, and 1



17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

(D) $f''(1) < f(1) < f'(1)$

(E) $f''(1) < f'(1) < f(1)$

18. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$ II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty} \frac{1}{n}$

(A) None
(B) II only
(C) III only
(D) I and II only
(E) I and III only

---

19. The area of the region inside the polar curve $r = 4\sin\theta$ and outside the polar curve $r = 2$ is given by

(A) $\frac{1}{2}\int_0^{\pi} (4\sin\theta - 2)^2\, d\theta$ (B) $\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (4\sin\theta - 2)^2\, d\theta$ (C) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (4\sin\theta - 2)^2\, d\theta$

(D) $\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(16\sin^2\theta - 4\right) d\theta$ (E) $\frac{1}{2}\int_0^{\pi} \left(16\sin^2\theta - 4\right) d\theta$

---

20. When $x = 8$, the rate at which $\sqrt[3]{x}$ is increasing is $\frac{1}{k}$ times the rate at which x is increasing. What is the value of k?

(A) 3 (B) 4 (C) 6 (D) 8 (E) 12

---

21. The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \le t \le 1$, is given by

(A) $\int_0^1 \sqrt{t^2 + 1}\, dt$

(B) $\int_0^1 \sqrt{t^2 + t}\, dt$

(C) $\int_0^1 \sqrt{t^4 + t^2}\, dt$

(D) $\frac{1}{2}\int_0^1 \sqrt{4 + t^4}\, dt$

(E) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2 + 9}\, dt$

22. If $\lim_{b\to\infty}\int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

(A) $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges

(B) $\sum_{n=1}^{\infty}\frac{1}{n^p}$ diverges

(C) $\sum_{n=1}^{\infty}\frac{1}{n^{p-2}}$ converges

(D) $\sum_{n=1}^{\infty}\frac{1}{n^{p-1}}$ converges

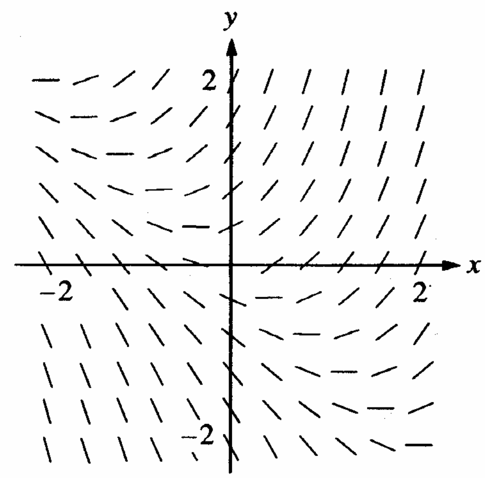
(E) $\sum_{n=1}^{\infty}\frac{1}{n^{p+1}}$ diverges

---

23. Let f be a function defined and continuous on the closed interval $[a,b]$. If f has a relative maximum at c and $a < c < b$, which of the following statements must be true?

I. $f'(c)$ exists.
II. If $f'(c)$ exists, then $f'(c) = 0$.
III. If $f''(c)$ exists, then $f''(c) \le 0$.

(A) II only (B) III only (C) I and II only (D) I and III only (E) II and III only



24. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = 1 + x$ (B) $\frac{dy}{dx} = x^2$ (C) $\frac{dy}{dx} = x + y$ (D) $\frac{dy}{dx} = \frac{x}{y}$ (E) $\frac{dy}{dx} = \ln y$

---

25. $\int_0^\infty x^2 e^{-x^3}\, dx$ is

(A) $-\frac{1}{3}$ (B) 0 (C) $\frac{1}{3}$ (D) 1 (E) divergent

---

26. The population $P(t)$ of a species satisfies the logistic differential equation $\frac{dP}{dt} = P\left(2 - \frac{P}{5000}\right)$, where the initial population $P(0) = 3{,}000$ and t is the time in years. What is $\lim_{t\to\infty} P(t)$?

(A) 2,500 (B) 3,000 (C) 4,200 (D) 5,000 (E) 10,000

---

27. If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1) =$

(A) 0 (B) a_1 (C) $\sum_{n=0}^{\infty} a_n$ (D) $\sum_{n=1}^{\infty} n a_n$ (E) $\sum_{n=1}^{\infty} n a_n^{\,n-1}$

---

28. $\lim_{x\to 1} \frac{\int_1^x e^{t^2}\, dt}{x^2 - 1}$ is

(A) 0 (B) 1 (C) $\frac{e}{2}$ (D) e (E) nonexistent

50 Minutes—Graphing Calculator Required

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

76. For what integer k, $k > 1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

---

77. If f is a vector-valued function defined by $f(t) = \left(e^{-t}, \cos t\right)$, then $f''(t) =$

(A) $-e^{-t} + \sin t$
(B) $e^{-t} - \cos t$
(C) $\left(-e^{-t}, -\sin t\right)$
(D) $\left(e^{-t}, \cos t\right)$
(E) $\left(e^{-t}, -\cos t\right)$

---

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference C, what is the rate of change of the area of the circle, in square centimeters per second?

(A) $-(0.2)\pi C$

(B) $-(0.1)C$

(C) $-\frac{(0.1)C}{2\pi}$

(D) $(0.1)^2 C$

(E) $(0.1)^2 \pi C$

79. Let f be the function given by $f(x)=\frac{(x-1)(x^2-4)}{x^2-a}$. For what positive values of a is f continuous for all real numbers x?

(A) None
(B) 1 only
(C) 2 only
(D) 4 only
(E) 1 and 4 only

---

80. Let R be the region enclosed by the graph of $y=1+\ln\left(\cos^4 x\right)$, the x-axis, and the lines $x=-\frac{2}{3}$ and $x=\frac{2}{3}$. The closest integer approximation of the area of R is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

---

81. If $\frac{dy}{dx}=\sqrt{1-y^2}$, then $\frac{d^2y}{dx^2}=$

(A) $-2y$ (B) $-y$ (C) $\frac{-y}{\sqrt{1-y^2}}$ (D) y (E) $\frac{1}{2}$

---

82. If $f(x)=g(x)+7$ for $3\le x\le 5$, then $\int_3^5 \left[f(x)+g(x)\right]dx=$

(A) $2\int_3^5 g(x)\,dx+7$

(B) $2\int_3^5 g(x)\,dx+14$

(C) $2\int_3^5 g(x)\,dx+28$

(D) $\int_3^5 g(x)\,dx+7$

(E) $\int_3^5 g(x)\,dx+14$

83. The Taylor series for $\ln x$, centered at $x=1$, is $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $0.3 \le x \le 1.7$ is

(A) 0.030 (B) 0.039 (C) 0.145 (D) 0.153 (E) 0.529

---

84. What are all values of x for which the series $\sum_{n=1}^{\infty}\frac{(x+2)^n}{\sqrt{n}}$ converges?

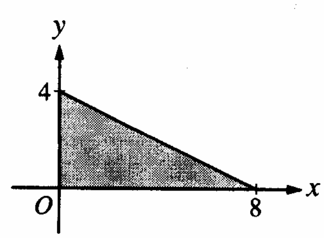
(A) $-3 < x < -1$ (B) $-3 \le x < -1$ (C) $-3 \le x \le -1$ (D) $-1 \le x < 1$ (E) $-1 \le x \le 1$

---

x	2	5	7	8
$f(x)$	10	30	40	20

85. The function f is continuous on the closed interval $[2,8]$ and has values that are given in the table above. Using the subintervals $[2,5]$, $[5,7]$, and $[7,8]$, what is the trapezoidal approximation of $\int_2^8 f(x)\,dx$?

(A) 110 (B) 130 (C) 160 (D) 190 (E) 210

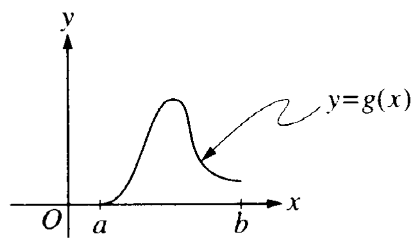


86. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x+2y=8$, as shown in the figure above. If cross sections of the solid perpendicular to the x-axis are semicircles, what is the volume of the solid?

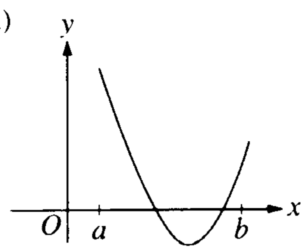
(A) 12.566 (B) 14.661 (C) 16.755 (D) 67.021 (E) 134.041

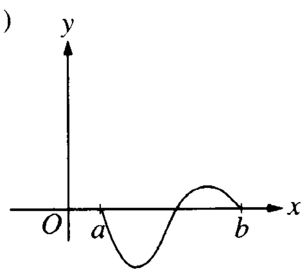
87. Which of the following is an equation of the line tangent to the graph of $f(x) = x^4 + 2x^2$ at the point where $f'(x) = 1$?

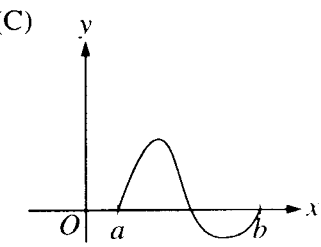
(A) $y = 8x - 5$
(B) $y = x + 7$
(C) $y = x + 0.763$
(D) $y = x - 0.122$
(E) $y = x - 2.146$

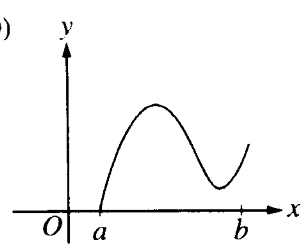


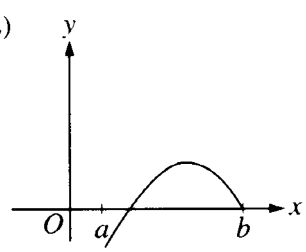
88. Let $g(x) = \int_a^x f(t)\,dt$, where $a \le x \le b$. The figure above shows the graph of g on $[a, b]$. Which of the following could be the graph of f on $[a, b]$?

(A)


(B)


(C)


(D)


(E)


89. The graph of the function represented by the Maclaurin series

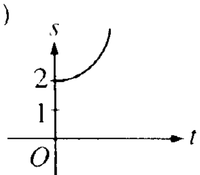
$$1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots + \frac{(-1)^n x^n}{n!} + \ldots$$ intersects the graph of $y = x^3$ at $x =$

(A) 0.773 (B) 0.865 (C) 0.929 (D) 1.000 (E) 1.857

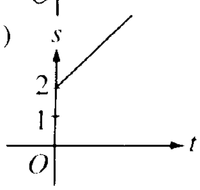
---

90. A particle starts from rest at the point $(2,0)$ and moves along the x-axis with a constant positive acceleration for time $t \geq 0$. Which of the following could be the graph of the distance $s(t)$ of the particle from the origin as a function of time t?

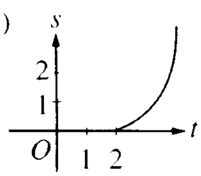
(A)



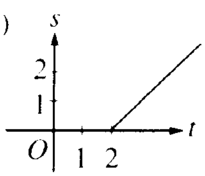
(B)



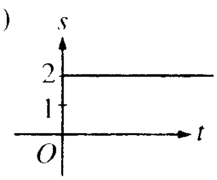
(C)



(D)



E)



t (sec)	0	2	4	6
$a(t)$ (ft/sec^2)	5	2	8	3

91. The data for the acceleration $a(t)$ of a car from 0 to 6 seconds are given in the table above. If the velocity at $t = 0$ is 11 feet per second, the approximate value of the velocity at $t = 6$, computed using a left-hand Riemann sum with three subintervals of equal length, is

(A) 26 ft/sec (B) 30 ft/sec (C) 37 ft/sec (D) 39 ft/sec (E) 41 ft/sec

---

92. Let f be the function given by $f(x) = x^2 - 2x + 3$. The tangent line to the graph of f at $x = 2$ is used to approximate values of $f(x)$. Which of the following is the greatest value of x for which the error resulting from this tangent line approximation is less than 0.5?

(A) 2.4 (B) 2.5 (C) 2.6 (D) 2.7 (E) 2.8

1969 Answer Key

1969 AB

1. B
2. C
3. B
4. D
5. E
6. B
7. D
8. B
9. C
10. E
11. B
12. A
13. C
14. E
15. B
16. B
17. B
18. E
19. C
20. A
21. B
22. E
23. C
24. C
25. A
26. C
27. C
28. C
29. A
30. E
31. C
32. B
33. A
34. D
35. A
36. B
37. D
38. C
39. D
40. E
41. D
42. D
43. D
44. C
45. D

1969 BC

1. C
2. E
3. B
4. D
5. E
6. B
7. D
8. C
9. D
10. A
11. B
12. E
13. C
14. D
15. B
16. B
17. B
18. E
19. C
20. A
21. B
22. E
23. D
24. C
25. A
26. C
27. C
28. D
29. C
30. D
31. C
32. B
33. A
34. D
35. A
36. B
37. D
38. A
39. D
40. E
41. D
42. B
43. E
44. E
45. E

1973 Answer Key

1973 AB

1. E
2. E
3. B
4. A
5. A
6. D
7. B
8. B
9. A
10. C
11. B
12. C
13. D
14. D
15. C
16. C
17. C
18. D
19. D
20. D
21. B
22. B
23. C
24. B
25. B
26. E
27. E
28. C
29. C
30. B
31. D
32. D
33. A
34. C
35. C
36. A
37. A
38. B
39. B
40. E
41. D
42. D
43. E
44. B
45. C

1973 BC

1. A
2. D
3. A
4. C
5. B
6. D
7. D
8. B
9. A
10. A
11. E
12. D
13. D
14. A
15. C
16. A
17. C
18. D
19. D
20. E
21. B
22. C
23. C
24. A
25. B
26. D
27. E
28. C
29. A
30. B
31. E
32. C
33. A
34. C
35. C
36. E
37. E
38. B
39. D
40. C
41. D
42. D
43. E
44. A
45. E

1985 Answer Key

1985 AB

1. D
2. E
3. A
4. C
5. D
6. C
7. E
8. B
9. D
10. D
11. B
12. C
13. A
14. D
15. C
16. B
17. C
18. C
19. B
20. A
21. B
22. A
23. B
24. D
25. E
26. E
27. D
28. C
29. D
30. B
31. C
32. D
33. B
34. A
35. D
36. B
37. D
38. C
39. E
40. D
41. E
42. C
43. B
44. A
45. A

1985 BC

1. D
2. A
3. B
4. D
5. D
6. E
7. A
8. C
9. B
10. A
11. A
12. A
13. B
14. C
15. C
16. C
17. B
18. C
19. D
20. C
21. B
22. A
23. C
24. D
25. C
26. E
27. E
28. E
29. D
30. B
31. D
32. E
33. C
34. A
35. B
36. E
37. A
38. C
39. A
40. A
41. C
42. E
43. E
44. A
45. D

1988 AB

1. C
2. D
3. A
4. E
5. A
6. D
7. D
8. B
9. E
10. C
11. A
12. B
13. A
14. D
15. B
16. C
17. D
18. E
19. B
20. C
21. C
22. C
23. B
24. C
25. B
26. E
27. E
28. C
29. B
30. A
31. C
32. A
33. A
34. D
35. B
36. C
37. D
38. E
39. E
40. B
41. A
42. C
43. B
44. C
45. D

1988 BC

1. A
2. D
3. B
4. E
5. C
6. C
7. A
8. A
9. D
10. D
11. A
12. B
13. B
14. A
15. E
16. A
17. D
18. E
19. B
20. E
21. D
22. E
23. E
24. D
25. D
26. C
27. B
28. E
29. B
30. C
31. C
32. E
33. E
34. C
35. A
36. E or D
37. D
38. C
39. C
40. E
41. B
42. A
43. A
44. A
45. B

1993 Answer Key

1993 AB

1. C
2. B
3. D
4. A
5. A
6. D
7. B
8. E
9. E
10. D
11. C
12. B
13. A
14. A
15. D
16. B
17. E
18. D
19. E
20. B
21. C
22. E
23. C
24. A
25. C
26. D
27. C
28. B
29. C
30. C
31. E
32. A
33. B
34. D
35. E
36. D
37. C
38. A
39. D
40. C
41. D
42. B
43. B
44. C
45. B

1993 BC

1. A
2. C
3. E
4. B
5. D
6. A
7. A
8. B
9. D
10. E
11. E
12. E
13. C
14. B
15. D
16. A
17. A
18. B
19. B
20. E
21. A
22. B
23. D
24. C
25. D
26. B
27. C
28. A
29. E
30. C
31. A
32. B
33. A
34. E
35. A
36. E
37. B
38. C
39. C
40. C
41. C
42. E
43. A
44. E
45. D

1997 Answer Key

1997 AB

1. C	21. E
2. A	22. D
3. C	23. A
4. D	24. B
5. E	25. A
6. C	76. E
7. D	77. D
8. C	78. D
9. B	79. C
10. E	80. A
11. E	81. A
12. B	82. B
13. A	83. C
14. C	84. C
15. B	85. C
16. D	86. A
17. A	87. B
18. C	88. E
19. D	89. B
20. E	90. D

1997 BC

1. C	21. A
2. E	22. C
3. A	23. E
4. C	24. D
5. C	25. A
6. A	76. D
7. C	77. E
8. E	78. A
9. A	79. D
10. B	80. B
11. C	81. D
12. A	82. B
13. B	83. E
14. C	84. C
15. D	85. D
16. B	86. A
17. B	87. B
18. C	88. C
19. D	89. D
20. E	90. B

1998 Answer Key

1998 AB

1. D
2. B
3. C
4. B
5. E
6. A
7. E
8. E
9. D
10. D
11. A
12. E
13. B
14. C
15. D
16. E
17. D
18. B
19. C
20. A
21. B
22. C
23. A
24. D
25. D
26. A
27. A
28. E
76. A
77. C
78. B
79. A
80. B
81. D
82. E
83. B
84. A
85. C
86. C
87. D
88. C
89. B
90. D
91. E
92. D

1998 BC

1. C
2. A
3. D
4. A
5. A
6. E
7. E
8. B
9. D
10. E
11. A
12. E
13. B
14. E
15. B
16. C
17. D
18. B
19. D
20. E
21. C
22. A
23. E
24. C
25. C
26. E
27. D
28. C
76. D
77. E
78. B
79. A
80. B
81. B
82. B
83. C
84. B
85. C
86. C
87. D
88. C
89. A
90. A
91. E
92. D

1. B Sine is the only odd function listed. $\sin(-x) = -\sin(x)$.

2. C $\ln t < 0$ for $0 < t < 1 \Rightarrow \ln(x-2) < 0$ for $2 < x < 3$.

3. B Need to have $\lim_{x\to 2} f(x) = f(2) = k$.

$$k = \lim_{x\to 2} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} = \lim_{x\to 2} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} \cdot \frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}$$

$$= \lim_{x\to 2} \frac{2x+5-(x+7)}{x-2} \cdot \frac{1}{\sqrt{2x+5}+\sqrt{x+7}} = \lim_{x\to 2} \frac{1}{\sqrt{2x+5}+\sqrt{x+7}} = \frac{1}{6}$$

4. D $\int_0^8 \frac{dx}{\sqrt{1+x}} = 2\sqrt{1+x}\Big|_0^8 = 2(3-1) = 4$

5. E Using implicit differentiation, $6x + 2xy' + 2y + 2y \cdot y' = 0$. Therefore $y' = \frac{-2y-6x}{2x+2y}$.

When $x = 1$, $3 + 2y + y^2 = 2 \Rightarrow 0 = y^2 + 2y + 1 = (y+1)^2 \Rightarrow y = -1$

Therefore $2x + 2y = 0$ and so $\frac{dy}{dx}$ is not defined at $x = 1$.

6. B This is the derivative of $f(x) = 8x^8$ at $x = \frac{1}{2}$

$$f'\left(\frac{1}{2}\right) = 64\left(\frac{1}{2}\right)^7 = \frac{1}{2}$$

7. D With $f(x) = x + \frac{k}{x}$, we need $0 = f'(-2) = 1 - \frac{k}{4}$ and so $k = 4$. Since $f''(-2) < 0$ for $k = 4$, f does have a relative maximum at $x = -2$.

8. B $p(x) = q(x)(x-1) + 12$ for some polynomial $q(x)$ and so $12 = p(1) = (1+2)(1+k) \Rightarrow k = 3$

9. C $A = \pi r^2$, $\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$ and from the given information in the problem $\frac{dA}{dt} = 2\frac{dr}{dt}$.

So, $2\frac{dr}{dt} = 2\pi r \cdot \frac{dr}{dt} \Rightarrow r = \frac{1}{\pi}$

10. E $x = e^y \Rightarrow y = \ln x$

11. B Let L be the distance from $\left(x, -\frac{x^2}{2}\right)$ and $\left(0, -\frac{1}{2}\right)$.

$$L^2 = (x-0)^2 + \left(\frac{x^2}{2} - \frac{1}{2}\right)^2$$

$$2L \cdot \frac{dL}{dx} = 2x + 2\left(\frac{x^2}{2} - \frac{1}{2}\right)(x)$$

$$\frac{dL}{dx} = \frac{2x + 2\left(\frac{x^2}{2} - \frac{1}{2}\right)(x)}{2L} = \frac{2x + x^3 - x}{2L} = \frac{x^3 + x}{2L} = \frac{x\left(x^2+1\right)}{2L}$$

$\frac{dL}{dx} < 0$ for all $x < 0$ and $\frac{dL}{dx} > 0$ for all $x > 0$, so the minimum distance occurs at $x = 0$.

The nearest point is the origin.

12. A $\frac{4}{2x-1} = 2\left(\frac{4}{x-1}\right) \Rightarrow x - 1 = 4x - 2;\ x = \frac{1}{3}$

13. C $\int_{-\pi/2}^{k} \cos x\, dx = 3\int_{k}^{\pi/2} \cos x\, dx;\ \sin k - \sin\left(-\frac{\pi}{2}\right) = 3\left(\sin\frac{\pi}{2} - \sin k\right)$

$\sin k + 1 = 3 - 3\sin k;\ 4\sin k = 2 \Rightarrow k = \frac{\pi}{6}$

14. E $y = x^5 - 1$ has an inverse $x = y^5 - 1 \Rightarrow y = \sqrt[5]{x+1}$

15. B The graphs do not need to intersect (eg. $f(x) = -e^{-x}$ and $g(x) = e^{-x}$). The graphs could intersect (e.g. $f(x) = 2x$ and $g(x) = x$). However, if they do intersect, they will intersect no more than once because $f(x)$ grows faster than $g(x)$.

16. B $y' > 0 \Rightarrow y$ is increasing; $y'' < 0 \Rightarrow$ the graph is concave down. Only B meets these conditions.

17. B $y' = 20x^3 - 5x^4$, $y'' = 60x^2 - 20x^3 = 20x^2(3-x)$. The only sign change in y'' is at $x = 3$. The only point of inflection is $(3,162)$.

18. E There is no derivative at the vertex which is located at $x = 3$.

19. C $\frac{dv}{dt} = \frac{1 - \ln t}{t^2} > 0$ for $0 < t < e$ and $\frac{dv}{dt} < 0$ for $t > e$, thus v has its maximum at $t = e$.

20. A $y(0) = 0$ and $y'(0) = \left. \frac{1/2}{\sqrt{1 - \frac{x^2}{4}}} \right|_{x=0} = \left. \frac{1}{\sqrt{4 - x^2}} \right|_{x=0} = \frac{1}{2}$. The tangent line is

$y = \frac{1}{2}x \Rightarrow x - 2y = 0$.

21. B $f'(x) = 2x - 2e^{-2x}$, $f'(0) = -2$, so f is decreasing

22. E $\ln e^{2x} = 2x \Rightarrow \frac{d}{dx}\left(\ln e^{2x}\right) = \frac{d}{dx}(2x) = 2$

23. C $\int_0^2 e^{2x}\,dx = \frac{1}{2}e^{2x}\Big|_0^2 = \frac{1}{2}\left(e^4 - 1\right)$

24. C $y = \ln \sin x$, $y' = \frac{\cos x}{\sin x} = \cot x$

25. A $\int_m^{2m} \frac{1}{x}\,dx = \ln x\Big|_m^{2m} = \ln(2m) - \ln(m) = \ln 2$ so the area is independent of m.

26. C $\int_0^1 \sqrt{x^2 - 2x + 1}\,dx = \int_0^1 |x - 1|\,dx = \int_0^1 -(x-1)\,dx = -\frac{1}{2}(x-1)^2\Big|_0^1 = \frac{1}{2}$

Alternatively, the graph of the region is a right triangle with vertices at (0,0), (0,1), and (1,0). The area is $\frac{1}{2}$.

27. C $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\ln|\cos x| + C = \ln|\sec x| + C$

28. C $\sqrt{3}\cos x + 3\sin x$ can be thought of as the expansion of $\sin(x+y)$. Since $\sqrt{3}$ and 3 are too large for values of $\sin y$ and $\cos y$, multiply and divide by the result of the Pythagorean Theorem used on those values, i.e. $2\sqrt{3}$. Then

$$\sqrt{3}\cos x + 3\sin x = 2\sqrt{3}\left(\frac{\sqrt{3}}{2\sqrt{3}}\cos x + \frac{3}{2\sqrt{3}}\sin x\right) = 2\sqrt{3}\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)$$
$$= 2\sqrt{3}\left(\sin y\cos x + \cos y\sin x\right) = 2\sqrt{3}\sin\left(y+x\right)$$

where $y = \sin^{-1}\left(\frac{1}{2}\right)$. The amplitude is $2\sqrt{3}$.

Alternatively, the function $f(x)$ is periodic with period 2π. $f'(x) = -\sqrt{3}\sin x + 3\cos x = 0$ when $\tan x = \sqrt{3}$. The solutions over one period are $x = \frac{\pi}{3}, \frac{4\pi}{3}$. Then $f\left(\frac{\pi}{3}\right) = 2\sqrt{3}$ and $f\left(\frac{4\pi}{3}\right) = -2\sqrt{3}$. So the amplitude is $2\sqrt{3}$.

29. A $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x}\,dx = \ln\left(\sin x\right)\Big|_{\pi/4}^{\pi/2} = \ln 1 - \ln\frac{1}{\sqrt{2}} = \ln\sqrt{2}$

30. E Because f is continuous for all x, the Intermediate Value Theorem implies that the graph of f must intersect the x-axis. The graph must also intersect the y-axis since f is defined for all x, in particular, at $x = 0$.

31. C $\frac{dy}{dx} = -y \Rightarrow y = ce^{-x}$ and $1 = ce^{-1} \Rightarrow c = e$; $y = e\cdot e^{-x} = e^{1-x}$

32. B If $a < 0$ then $\lim_{x\to-\infty} y = \infty$ and $\lim_{x\to\infty} y = -\infty$ which would mean that there is at least one root. If $a > 0$ then $\lim_{x\to-\infty} y = -\infty$ and $\lim_{x\to\infty} y = \infty$ which would mean that there is at least one root. In both cases the equation has at least one root.

33. A $\frac{1}{3}\int_{-1}^{2} 3t^3 - t^2\,dt = \frac{1}{3}\left(\frac{3}{4}t^4 - \frac{1}{3}t^3\right)\Big|_{-1}^{2} = \frac{1}{3}\left(\left(12 - \frac{8}{3}\right) - \left(\frac{3}{4} + \frac{1}{3}\right)\right) = \frac{11}{4}$

34. D $y' = -\frac{1}{x^2}$, so the desired curve satisfies $y' = x^2 \Rightarrow y = \frac{1}{3}x^3 + C$

35. A $a(t)=24t^2$, $v(t)=8t^3+C$ and $v(0)=0 \Rightarrow C=0$. The particle is always moving to the right, so distance = $\int_0^2 8t^3 dt = 2t^4\Big|_0^2 = 32$.

36. B $y=\sqrt{4+\sin x}$, $y(0)=2$, $y'(0)=\dfrac{\cos 0}{2\sqrt{4+\sin 0}}=\dfrac{1}{4}$. The linear approximation to y is $L(x)=2+\dfrac{1}{4}x$. $L(1.2)=2+\dfrac{1}{4}(1.2)=2.03$

37. D All options have the same value at $x=0$. We want the one that has the same first and second derivatives at $x=0$ as $y=\cos 2x$: $y'(0)=-2\sin 2x\Big|_{x=0}=0$ and $y''(0)=-4\cos 2x\Big|_{x=0}=-4$.

For $y=1-2x^2$, $y'(0)=-4x\Big|_{x=0}=0$ and $y''(0)=-4$ and no other option works.

38. C $\displaystyle\int \frac{x^2}{e^{x^3}}dx=-\frac{1}{3}\int e^{-x^3}(-3x^2dx)=-\frac{1}{3}e^{-x^3}+C=-\frac{1}{3e^{-x^3}}+C$

39. D $x=e \Rightarrow v=1,\ u=0,\ y=0;\ \dfrac{dy}{dx}=\dfrac{dy}{du}\cdot\dfrac{du}{dv}\cdot\dfrac{dv}{dx}=\left(\sec^2 u\right)\left(1+\dfrac{1}{v^2}\right)\left(\dfrac{1}{x}\right)=(1)(2)\left(e^{-1}\right)=\dfrac{2}{e}$

40. E One solution technique is to evaluate each integral and note that the value is $\dfrac{1}{n+1}$ for each.

Another technique is to use the substitution $u=1-x$; $\int_0^1(1-x)^n\,dx=\int_1^0 u^n(-du)=\int_0^1 u^n\,du$.
Integrals do not depend on the variable that is used and so $\int_0^1 u^n\,du$ is the same as $\int_0^1 x^n\,dx$.

41. D $\int_{-1}^3 f(x)\,dx=\int_{-1}^2\left(8-x^2\right)dx+\int_2^3 x^2dx=\left(8x-\dfrac{1}{3}x^3\right)\Big|_{-1}^{2}+\dfrac{1}{3}x^3\Big|_2^3=27\tfrac{1}{3}$

42. D $y=x^3-3x^2+k$, $y'=3x^2-6x=3x(x-2)$. So f has a relative maximum at $(0,k)$ and a relative minimum at $(2,k-4)$. There will be 3 distinct x-intercepts if the maximum and minimum are on the opposite sides of the x-axis. We want $k-4<0<k \Rightarrow 0<k<4$.

43. D $\int \sin(2x+3)\,dx=-\dfrac{1}{2}\cos(2x+3)+C$

44. C Since $\cos 2A = 2\cos^2 A - 1$, we have $3 - 2\cos^2 \frac{\pi x}{3} = 3 - (1 + \cos \frac{2\pi x}{3})$ and the latter expression has period $\frac{2\pi}{\left(\frac{2\pi}{3}\right)} = 3$

45. D Let $y = f(x^3)$. We want y'' where $f'(x) = g(x)$ and $f''(x) = g'(x) = f(x^2)$

$$
\begin{aligned}
y &= f(x^3) \\
y' &= f'(x^3) \cdot 3x^2 \\
y'' &= 3x^2 \left(f''(x^3) \cdot 3x^2 \right) + f'(x^3) \cdot 6x \\
&= 9x^4 f''(x^3) + 6x\, f'(x^3) = 9x^4 f((x^3)^2) + 6x\, g(x^3) = 9x^4 f(x^6) + 6x\, g(x^3)
\end{aligned}
$$

1969 Calculus BC Solutions

1. C For horizontal asymptotes consider the limit as $x \to \pm\infty$: $t \to 0 \Rightarrow y = 0$ is an asymptote
For vertical asymptotes consider the limit as $y \to \pm\infty$: $t \to -1 \Rightarrow x = -1$ is an asymptote

2. E $y = (x+1)\tan^{-1} x$, $y' = \dfrac{x+1}{1+x^2} + \tan^{-1} x$

$$y'' = \frac{\left(1+x^2\right)(1) - (x+1)(2x)}{\left(1+x^2\right)^2} + \frac{1}{1+x^2} = \frac{2-2x}{\left(1+x^2\right)^2}$$

y'' changes sign at $x = 1$ only. The point of inflection is $\left(1, \pi/2\right)$

3. B $y = \sqrt{x}$, $y' = \dfrac{1}{2\sqrt{x}}$. By the Mean Value Theorem we have $\dfrac{1}{2\sqrt{c}} = \dfrac{2}{4} \Rightarrow c = 1$.

The point is $(1,1)$.

4. D $\displaystyle\int_0^8 \frac{dx}{\sqrt{1+x}} dx = 2\sqrt{1+x}\,\Big|_0^8 = 2(3-1) = 4$

5. E Using implicit differentiation, $6x + 2xy' + 2y + 2y \cdot y' = 0$. Therefore $y' = \dfrac{-2y-6x}{2x+2y}$.

When $x = 1$, $3 + 2y + y^2 = 2 \Rightarrow 0 = y^2 + 2y + 1 = (y+1)^2 \Rightarrow y = -1$

Therefore $2x + 2y = 0$ and so $\dfrac{dy}{dx}$ is not defined at $x = 1$.

6. B This is the derivative of $f(x) = 8x^8$ at $x = \dfrac{1}{2}$.

$$f'\left(\frac{1}{2}\right) = 64\left(\frac{1}{2}\right)^7 = \frac{1}{2}$$

7. D With $f(x) = x + \dfrac{k}{x}$, we need $0 = f'(-2) = 1 - \dfrac{k}{4}$ and so $k = 4$. Since $f''(-2) < 0$ for $k = 4$, f does have a relative maximum at $x = -2$.

8. C $h'(x) = 2f(x)\cdot f'(x) - 2g(x)\cdot g'(x) = 2f(x)\cdot\left(-g(x)\right) - 2g(x)\cdot f(x) = -4f(x)\cdot g(x)$

9. D $A=\frac{1}{2}\int_0^{2\pi}\left(\sqrt{3+\cos\theta}\right)^2 d\theta = 2\cdot\frac{1}{2}\int_0^{\pi}\left(\sqrt{3+\cos\theta}\right)^2 d\theta=\int_0^{\pi}(3+\cos\theta)\,d\theta$

10. A $\int_0^1 \frac{x^2}{x^2+1}dx=\int_0^1\frac{x^2+1-1}{x^2+1}dx=\int_0^1\left(\frac{x^2+1}{x^2+1}-\frac{1}{x^2+1}\right)dx=\left(x-\tan^{-1}x\right)\Big|_0^1=1-\frac{\pi}{4}=\frac{4-\pi}{4}$

11. B Let L be the distance from $\left(x,-\frac{x^2}{2}\right)$ and $\left(0,-\frac{1}{2}\right)$.

$$L^2=(x-0)^2+\left(\frac{x^2}{2}-\frac{1}{2}\right)^2$$

$$2L\cdot\frac{dL}{dx}=2x+2\left(\frac{x^2}{2}-\frac{1}{2}\right)(x)$$

$$\frac{dL}{dx}=\frac{2x+2\left(\frac{x^2}{2}-\frac{1}{2}\right)(x)}{2L}=\frac{2x+x^3-x}{2L}=\frac{x^3+x}{2L}=\frac{x\left(x^2+1\right)}{2L}$$

$\frac{dL}{dx}<0$ for all $x<0$ and $\frac{dL}{dx}>0$ for all $x>0$, so the minimum distance occurs at $x=0$.

The nearest point is the origin.

12. E By the Fundamental Theorem of Calculus, if $F(x)=\int_0^x e^{-t^2}dt$ then $F'(x)=e^{-x^2}$.

13. C $\int_{-\pi/2}^{k}\cos x\,dx=3\int_k^{\pi/2}\cos x\,dx$; $\sin k-\sin\left(-\frac{\pi}{2}\right)=3\left(\sin\frac{\pi}{2}-\sin k\right)$

$\sin k+1=3-3\sin k$; $4\sin k=2\Rightarrow k=\pi/6$

14. D $y=x^2+2$ and $u=2x-1$, $\frac{dy}{du}=\frac{dy}{dx}\cdot\frac{dx}{du}=(2x)\left(\frac{1}{2}\right)=x$

15. B The graphs do not need to intersect (eg. $f(x)=-e^{-x}$ and $g(x)=e^{-x}$). The graphs could intersect (e.g. $f(x)=2x$ and $g(x)=x$). However, if they do intersect, they will intersect no more than once because $f(x)$ grows faster than $g(x)$.

16. B $y' > 0 \Rightarrow y$ is increasing; $y'' < 0 \Rightarrow$ the graph is concave down . Only B meets these conditions.

17. B $y' = 20x^3 - 5x^4$, $y'' = 60x^2 - 20x^3 = 20x^2(3-x)$. The only sign change in y'' is at $x = 3$. The only point of inflection is $(3,162)$.

18. E There is no derivative at the vertex which is located at $x = 3$.

19. C $\dfrac{dv}{dt} = \dfrac{1-\ln t}{t^2} > 0$ for $0 < t < e$ and $\dfrac{dv}{dt} < 0$ for $t > e$, thus v has its maximum at $t = e$.

20. A $y(0) = 0$ and $y'(0) = \left.\dfrac{1/2}{\sqrt{1-\dfrac{x^2}{4}}}\right|_{x=0} = \left.\dfrac{1}{\sqrt{4-x^2}}\right|_{x=0} = \dfrac{1}{2}$. The tangent line is

$y = \dfrac{1}{2}x \Rightarrow x - 2y = 0$.

21. B $f'(x) = 2x - 2e^{-2x}$, $f'(0) = -2$, so f is decreasing

22. E $f(x) = \int_0^x \dfrac{1}{\sqrt{t^3+2}}\,dt$, $f(-1) = \int_0^{-1} \dfrac{1}{\sqrt{t^3+2}}\,dt = -\int_{-1}^{0} \dfrac{1}{\sqrt{t^3+2}}\,dt < 0$

$f(-1) < 0$ so E is false.

23. D $\dfrac{dy}{dx} = \dfrac{-xe^{-x^2}}{y} \Rightarrow 2y\,dy = -2xe^{-x^2}dx \Rightarrow y^2 = e^{-x^2} + C$

$4 = 1 + C \Rightarrow C = 3$; $y^2 = e^{-x^2} + 3 \Rightarrow y = \sqrt{e^{-x^2}+3}$

24. C $y = \ln\sin x$, $y' = \dfrac{\cos x}{\sin x} = \cot x$

25. A $\int_m^{2m} \dfrac{1}{x}\,dx = \ln x\Big|_m^{2m} = \ln(2m) - \ln(m) = \ln 2$ so the area is independent of m.

26. C $\int_0^1 \sqrt{x^2-2x+1}\,dx = \int_0^1 |x-1|\,dx = \int_0^1 -(x-1)\,dx = -\frac{1}{2}(x-1)^2\Big|_0^1 = \frac{1}{2}$

Alternatively, the graph of the region is a right triangle with vertices at (0,0), (0,1), and (1,0). The area is $\frac{1}{2}$.

27. C $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\ln|\cos x| + C = \ln|\sec x| + C$

28. D Use L'Hôpital's Rule: $\lim_{x\to 0} \frac{e^{2x}-1}{\tan x} = \lim_{x\to 0} \frac{2e^{2x}}{\sec^2 x} = 2$

29. C Make the subsitution $x = 2\sin\theta \Rightarrow dx = 2\cos\theta\,d\theta$.

$\int_0^1 \left(4-x^2\right)^{-3/2} dx = \int_0^{\pi/6} \frac{2\cos\theta}{8\cos^3\theta}\,d\theta = \frac{1}{4}\int_0^{\pi/6} \sec^2\theta\,d\theta = \frac{1}{4}\tan\theta\Big|_0^{\pi/6} = \frac{1}{4}\cdot\frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{12}$

30. D Substitute $-x$ for x in $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$ to get $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!} = e^{-x}$

31. C $\frac{dy}{dx} = -y \Rightarrow y = ce^{-x}$ and $1 = ce^{-1} \Rightarrow c = e;\ y = e\cdot e^{-x} = e^{1-x}$

32. B $1+2^x+3^x+4^x+\cdots+n^x+\cdots = \sum_{n=1}^{\infty} \frac{1}{n^p}$ where $p=-x$. This is a *p*-series and is convergent if $p>1 \Rightarrow -x>1 \Rightarrow x<-1$.

33. A $\frac{1}{3}\int_{-1}^{2} 3t^3 - t^2\,dt = \frac{1}{3}\left(\frac{3}{4}t^4 - \frac{1}{3}t^3\right)\Big|_{-1}^{2} = \frac{1}{3}\left(\left(12-\frac{8}{3}\right)-\left(\frac{3}{4}+\frac{1}{3}\right)\right) = \frac{11}{4}$

34. D $y' = -\frac{1}{x^2}$, so the desired curve satisfies $y' = x^2 \Rightarrow y = \frac{1}{3}x^3 + C$

35. A $a(t) = 24t^2$, $v(t) = 8t^3 + C$ and $v(0) = 0 \Rightarrow C = 0$. The particle is always moving to the right, so distance = $\int_0^2 8t^3 dt = 2t^4\Big|_0^2 = 32$.

36. B $y=\sqrt{4+\sin x}$, $y(0)=2$, $y'(0)=\dfrac{\cos 0}{2\sqrt{4+\sin 0}}=\dfrac{1}{4}$. The linear approximation to y is

$L(x)=2+\frac{1}{4}x$. $L(1.2)=2+\frac{1}{4}(1.2)=2.03$

37. D This item uses the formal definition of a limit and is no longer part of the AP Course Description. Need to have $|(1-3x)-(-5)|<\varepsilon$ whenever $0<|x-2|<\delta$.
$|(1-3x)-(-5)|=|6-3x|=3|x-2|<\varepsilon$ if $|x-2|<\varepsilon/3$.
Thus we can use any $\delta<\varepsilon/3$. Of the five choices, the largest satisfying this condition is $\delta=\varepsilon/4$.

38. A Note $f(1)=\frac{1}{2}$. Take the natural logarithm of each side of the equation and then differentiate.

$$\ln f(x)=(2-3x)\ln\left(x^2+1\right);\ \frac{f'(x)}{f(x)}=(2-3x)\cdot\frac{2x}{x^2+1}-3\ln\left(x^2+1\right)$$

$$f'(1)=f(1)\left((-1)\cdot\frac{2}{2}-3\ln(2)\right)\Rightarrow f'(1)=\frac{1}{2}\left(-1-3\ln 2\right)=-\frac{1}{2}\left(\ln e+\ln 2^3\right)=-\frac{1}{2}\ln 8e$$

39. D $x=e\Rightarrow v=1,\ u=0,\ y=0;\ \dfrac{dy}{dx}=\dfrac{dy}{du}\cdot\dfrac{du}{dv}\cdot\dfrac{dv}{dx}=\left(\sec^2 u\right)\left(1+\dfrac{1}{v^2}\right)\left(\dfrac{1}{x}\right)=(1)(2)\left(e^{-1}\right)=\dfrac{2}{e}$

40. E One solution technique is to evaluate each integral and note that the value is $\dfrac{1}{n+1}$ for each.

Another technique is to use the substitution $u=1-x$; $\int_0^1(1-x)^n\,dx=\int_1^0 u^n\,(-du)=\int_0^1 u^n\,du$.

Integrals do not depend on the variable that is used and so $\int_0^1 u^n\,du$ is the same as $\int_0^1 x^n\,dx$.

41. D $\int_{-1}^{3} f(x)\,dx=\int_{-1}^{2}\left(8-x^2\right)dx+\int_2^3 x^2dx=\left(8x-\frac{1}{3}x^3\right)\Big|_{-1}^{2}+\frac{1}{3}x^3\Big|_2^3=27\tfrac{1}{3}$

42. B Use the technique of antiderivatives by parts to evaluate $\int x^2 \cos x \, dx$

$u = x^2 \qquad dv = \cos x \, dx$

$du = 2x \, dx \qquad v = \sin x$

$f(x) - \int 2x \sin x \, dx = \int x^2 \cos x \, dx = x^2 \sin x - \int 2x \sin x \, dx + C$

$f(x) = x^2 \sin x + C$

43. E $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx = \int_a^b \sqrt{1 + \left(\sec^2 x\right)^2} \, dx = \int_a^b \sqrt{1 + \sec^4 x} \, dx$

44. E $y'' - y' - 2y = 0$, $y'(0) = -2$, $y(0) = 2$; the characteristic equation is $r^2 - r - 2 = 0$.

The solutions are $r = -1$, $r = 2$ so the general solution to the differential equation is

$y = c_1 e^{-x} + c_2 e^{2x}$ with $y' = -c_1 e^{-x} + 2c_2 e^{2x}$. Using the initial conditions we have the system:

$2 = c_1 + c_2$ and $-2 = -c_1 + 2c_2 \Rightarrow c_2 = 0$, $c_1 = 2$. The solution is $f(x) = 2e^{-x} \Rightarrow f(1) = 2e^{-1}$.

45. E The ratio test shows that the series is convergent for any value of x that makes $|x+1| < 1$. The solutions to $|x+1| = 1$ are the endpoints of the interval of convergence. Test $x = -2$ and $x = 0$ in the series. The resulting series are $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$ and $\sum_{k=1}^{\infty} \frac{1}{k^2}$ which are both convergent. The interval is $-2 \le x \le 0$.

1973 Calculus AB Solutions

1. E $\int (x^3 - 3x)\,dx = \frac{1}{4}x^4 - \frac{3}{2}x^2 + C$

2. E $g(x) = 5 \Rightarrow g\left(f(x)\right) = 5$

3. B $y = \ln x^2$; $y' = \frac{2x}{x^2} = \frac{2}{x}$. At $x = e^2$, $y' = \frac{2}{e^2}$.

4. A $f(x) = x + \sin x$; $f'(x) = 1 - \cos x$

5. A $\lim_{x\to-\infty} e^x = 0 \Rightarrow y = 0$ is a horizontal asymptote

6. D $f'(x) = \frac{(1)(x+1) - (x-1)(1)}{(x+1)^2}$, $f'(1) = \frac{2}{4} = \frac{1}{2}$

7. B Replace x with $(-x)$ and see if the result is the opposite of the original. This is true for B.
$-(-x)^5 + 3(-x) = x^5 - 3x = -(-x^5 + 3x)$.

8. B Distance $= \int_1^2 \left| t^2 \right| dx = \int_1^2 t^2\,dt = \frac{1}{3}t^3 \Big|_1^2 = \frac{1}{3}(2^3 - 1^3) = \frac{7}{3}$

9. A $y' = 2\cos 3x \cdot \frac{d}{dx}(\cos 3x) = 2\cos 3x \cdot (-\sin 3x) \cdot \frac{d}{dx}(3x) = 2\cos 3x \cdot (-\sin 3x) \cdot (3)$
$y' = -6\sin 3x \cos 3x$

10. C $f(x) = \frac{x^4}{3} - \frac{x^5}{5}$; $f'(x) = \frac{4x^3}{3} - x^4$; $f''(x) = 4x^2 - 4x^3 = 4x^2(1-x)$
$f'' > 0$ for $x < 1$ and $f'' < 0$ for $x > 1 \Rightarrow f'$ has its maximum at $x = 1$.

11. B Curve and line have the same slope when $3x^2 = \frac{3}{4} \Rightarrow x = \frac{1}{2}$. Using the line, the point of tangency is $\left(\frac{1}{2}, \frac{3}{8}\right)$. Since the point is also on the curve, $\frac{3}{8} = \left(\frac{1}{2}\right)^3 + k \Rightarrow k = \frac{1}{4}$.

12. C Substitute the points into the equation and solve the resulting linear system.
$3 = 16 + 4A + 2B - 5$ and $-37 = -16 + 4A - 2B - 5$; $A = -3$, $B = 2 \Rightarrow A + B = -1$.

13. D $v(t) = 8t - 3t^2 + C$ and $v(1) = 25 \Rightarrow C = 20$ so $v(t) = 8t - 3t^2 + 20$.

$s(4) - s(2) = \int_2^4 v(t)\,dt = (4t^2 - t^3 + 20t)\Big|_2^4 = 32$

14. D $f(x) = x^{1/3}(x-2)^{2/3}$

$f'(x) = x^{1/3} \cdot \frac{2}{3}(x-2)^{-1/3} + (x-2)^{2/3} \cdot \frac{1}{3}x^{-2/3} = \frac{1}{3}x^{-2/3}(x-2)^{-1/3}(3x-2)$

f' is not defined at $x = 0$ and at $x = 2$.

15. C Area $= \int_0^2 e^{\frac{x}{2}}\,dx = 2e^{\frac{x}{2}}\Big|_0^2 = 2(e-1)$

16. C $\frac{dN}{dt} = 3000e^{\frac{2}{5}t}$, $N = 7500e^{\frac{2}{5}t} + C$ and $N(0) = 7500 \Rightarrow C = 0$

$N = 7500e^{\frac{2}{5}t}$, $N(5) = 7500e^2$

17. C Determine where the curves intersect. $-x^2 + x + 6 = 4 \Rightarrow x^2 - x - 2 = 0$

$(x-2)(x+1) = 0 \Rightarrow x = -1,\ x = 2$. Between these two x values the parabola lies above the line $y = 4$.

Area $= \int_{-1}^2 \left((-x^2 + x + 6) - 4\right)dx = \left(-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x\right)\Big|_{-1}^2 = \frac{9}{2}$

18. D $\frac{d}{dx}(\arcsin 2x) = \frac{1}{\sqrt{1-(2x)^2}} \cdot \frac{d}{dx}(2x) = \frac{2}{\sqrt{1-(2x)^2}} = \frac{2}{\sqrt{1-4x^2}}$

19. D If f is strictly increasing then it must be one to one and therefore have an inverse.

20. D By the Fundamental Theorem of Calculus, $\int_a^b f(x)\,dx = F(b) - F(a)$ where $F'(x) = f(x)$.

21. B $\int_0^1 (x+1)e^{x^2+2x}\,dx = \frac{1}{2}\int_0^1 e^{x^2+2x}\left((2x+2)\,dx\right) = \frac{1}{2}\left(e^{x^2+2x}\right)\Big|_0^1 = \frac{1}{2}\left(e^3 - e^0\right) = \frac{e^3-1}{2}$

22. B $f(x) = 3x^5 - 20x^3$; $f'(x) = 15x^4 - 60x^2$; $f''(x) = 60x^3 - 120x = 60x\left(x^2 - 2\right)$

The graph of f is concave up where $f'' > 0$: $f'' > 0$ for $x > \sqrt{2}$ and for $-\sqrt{2} < x < 0$.

23. C $\lim_{h\to 0}\frac{\ln(2+h)-\ln 2}{h}=f'(2)$ where $f(x)=\ln x$; $f'(x)=\frac{1}{x}\Rightarrow f'(2)=\frac{1}{2}$

24. B $f(x)=\cos(\arctan x)$; $-\frac{\pi}{2}<\arctan x<\frac{\pi}{2}$ and the cosine in this domain takes on all values in the interval $(0,1]$.

25. B $\int_0^{\frac{\pi}{4}}\tan^2 x\,dx=\int_0^{\frac{\pi}{4}}(\sec^2 x-1)\,dx=(\tan x-x)\Big|_0^{\pi/4}=1-\frac{\pi}{4}$

26. E $\frac{dV}{dt}=4\pi r^2\cdot\frac{dr}{dt}=S\cdot\frac{dr}{dt}=100\pi(0.3)=30\pi$

27. E $\int_0^{1/2}\frac{2x}{\sqrt{1-x^2}}dx=-\int_0^{1/2}\left(1-x^2\right)^{-\frac{1}{2}}(-2x\,dx)=-2\left(1-x^2\right)^{\frac{1}{2}}\Big|_0^{\frac{1}{2}}=2-\sqrt{3}$

28. C $v(t)=8-6t$ changes sign at $t=\frac{4}{3}$. Distance $=\left|x(1)-x\left(\frac{4}{3}\right)\right|+\left|x(2)-x\left(\frac{4}{3}\right)\right|=\frac{5}{3}$.

Alternative Solution: Distance $=\int_1^2\left|v(t)\right|dt=\int_1^2\left|8-6t\right|dt=\frac{5}{3}$

29. C $-1\le\sin x\le 1\Rightarrow -\frac{3}{2}\le\sin x-\frac{1}{2}\le\frac{1}{2}$; The maximum for $\left|\sin x-\frac{1}{2}\right|$ is $\frac{3}{2}$.

30. B $\int_1^2\frac{x-4}{x^2}dx=\int_1^2\left(\frac{1}{x}-4x^{-2}\right)dx=\left(\ln x+\frac{4}{x}\right)\Big|_1^2=(\ln 2+2)-(\ln 1+4)=\ln 2-2$

31. D $\log_a\left(2^a\right)=\frac{a}{4}\Rightarrow\log_a 2=\frac{1}{4}\Rightarrow 2=a^{\frac{1}{4}}$; $a=16$

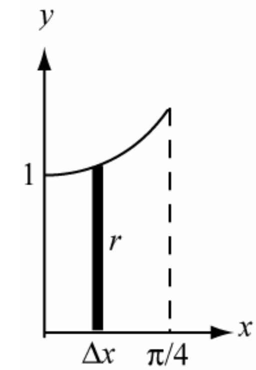
32. D $\int\frac{5}{1+x^2}dx=5\int\frac{1}{1+x^2}dx=5\tan^{-1}(x)+C$

33. A $f(-x)=-f(x)\Rightarrow f'(-x)\cdot(-1)=-f'(x)\Rightarrow f'(-x)=-f'(x)$ thus $f'(-x_0)=-f'(x_0)$.

34. C $\frac{1}{2}\int_0^2 \sqrt{x}\,dx = \frac{1}{2}\cdot\frac{2}{3}x^{\frac{3}{2}}\Big|_0^2 = \frac{1}{3}\cdot 2^{\frac{3}{2}} = \frac{2}{3}\sqrt{2}$

35. C Washers: $\sum \pi r^2 \Delta x$ where $r = y = \sec x$.

Volume $= \pi\int_0^{\frac{\pi}{4}} \sec^2 x\,dx = \pi \tan x\Big|_0^{\pi/4} = \pi(\tan\frac{\pi}{4} - \tan 0) = \pi$



36. A $y = e^{nx}$, $y' = ne^{nx}$, $y'' = n^2 e^{nx}, \cdots, y^{(n)} = n^n e^{nx}$

37. A $\frac{dy}{dx} = 4y$, $y(0) = 4$. This is exponential growth. The general solution is $y = Ce^{4x}$. Since $y(0) = 4$, $C = 4$ and so the solution is $y = 4e^{4x}$.

38. B Let $z = x - c$. Then $5 = \int_1^2 f(x-c)\,dx = \int_{1-c}^{2-c} f(z)\,dz$

39. B Use the distance formula to determine the distance, L, from any point $(x, y) = (x, \frac{1}{2}x^2)$ on the curve to the point $(4,1)$. The distance L satisfies the equation $L^2 = (x-4)^2 + \left(\frac{1}{2}x^2 - 1\right)$. Determine where L is a maximum by examining critical points. Differentiating with respect to x, $2L\cdot\frac{dL}{dx} = 2(x-4) + 2\left(\frac{1}{2}x^2 - 1\right)x = x^3 - 8$. $\frac{dL}{dx}$ changes sign from positive to negative at $x = 2$ only. The point on the curve has coordinates $(2,2)$.

40. E $\sec^2(xy)\cdot(xy' + y) = 1$, $xy'\sec^2(xy) + y\sec^2(xy) = 1$, $y' = \frac{1 - y\sec^2(xy)}{x\sec^2(xy)} = \frac{\cos^2(xy) - y}{x}$

41. D $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0}(x+1)\,dx + \int_0^1 \cos(\pi x)\,dx = \frac{1}{2}(x+1)^2\Big|_{-1}^{0} + \frac{1}{\pi}\sin(\pi x)\Big|_0^1$

$= \frac{1}{2} + \frac{1}{\pi}(\sin\pi - \sin 0) = \frac{1}{2}$

42. D $\Delta x = \frac{1}{3};\ T = \frac{1}{2}\cdot\frac{1}{3}\left(1^2 + 2\left(\frac{4}{3}\right)^2 + 2\left(\frac{5}{3}\right)^2 + 2^2\right) = \frac{127}{54}$

43. E Solve $\frac{x}{2} = -1$ and $\frac{x}{2} = 2$; $x = -2,\ 4$

44. B Use the linearization of $f(x) = \sqrt[4]{x}$ at $x = 16$. $f'(x) = \frac{1}{4}x^{-\frac{3}{4}}$, $f'(16) = \frac{1}{32}$

$L(x) = 2 + \frac{1}{32}(x - 16);\ f(16+h) \approx L(16+h) = 2 + \frac{h}{32}$

45. C This uses the definition of continuity of f at $x = x_0$.

1. A $f'(x) = e^{\frac{1}{x}} \cdot \dfrac{d\left(\frac{1}{x}\right)}{dx} = e^{\frac{1}{x}}\left(-\dfrac{1}{x^2}\right) = -\dfrac{e^{\frac{1}{x}}}{x^2}$

2. D $\int_0^3 (x+1)^{\frac{1}{2}}\,dx = \dfrac{2}{3}(x+1)^{\frac{3}{2}}\Big|_0^3 = \dfrac{2}{3}\left(4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) = \dfrac{2}{3}(8-1) = \dfrac{14}{3}$

3. A $f'(x) = 1 - \dfrac{1}{x^2} = \dfrac{(x+1)(x-1)}{x^2}$. $f'(x) > 0$ for $x < -1$ and for $x > 1$.

f is increasing for $x \le -1$ and for $x \ge 1$.

4. C The slopes will be negative reciprocals at the point of intersection.

$3x^2 = 3 \Rightarrow x = \pm 1$ and $x \ge 0$, thus $x = 1$ and the y values must be the same at $x = 1$.

$-\dfrac{1}{3} + b = 1 \Rightarrow b = \dfrac{4}{3}$

5. B $\int_{-1}^{2} \dfrac{|x|}{x}\,dx = \int_{-1}^{0} -1\,dx + \int_0^2 dx = -1 + 2 = 1$

6. D $f'(x) = \dfrac{(1)(x+1) - (x-1)(1)}{(x+1)^2}$, $f'(1) = \dfrac{2}{4} = \dfrac{1}{2}$

7. D $\dfrac{dy}{dx} = \dfrac{2x + 2y \cdot \frac{dy}{dx}}{x^2 + y^2}$ at $(1,0) \Rightarrow y' = \dfrac{2}{1} = 2$

8. B $y = \sin x$, $y' = \cos x$, $y'' = -\sin x$, $y''' = -\cos x$, $y^{(4)} = \sin x$

9. A $y' = 2\cos 3x \cdot \dfrac{d}{dx}(\cos 3x) = 2\cos 3x \cdot (-\sin 3x) \cdot \dfrac{d}{dx}(3x) = 2\cos 3x \cdot (-\sin 3x) \cdot 3$

$y' = -6 \sin 3x \cos 3x$

10. A $L=\int_0^b \sqrt{1+\left(y'\right)^2}\,dx=\int_0^b \sqrt{1+\left(\frac{\sec x\tan x}{\sec x}\right)^2}\,dx$

$=\int_0^b \sqrt{1+\left(\tan x\right)^2}\,dx=\int_0^b \sqrt{\sec^2 x}\,dx=\int_0^b \sec x\,dx$

11. E $dy=\left(x\cdot\frac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}}(2x)+\left(1+x^2\right)^{\frac{1}{2}}\right)dx$; $dy=(0+1)(2)=2$

12. D $\frac{1}{n}=\int_1^k x^{n-1}\,dx=\left.\frac{x^n}{n}\right|_1^k \Rightarrow \frac{1}{n}=\frac{k^n}{n}-\frac{1}{n}$; $\frac{k^n}{n}=\frac{2}{n}\Rightarrow k=2^{\frac{1}{n}}$

13. D $v(t)=8t-3t^2+C$ and $v(1)=25\Rightarrow C=20$ so $v\left(t\right)=8t-3t^2+20$.

$s(4)-s(2)=\int_2^4 v(t)\,dt=\left.\left(4t^2-t^3+20t\right)\right|_2^4=32$

14. A $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2e^t}{2t}=\frac{e^t}{t}$

15. C Area $=\int_0^2 e^{\frac{1}{2}x}\,dx=\left.2e^{\frac{1}{2}x}\right|_0^2=2(e-1)$

16. A $\sin t=t-\frac{t^3}{3!}+\frac{t^5}{5!}-\frac{t^7}{7!}+\cdots\Rightarrow\frac{\sin t}{t}=1-\frac{t^2}{3!}+\frac{t^4}{5!}-\frac{t^6}{7!}+\cdots$

17. C $\frac{dN}{dt}=3000e^{\frac{2}{5}t}$, $N=7500e^{\frac{2}{5}t}+C$ and $N(0)=7500\Rightarrow C=0$

$N=7500e^{\frac{2}{5}t}$, $N(5)=7500e^2$

18. D D could be false, consider $g(x)=1-x$ on $[0,1]$. A is true by the Extreme Value Theorem, B is true because g is a function, C is true by the Intermediate Value Theorem, and E is true because g is continuous.

19. D I is a convergent p-series, $p = 2 > 1$
II is the Harmonic series and is known to be divergent,
III is convergent by the Alternating Series Test.

20. E $\int x\sqrt{4-x^2}\,dx = -\frac{1}{2}\int \left(4-x^2\right)^{\frac{1}{2}}\left(-2x\,dx\right) = -\frac{1}{2}\cdot\frac{2}{3}\left(4-x^2\right)^{\frac{3}{2}} + C = -\frac{1}{3}\left(4-x^2\right)^{\frac{3}{2}} + C$

21. B $\int_0^1 (x+1)e^{x^2+2x}\,dx = \frac{1}{2}\int_0^1 e^{x^2+2x}\left((2x+2)\,dx\right) = \frac{1}{2}\left(e^{x^2+2x}\right)\Big|_0^1 = \frac{1}{2}\left(e^3 - e^0\right) = \frac{e^3-1}{2}$

22. C $x'(t) = t+1 \Rightarrow x(t) = \frac{1}{2}(t+1)^2 + C$ and $x(0) = 1 \Rightarrow C = \frac{1}{2} \Rightarrow x(t) = \frac{1}{2}(t+1)^2 + \frac{1}{2}$

$x(1) = \frac{5}{2},\ y(1) = \ln\frac{5}{2};\qquad \left(\frac{5}{2},\ \ln\frac{5}{2}\right)$

23. C $\lim_{h\to 0}\frac{\ln(2+h)-\ln 2}{h} = f'(2)$ where $f(x) = \ln x$; $f'(x) = \frac{1}{x} \Rightarrow f'(2) = \frac{1}{2}$

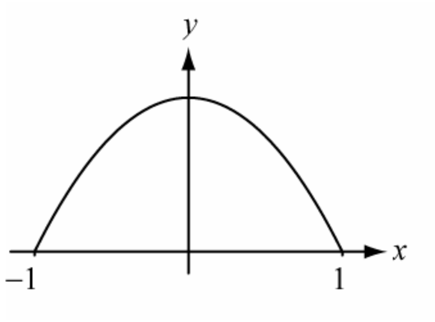
24. A This item uses the formal definition of a limit and is no longer part of the AP Course Description. $\left|f(x)-7\right| = \left|(3x+1)-7\right| = \left|3x-6\right| = 3\left|x-2\right| < \varepsilon$ whenever $\left|x-2\right| < \frac{\varepsilon}{3}$.
Any $\delta < \frac{\varepsilon}{3}$ will be sufficient and $\frac{\varepsilon}{4} < \frac{\varepsilon}{3}$, thus the answer is $\frac{\varepsilon}{4}$.

25. B $\int_0^{\frac{\pi}{4}} \tan^2 x\,dx = \int_0^{\frac{\pi}{4}}\left(\sec^2 x - 1\right)dx = \left(\tan x - x\right)\Big|_0^{\pi/4} = 1 - \frac{\pi}{4}$

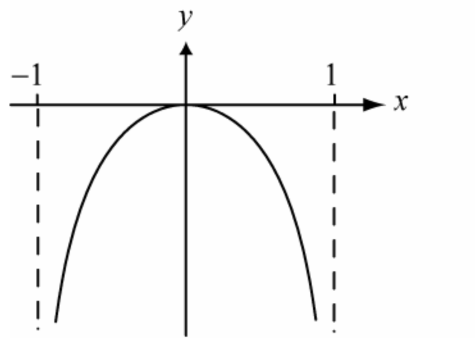
26. D For x in the interval (–1, 1), $g(x) = \left|x^2-1\right| = -(x^2-1)$ and so $y = \ln g(x) = \ln(-(x^2-1))$.
Therefore

$$y' = \frac{2x}{x^2-1},\quad y'' = \frac{\left(x^2-1\right)(2) - (2x)(2x)}{\left(x^2-1\right)^2} = \frac{-2x^2-2}{\left(x^2-1\right)^2} < 0$$

Alternative graphical solution: Consider the graphs of $g(x) = |x^2 - 1|$ and $\ln g(x)$.



$g(x) = |x^2 - 1|$



$\ln|x^2 - 1|$

concave down

27. E $f'(x) = x^2 - 8x + 12 = (x-2)(x-6)$; the candidates are: $x = 0, 2, 6, 9$

x	0	2	6	9
$f(x)$	−5	$17/3$	−5	22

the maximum is at $x = 9$

28. C $x = \sin^2 y \Rightarrow dx = 2\sin y \cos y\, dy$; when $x = 0$, $y = 0$ and when $x = \frac{1}{2}$, $y = \frac{\pi}{4}$

$$\int_0^{\frac{1}{2}} \frac{\sqrt{x}}{\sqrt{1-x}}\,dx = \int_0^{\frac{\pi}{4}} \frac{\sin y}{\sqrt{1-\sin^2 y}} \cdot 2\sin y \cos y\, dy = \int_0^{\frac{\pi}{4}} 2\sin^2 y\, dy$$

29. A Let $z = y'$. Then $z = e$ when $x = 0$. Thus $y'' = 2y' \Rightarrow z' = 2z$. Solve this differential equation.

$z = Ce^{2x}$; $e = Ce^0 \Rightarrow C = e \Rightarrow y' = z = e^{2x+1}$. Solve this differential equation.

$$y = \frac{1}{2}e^{2x+1} + K;\ e = \frac{1}{2}e^1 + K \Rightarrow K = \frac{1}{2}e;\ y = \frac{1}{2}e^{2x+1} + \frac{1}{2}e,\ y(1) = \frac{1}{2}e^3 + \frac{1}{2}e = \frac{1}{2}e\left(e^2 + 1\right)$$

Alternative Solution: $y'' = 2y' \Rightarrow y' = Ce^{2x} = e \cdot e^{2x}$. Therefore $y'(1) = e^3$.

$y'(1) - y'(0) = \int_0^1 y''(x)dx = \int_0^1 2y'(x)dx = 2y(1) - 2y(0)$ and so

$$y(1) = \frac{y'(1) - y'(0) + 2y(0)}{2} = \frac{e^3 + e}{2}.$$

30. B $\int_1^2 \frac{x-4}{x^2}\,dx = \int_1^2 \left(\frac{1}{x} - 4x^{-2}\right)dx = \left(\ln x + \frac{4}{x}\right)\Big|_1^2 = (\ln 2 + 2) - (\ln 1 + 4) = \ln 2 - 2$

31. E $f'(x) = \frac{\frac{d}{dx}(\ln x)}{\ln x} = \frac{\frac{1}{x}}{\ln x} = \frac{1}{x \ln x}$

32. C Take the log of each side of the equation and differentiate. $\ln y = \ln x^{\ln x} = \ln x \cdot \ln x = (\ln x)^2$

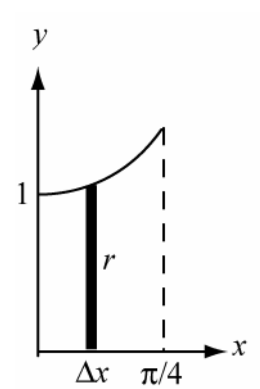
$$\frac{y'}{y} = 2\ln x \cdot \frac{d}{dx}(\ln x) = \frac{2}{x}\ln x \Rightarrow y' = x^{\ln x}\left(\frac{2}{x}\ln x\right)$$

33. A $f(-x) = -f(x) \Rightarrow f'(-x)\cdot(-1) = -f'(x) \Rightarrow f'(-x) = -f'(x)$ thus $f'(-x_0) = -f'(x_0)$.

34. C $\frac{1}{2}\int_0^2 \sqrt{x}\,dx = \frac{1}{2}\cdot\frac{2}{3}x^{\frac{3}{2}}\Big|_0^2 = \frac{1}{3}\cdot 2^{\frac{3}{2}} = \frac{2}{3}\sqrt{2}$

35. C Washers: $\sum \pi r^2 \Delta x$ where $r = y = \sec x$.

Volume $= \pi\int_0^{\frac{\pi}{4}} \sec^2 x\,dx = \pi \tan x\Big|_0^{\frac{\pi}{4}} = \pi\left(\tan\frac{\pi}{4} - \tan 0\right) = \pi$



36. E $\int_0^1 \frac{x+1}{x^2+2x-3}\,dx = \frac{1}{2}\lim_{L\to 1^-}\int_0^L \frac{2x+2}{x^2+2x-3}\,dx = \frac{1}{2}\lim_{L\to 1^-} \ln\left|x^2+2x-3\right|\Big|_0^L$

$$= \frac{1}{2}\lim_{L\to 1^-}\left(\ln\left|L^2+2L-3\right| - \ln\left|-3\right|\right) = -\infty.$$ Divergent

37. E $\lim_{x\to 0}\frac{1-\cos^2 2x}{x^2} = \lim_{x\to 0}\frac{\sin^2 2x}{x^2} = \lim_{x\to 0}\frac{\sin 2x}{2x}\cdot\frac{\sin 2x}{2x}\cdot 4 = 1\cdot 1\cdot 4 = 4$

38. B Let $z = x - c$. $5 = \int_1^2 f(x-c)\,dx = \int_{1-c}^{2-c} f(z)\,dz$

39. D $h'(x) = f'(g(x))\cdot g'(x)$; $h'(1) = f'(g(1))\cdot g'(1) = f'(2)\cdot g'(1) = (-4)(-3) = 12$

40. C Area $=\frac{1}{2}\int_0^{2\pi}(1-\cos\theta)^2 d\theta=\int_0^{\pi}\left(1-2\cos\theta+\cos^2\theta\right)d\theta$; $\cos^2\theta=\frac{1}{2}(1+\cos 2\theta)$

$$\text{Area}=\int_0^{\pi}\left(1-2\cos\theta+\frac{1}{2}(1+\cos 2\theta)\right)d\theta=\left(\frac{3}{2}\theta-2\sin\theta+\frac{1}{4}\sin 2\theta\right)\Bigg|_0^{\pi}=\frac{3}{2}\pi$$

41. D $\int_{-1}^{1} f(x)\,dx=\int_{-1}^{0}(x+1)\,dx+\int_0^1\cos(\pi x)\,dx$

$$=\frac{1}{2}(x+1)^2\Bigg|_{-1}^{0}+\frac{1}{\pi}\sin(\pi x)\Bigg|_0^1=\frac{1}{2}+\frac{1}{\pi}(\sin\pi-\sin 0)=\frac{1}{2}$$

42. D $\Delta x=\frac{1}{3}$; $T=\frac{1}{2}\cdot\frac{1}{3}\left(1^2+2\left(\frac{4}{3}\right)^2+2\left(\frac{5}{3}\right)^2+2^2\right)=\frac{127}{54}$

43. E Use the technique of antiderivatives by part:

$u=\sin^{-1}x \qquad dv=dx$

$du=\dfrac{dx}{\sqrt{1-x^2}} \qquad v=x$

$$\int\sin^{-1}x\,dx=x\sin^{-1}x-\int\frac{x}{\sqrt{1-x^2}}dx$$

44. A Multiply both sides of $x=xf'(x)-f(x)$ by $\frac{1}{x^2}$. Then $\frac{1}{x}=\frac{xf'(x)-f(x)}{x^2}=\frac{d}{dx}\left(\frac{f(x)}{x}\right)$.

Thus we have $\frac{f(x)}{x}=\ln|x|+C\Rightarrow f(x)=x\left(\ln|x|+C\right)=x\left(\ln|x|-1\right)$ since $f(-1)=1$.

Therefore $f(e^{-1})=e^{-1}\left(\ln\left|e^{-1}\right|-1\right)=e^{-1}(-1-1)=-2e^{-1}$

This was most likely the solution students were expected to produce while solving this problem on the 1973 multiple-choice exam. However, the problem itself is not well-defined. A solution to an initial value problem should be a function that is differentiable on an interval containing the initial point. In this problem that would be the domain $x<0$ since the solution requires the choice of the branch of the logarithm function with $x<0$. Thus one cannot ask about the value of the function at $x=e^{-1}$.

45. E $F'(x)=xg'(x)$ with $x\geq 0$ and $g'(x)<0\Rightarrow F'(x)<0\Rightarrow F$ is not increasing.

1. D $\int_1^2 x^{-3}\,dx = -\frac{1}{2}x^{-2}\Big|_1^2 = -\frac{1}{2}\left(\frac{1}{4}-1\right) = \frac{3}{8}$.

2. E $f'(x) = 4(2x+1)^3\cdot 2$, $f''(1) = 4\cdot 3(2x+1)^2\cdot 2^2$, $f'''(1) = 4\cdot 3\cdot 2(2x+1)^1\cdot 2^3$,
$f^{(4)}(1) = 4!\cdot 2^4 = 384$

3. A $y = 3(4+x^2)^{-1}$ so $y' = -3(4+x^2)^{-2}(2x) = \frac{-6x}{(4+x^2)^2}$

Or using the quotient rule directly gives $y' = \frac{\left(4+x^2\right)(0)-3(2x)}{\left(4+x^2\right)^2} = \frac{-6x}{\left(4+x^2\right)^2}$

4. C $\int \cos(2x)\,dx = \frac{1}{2}\int \cos(2x)\,(2\,dx) = \frac{1}{2}\sin(2x)+C$

5. D $\lim_{n\to\infty}\frac{4n^2}{n^2+10000n} = \lim_{n\to\infty}\frac{4}{1+\frac{10000}{n}} = 4$

6. C $f'(x) = 1 \Rightarrow f'(5) = 1$

7. E $\int_1^4 \frac{1}{t}\,dt = \ln t\big|_1^4 = \ln 4 - \ln 1 = \ln 4$

8. B $y = \ln\left(\frac{x}{2}\right) = \ln x - \ln 2$, $y' = \frac{1}{x}$, $y'(4) = \frac{1}{4}$

9. D Since e^{-x^2} is even, $\int_{-1}^0 e^{-x^2}\,dx = \frac{1}{2}\int_{-1}^1 e^{-x^2}\,dx = \frac{1}{2}k$

10. D $y' = 10^{(x^2-1)}\cdot\ln(10)\cdot\frac{d}{dx}\left((x^2-1)\right) = 2x\cdot 10^{(x^2-1)}\cdot\ln(10)$

11. B $v(t) = 2t+4 \Rightarrow a(t) = 2 \therefore a(4) = 2$

12. C $f\left(g(x)\right) = \ln\left(g(x)^2\right) = \ln\left(x^2+4\right) \Rightarrow g(x) = \sqrt{x^2+4}$

13. A $2x + x \cdot y' + y + 3y^2 \cdot y' = 0 \Rightarrow y' = -\dfrac{2x+y}{x+3y^2}$

14. D Since $v(t) \geq 0$, distance = $\int_0^4 \left| v(t) \right| dt = \int_0^4 \left(3t^{\frac{1}{2}} + 5t^{\frac{3}{2}} \right) dt = \left(2t^{\frac{3}{2}} + 2t^{\frac{5}{2}} \right)\Bigg|_0^4 = 80$

15. C $x^2 - 4 > 0 \Rightarrow |x| > 2$

16. B $f'(x) = 3x^2 - 6x = 3x(x-2)$ changes sign from positive to negative only at $x = 0$.

17. C Use the technique of antiderivatives by parts:

$u = x \qquad dv = e^{-x}\,dx$

$du = dx \qquad v = -e^{-x}$

$-xe^{-x} + \int e^{-x}\,dx = \left(-xe^{-x} - e^{-x} \right)\Big|_0^1 = 1 - 2e^{-1}$

18. C $y = \cos^2 x - \sin^2 x = \cos 2x$, $y' = -2\sin 2x$

19. B Quick solution: lines through the origin have this property.

Or, $f(x_1) + f(x_2) = 2x_1 + 2x_2 = 2(x_1 + x_2) = f(x_1 + x_2)$

20. A $\dfrac{dy}{dx} = \dfrac{1}{1+\cos^2 x} \cdot \dfrac{d}{dx}(\cos x) = \dfrac{-\sin x}{1+\cos^2 x}$

21. B $|x| > 1 \Rightarrow x^2 > 1 \Rightarrow f(x) < 0$ for all x in the domain. $\lim\limits_{|x|\to\infty} f(x) = 0$. $\lim\limits_{|x|\to 1} f(x) = -\infty$. The only option that is consistent with these statements is (B).

22. A $\int_1^2 \dfrac{x^2-1}{x+1}\,dx = \int_1^2 \dfrac{(x+1)(x-1)}{x+1}\,dx = \int_1^2 (x-1)\,dx = \dfrac{1}{2}(x-1)^2 \Big|_1^2 = \dfrac{1}{2}$

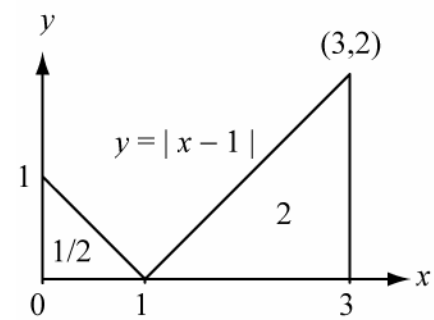
23. B $\dfrac{d}{dx}\left(x^{-3} - x^{-1} + x^2 \right)\Big|_{x=-1} = \left(-3x^{-4} + x^{-2} + 2x \right)\Big|_{x=-1} = -3 + 1 - 2 = -4$

24. D $16 = \int_{-2}^{2} (x^7 + k)\,dx = \int_{-2}^{2} x^7\,dx + \int_{-2}^{2} k\,dx = 0 + \left(2 - (-2) \right)k = 4k \Rightarrow k = 4$

25. E $f'(e) = \lim_{h \to 0} \frac{f(e+h) - f(e)}{h} = \lim_{h \to 0} \frac{e^{e+h} - e^e}{h}$

26. E I: Replace y with $(-y)$: $(-y)^2 = x^2 + 9 \Rightarrow y^2 = x^2 + 9$, no change, so yes.
II: Replace x with $(-x)$: $y^2 = (-x)^2 + 9 \Rightarrow y^2 = x^2 + 9$, no change, so yes.
III: Since there is symmetry with respect to both axes there is origin symmetry.

27. D The graph is a V with vertex at $x = 1$. The integral gives the sum of the areas of the two triangles that the V forms with the horizontal axis for x from 0 to 3. These triangles have areas of 1/2 and 2 respectively.



28. C Let $x(t) = -5t^2$ be the position at time t. Average velocity $= \frac{x(3) - x(0)}{3 - 0} = \frac{-45 - 0}{3} = -15$

29. D The tangent function is not defined at $x = \pi/2$ so it cannot be continuous for all real numbers. Option E is the only one that includes item III. In fact, the functions in I and II are a power and an exponential function that are known to be continuous for all real numbers x.

30. B $\int \tan(2x)\,dx = -\frac{1}{2}\int \frac{-2\sin(2x)}{\cos(2x)}\,dx = -\frac{1}{2}\ln\left|\cos(2x)\right| + C$

31. C $V = \frac{1}{3}\pi r^2 h$, $\frac{dV}{dt} = \frac{1}{3}\pi\left(2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right) = \frac{1}{3}\pi\left(2(6)(9)\left(\frac{1}{2}\right) + 6^2\left(\frac{1}{2}\right)\right) = 24\pi$

32. D $\int_0^{\pi/3} \sin(3x)\,dx = -\frac{1}{3}\cos(3x)\Big|_0^{\pi/3} = -\frac{1}{3}\left(\cos\pi - \cos 0\right) = \frac{2}{3}$

33. B f' changes sign from positive to negative at $x = -1$ and therefore f changes from increasing to decreasing at $x = -1$.

Or f' changes sign from positive to negative at $x = -1$ and from negative to positive at $x = 1$. Therefore f has a local maximum at $x = -1$ and a local minimum at $x = 1$.

34. A $\int_0^1 \left((x+8) - (x^3+8)\right)dx = \int_0^1 (x - x^3)\,dx = \left(\frac{1}{2}x^2 - \frac{1}{4}x^4\right)\Big|_0^1 = \frac{1}{4}$

35. D The amplitude is 2 and the period is 2.

$y = A\sin Bx$ where $|A| =$ amplitude $= 2$ and $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{2} = \pi$

36. B II is true since $|-7| = 7$ will be the maximum value of $|f(x)|$. To see why I and III do not have to be true, consider the following: $f(x) = \begin{cases} 5 & \text{if} \quad x \le -5 \\ -x & \text{if} \quad -5 < x < 7 \\ -7 & \text{if} \quad x \ge 7 \end{cases}$

For $f(|x|)$, the maximum is 0 and the minimum is –7.

37. D $\lim_{x\to 0} x\csc x = \lim_{x\to 0} \frac{x}{\sin x} = 1$

38. C To see why I and II do not have to be true consider $f(x) = \sin x$ and $g(x) = 1 + e^x$. Then $f(x) \le g(x)$ but neither $f'(x) \le g'(x)$ nor $f''(x) < g''(x)$ is true for all real values of x.

III is true, since

$f(x) \le g(x) \Rightarrow g(x) - f(x) \ge 0 \Rightarrow \int_0^1 \big(g(x) - f(x)\big)\,dx \ge 0 \Rightarrow \int_0^1 f(x)\,dx \le \int_0^1 g(x)\,dx$

39. E $f'(x) = \frac{1}{x}\cdot\frac{1}{x} - \frac{1}{x^2}\ln x = \frac{1}{x^2}(1 - \ln x) < 0$ for $x > e$. Hence f is decreasing. for $x > e$.

40. D $\int_0^2 f(x)\,dx \le \int_0^2 4\,dx = 8$

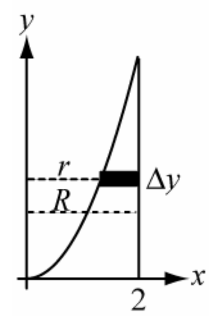
41. E Consider the function whose graph is the horizontal line $y = 2$ with a hole at $x = a$. For this function $\lim_{x\to a} f(x) = 2$ and none of the given statements are true.

42. C This is a direct application of the Fundamental Theorem of Calculus: $f'(x) = \sqrt{1 + x^2}$

43. B $y' = 3x^2 + 6x$, $y'' = 6x + 6 = 0$ for $x = -1$. $y'(-1) = -3$. Only option B has a slope of –3.

44. A $\frac{1}{2}\int_0^2 x^2\left(x^3+1\right)^{1/2} dx = \frac{1}{2}\cdot\frac{1}{3}\int_0^2 \left(x^3+1\right)^{1/2}\left(3x^2dx\right) = \frac{1}{6}\left(x^3+1\right)^{3/2}\cdot\frac{2}{3}\Big|_0^2 = \frac{26}{9}$

45. A Washers: $\sum \pi\left(R^2 - r^2\right)\Delta y$ where $R = 2,\ r = x$



$$\text{Volume} = \pi\int_0^4 \left(2^2 - x^2\right)dy = \pi\int_0^4 (4-y)\,dy = \pi\left(4y - \frac{1}{2}y^2\right)\Bigg|_0^4 = 8\pi$$

1. D $\int_0^2 (4x^3+2)\,dx = (x^4+2x)\Big|_0^2 = (16+4)-(1+2) = 17$

2. A $f'(x) = 15x^4 - 15x^2 = 15x^2(x^2-1) = 15x^2(x-1)(x+1)$, changes sign from positive to negative only at $x=-1$. So f has a relative maximum at $x=-1$ only.

3. B $\int_1^2 \frac{x+1}{x^2+2x}\,dx = \frac{1}{2}\int_1^2 \frac{(2x+2)\,dx}{x^2+2x} = \frac{1}{2}\ln\left|x^2+2x\right|\Big|_1^2 = \frac{1}{2}(\ln 8 - \ln 3)$

4. D $x(t) = t^2 - 1 \Rightarrow \frac{dx}{dt} = 2t$ and $\frac{d^2x}{dt^2} = 2$; $y(t) = t^4 - 2t^3 \Rightarrow \frac{dy}{dt} = 4t^3 - 6t^2$ and $\frac{d^2y}{dt^2} = 12t^2 - 12t$

$a(t) = \left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}\right) = (2,\, 12t^2 - 12t) \Rightarrow a(1) = (2,0)$

5. D Area $= \int_{x_1}^{x_2} (\text{top curve} - \text{bottom curve})\,dx$, $x_1 < x_2$; Area $= \int_{-1}^{a} (f(x) - g(x))\,dx$

6. E $f(x) = \frac{x}{\tan x}$, $f'(x) = \frac{\tan x - x\sec^2 x}{\tan^2 x}$, $f'\left(\frac{\pi}{4}\right) = \frac{1 - \frac{\pi}{4}\cdot\left(\sqrt{2}\right)^2}{1} = 1 - \frac{\pi}{2}$

7. A $\int \frac{du}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{a}\right) \Rightarrow \int \frac{dx}{\sqrt{25-x^2}}\,dx = \sin^{-1}\left(\frac{x}{5}\right) + C$

8. C $\lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = f'(2)$ so the derivative of f at $x=2$ is 0.

9. B Take the derivative of each side of the equation with respect to x.
$2xyy' + y^2 + 2xy' + 2y = 0$, substitute the point $(1,2)$
$(1)(4)y' + 2^2 + (2)(1)y' + (2)(2) = 0 \Rightarrow y = -\frac{4}{3}$

10. A Take the derivative of the general term with respect to x: $\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n-2}$

11. A $\frac{d}{dx}\left(\ln\left(\frac{1}{1-x}\right)\right) = \frac{d}{dx}\left(-\ln(1-x)\right) = -\left(\frac{-1}{1-x}\right) = \frac{1}{1-x}$

12. A Use partial fractions to rewrite $\dfrac{1}{(x-1)(x+2)}$ as $\dfrac{1}{3}\left(\dfrac{1}{x-1}-\dfrac{1}{x+2}\right)$

$$\int \frac{1}{(x-1)(x+2)}\,dx=\frac{1}{3}\int\left(\frac{1}{x-1}-\frac{1}{x+2}\right)dx=\frac{1}{3}\left(\ln|x-1|-\ln|x+2|\right)+C=\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right|+C$$

13. B $f(0)=0,\ f(3)=0,\ f'(x)=3x^2-6x;$ by the Mean Value Theorem,
$f'(c)=\dfrac{f(3)-f(0)}{3}=0$ for $c\in(0,3)$.
So, $0=3c^2-6c=3c(c-2)$. The only value in the open interval is 2.

14. C I. convergent: p-series with $p=2>1$
II. divergent: Harmonic series which is known to diverge
III. convergent: Geometric with $|r|=\dfrac{1}{3}<1$

15. C $x(t)=4+\int_0^t(2w-4)\,dw=4+(w^2-4w)\Big|_0^t=4+t^2-4t=t^2-4t+4$

or, $x(t)=t^2-4t+C$, $x(0)=4\Rightarrow C=4$ so, $x(t)=t^2-4t+4$

16. C For $f(x)=x^{\frac{1}{3}}$ we have continuity at $x=0$, however, $f'(x)=\dfrac{1}{3}x^{-\frac{2}{3}}$ is not defined at $x=0$.

17. B $f'(x)=(1)\cdot\ln(x^2)+x\cdot\dfrac{\frac{d}{dx}(x^2)}{x^2}=\ln(x^2)+\dfrac{2x^2}{x^2}=\ln(x^2)+2$

18. C $\int\sin(2x+3)\,dx=\dfrac{1}{2}\int\sin(2x+3)(2dx)=-\dfrac{1}{2}\cos(2x+3)+C$

19. D $g(x)=e^{f(x)},\ g'(x)=e^{f(x)}\cdot f'(x),\ g''(x)=e^{f(x)}\cdot f''(x)+f'(x)\cdot e^{f(x)}\cdot f'(x)$
$g''(x)=e^{f(x)}\left(f''(x)+\left(f'(x)^2\right)\right)=h(x)e^{f(x)}\Rightarrow h(x)=f''(x)+\left(f'(x)^2\right)$

20. C Look for concavity changes, there are 3.

21. B Use the technique of antiderivatives by parts:

$u = f(x) \qquad dv = \sin x\,dx$

$du = f'(x)\,dx \qquad v = -\cos x$

$\int f(x)\sin x\,dx = -f(x)\cos x + \int f'(x)\cos x\,dx$ and we are given that

$\int f(x)\sin x\,dx = -f(x)\cos x + \int 3x^2 \cos x\,dx \Rightarrow f'(x) = 3x^2 \Rightarrow f(x) = x^3$

22. A $A = \pi r^2$, $A = 64\pi$ when $r = 8$. Take the derivative with respect to *t*.

$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$; $96\pi = 2\pi(8)\cdot\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = 6$

23. C $\lim\limits_{h\to 0}\frac{\int_1^{1+h}\sqrt{x^5+8}\,dx}{h} = \lim\limits_{h\to 0}\frac{F(1+h)-F(1)}{h}$ $=F'(1)$ where $F'(x) = \sqrt{x^5+8}$. $F'(1)=3$

Alternate solution by L'Hôpital's Rule: $\lim\limits_{h\to 0}\frac{\int_1^{1+h}\sqrt{x^5+8}\,dx}{h} = \lim\limits_{h\to 0}\frac{\sqrt{(1+h)^5+8}}{1}$ $=\sqrt{9}=3$

24. D $\text{Area} = \frac{1}{2}\int_0^{\pi/2}\sin^2(2\theta)\,d\theta = \frac{1}{2}\int_0^{\pi/2}\frac{1}{2}\left(1-\cos 4\theta\right)d\theta = \frac{1}{4}\left(\theta - \frac{1}{4}\sin 4\theta\right)\Bigg|_0^{\pi/2} = \frac{\pi}{8}$

25. C At rest when $v(t) = 0$. $v(t) = e^{-2t} - 2te^{-2t} = e^{-2t}(1-2t)$, $v(t) = 0$ at $t = \frac{1}{2}$ only.

26. E Apply the log function, simplify, and differentiate. $\ln y = \ln(\sin x)^x = x\ln(\sin x)$

$\frac{y'}{y} = \ln(\sin x) + x\cdot\frac{\cos x}{\sin x} \Rightarrow y' = y\left(\ln(\sin x) + x\cdot\cot x\right) = (\sin x)^x\left(\ln(\sin x) + x\cdot\cot x\right)$

27. E Each of the right-hand sides represent the area of a rectangle with base length $(b-a)$.

I. Area under the curve is less than the area of the rectangle with height $f(b)$.

II. Area under the curve is more than the area of the rectangle with height $f(a)$.

III. Area under the curve is the same as the area of the rectangle with height $f(c)$, $a < c < b$. Note that this is the Mean Value Theorem for Integrals.

28. E $\int e^{x+e^x}\,dx = \int e^{e^x}(e^x\,dx)$. This is of the form $\int e^u\,du$, $u = e^x$, so $\int e^{x+e^x}\,dx = e^{e^x} + C$

29. D Let $x-\frac{\pi}{4}=t$. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin\left(x-\frac{\pi}{4}\right)}{x-\frac{\pi}{4}}=\lim\limits_{t\to 0}\dfrac{\sin t}{t}=1$

30. B At t = 1, $\dfrac{dy}{dx}=\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}=\left.\dfrac{\frac{3}{2\sqrt{3t+1}}}{3t^2-1}\right|_{t=1}=\dfrac{\frac{3}{4}}{3-1}=\dfrac{3}{8}$

31. D The center is $x=1$, so only C, D, or E are possible. Check the endpoints.

At $x=0$: $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n}{n}$ converges by alternating series test.

At $x=2$: $\sum\limits_{n=1}^{\infty}\dfrac{1}{n}$ which is the harmonic series and known to diverge.

32. E $y(-1)=-6$, $y'(-1)=3x^2+6x+7\Big|_{x=-1}=4$, the slope of the normal is $-\dfrac{1}{4}$ and an equation for the normal is $y+6=-\dfrac{1}{4}(x+1)\Rightarrow x+4y=-25$.

33. C This is the differential equation for exponential growth.

$y=y(0)e^{-2t}=e^{-2t}$; $\dfrac{1}{2}=e^{-2t}$; $-2t=\ln\left(\dfrac{1}{2}\right)\Rightarrow t=-\dfrac{1}{2}\ln\left(\dfrac{1}{2}\right)=\dfrac{1}{2}\ln 2$

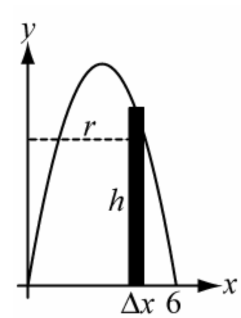
34. A This topic is no longer part of the AP Course Description. $\sum 2\pi\rho\,\Delta s$ where $\rho=x=y^3$

Surface Area $=\int_0^1 2\pi y^3\sqrt{1+\left(\dfrac{dx}{dy}\right)^2}\,dy=\int_0^1 2\pi y^3\sqrt{1+\left(3y^2\right)^2}\,dy=2\pi\int_0^1 y^3\sqrt{1+9y^4}\,dy$

35. B Use shells (which is no longer part of the AP Course Description)

$\sum 2\pi rh\,\Delta x$ where $r=x$ and $h=y=6x-x^2$

Volume $=2\pi\int_0^6 x\left(6x-x^2\right)dx$



36. E $\int_{-1}^{1} \frac{3}{x^2}\,dx = 2\int_{0}^{1} \frac{3}{x^2}\,dx = 2\lim_{L\to 0^+}\int_{L}^{1} \frac{3}{x^2}\,dx = 2\lim_{L\to 0^+} -\frac{3}{x}\Big|_{L}^{1}$ which does not exist.

37. A This topic is no longer part of the AP Course Description. $y = y_h + y_p$ where $y_h = ce^{-x}$ is the solution to the homogeneous equation $\frac{dy}{dx} + y = 0$ and $y_p = \left(Ax^2 + Bx\right)e^{-x}$ is a particular solution to the given differential equation. Substitute y_p into the differential equation to determine the values of *A* and *B*. The answer is $A = \frac{1}{2},\ B = 0$.

38. C $\lim_{x\to\infty}\left(1+5e^x\right)^{1/x} = \lim_{x\to\infty} e^{\ln\left(1+5e^x\right)^{1/x}} = e^{\lim_{x\to\infty}\ln\left(1+5e^x\right)^{1/x}} = e^{\lim_{x\to\infty}\frac{\ln\left(1+5e^x\right)}{x}} = e^{\lim_{x\to\infty}\frac{5e^x}{1+5e^x}} = e$

39. A Square cross sections: $\sum y^2\,\Delta x$ where $y = e^{-x}$. $V = \int_0^3 e^{-2x}\,dx = -\frac{1}{2}e^{-2x}\Big|_0^3 = \frac{1}{2}\left(1-e^{-6}\right)$

40. A $u = \frac{x}{2},\ du = \frac{1}{2}dx$; when $x = 2,\ u = 1$ and when $x = 4,\ u = 2$

$$\int_2^4 \frac{1-\left(\frac{x}{2}\right)^2}{x}\,dx = \int_1^2 \frac{1-u^2}{2u}\cdot 2\,du = \int_1^2 \frac{1-u^2}{u}\,du$$

41. C $y' = x^{\frac{1}{2}},\ L = \int_0^3 \sqrt{1+\left(y'\right)^2}\,dx = \int_0^3 \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2}\Big|_0^3 = \frac{2}{3}\left(4^{3/2}-1^{3/2}\right) = \frac{2}{3}(8-1) = \frac{14}{3}$

42. E Since $e^u = 1+u+\frac{u^2}{2!}+\frac{u^3}{3!}+\cdots$, then $e^{3x} = 1+3x+\frac{(3x)^2}{2!}+\frac{(3x)^3}{3!}+\cdots$

The coefficient we want is $\frac{3^3}{3!} = \frac{9}{2}$

43. E Graphs A and B contradict $f'' < 0$. Graph C contradicts $f'(0)$ does not exist. Graph D contradicts continuity on the interval $[-2,3]$. Graph E meets all given conditions.

44. A $\frac{dy}{dx} = 3x^2 y \Rightarrow \frac{dy}{y} = 3x^2 dx \Rightarrow \ln|y| = x^3 + K;\ y = Ce^{x^3}$ and $y(0) = 8$ so, $y = 8e^{x^3}$

45. D The expression is a Riemann sum with $\Delta x = \frac{1}{n}$ and $f(x) = x^2$.

The evaluation points are: $\frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \cdots, \frac{3n}{n}$

Thus the right Riemann sum is for $x = 0$ to $x = 3$. The limit is equal to $\int_0^3 x^2\,dx$.

1. C $\frac{dy}{dx} = x^2 \cdot \frac{d}{dx}(e^x) + e^x \cdot \frac{d}{dx}(x^2) = x^2e^x + 2xe^x = xe^x(x+2)$

2. D $x^2 - 4 \ge 0$ and $x \ne 3 \Rightarrow |x| \ge 2$ and $x \ne 3$

3. A Distance $= \int_0^2 |v(t)|\,dt = \int_0^2 e^t\,dt = e^t \Big|_0^2 = e^2 - e^0 = e^2 - 1$

4. E Students should know what the graph looks like without a calculator and choose option E. Or $y = -5(x-2)^{-1}$; $y' = 5(x-2)^{-2}$; $y'' = -10(x-2)^{-3}$. $y'' < 0$ for $x > 2$.

5. A $\int \sec^2 x\,dx = \int d(\tan x) = \tan x + C$

6. D $\frac{dy}{dx} = \frac{x \cdot \frac{d}{dx}(\ln x) - \ln x \cdot \frac{d}{dx}(x)}{x^2} = \frac{x \cdot \left(\frac{1}{x}\right) - \ln x \cdot (1)}{x^2} = \frac{1 - \ln x}{x^2}$

7. D $\int x(3x^2+5)^{-\frac{1}{2}}\,dx = \frac{1}{6}\int (3x^2+5)^{-\frac{1}{2}}(6x\,dx) = \frac{1}{6} \cdot 2(3x^2+5)^{\frac{1}{2}} + C = \frac{1}{3}(3x^2+5)^{\frac{1}{2}} + C$

8. B $\frac{dy}{dx} > 0 \Rightarrow y$ is increasing; $\frac{d^2y}{dx^2} < 0 \Rightarrow$ graph is concave down. This is only on $b < x < c$.

9. E $1 + (2x \cdot y' + 2y) - 2y \cdot y' = 0$; $y' = \frac{1+2y}{2y-2x}$. This cannot be evaluated at $(1,1)$ and so y' does not exist at $(1,1)$.

10. C $18 = \left(kx^2 - \frac{1}{3}x^3\right)\Big|_0^k = \frac{2}{3}k^3 \Rightarrow k^3 = 27$, so $k = 3$

11. A $f'(x) = x \cdot 3(1-2x)^2(-2) + (1-2x)^3$; $f'(1) = -7$. Only option A has a slope of –7.

12. B $f'\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

13. A By the Fundamental Theorem of Calculus $\int_0^c f'(x)\,dx = f(x)\Big|_0^c = f(c) - f(0)$

14. D $\int_0^{\frac{\pi}{2}} (1+\sin\theta)^{-1/2} (\cos\theta\, d\theta) = 2(1+\sin\theta)^{1/2}\Big|_0^{\frac{\pi}{2}} = 2\left(\sqrt{2}-1\right)$

15. B $f(x) = \sqrt{2x} = \sqrt{2}\cdot\sqrt{x}$; $f'(x) = \sqrt{2}\cdot\dfrac{1}{2\sqrt{x}}$; $f'(2) = \sqrt{2}\cdot\dfrac{1}{2\sqrt{2}} = \dfrac{1}{2}$

16. C At rest when $0 = v(t) = x'(t) = 3t^2 - 6t - 9 = 3\left(t^2 - 2t - 3\right) = 3(t-3)(t+1)$

$t = -1,\ 3$ and $t \geq 0 \Rightarrow t = 3$

17. D $\int_0^1 (3x-2)^2\,dx = \dfrac{1}{3}\int_0^1 (3x-2)^2 (3\,dx) = \dfrac{1}{3}\cdot\dfrac{1}{3}(3x-2)^3\Big|_0^1 = \dfrac{1}{9}\left(1-(-8)\right) = 1$

18. E $y' = 2\cdot\left(-\sin\left(\dfrac{x}{2}\right)\cdot\dfrac{1}{2}\right) = -\sin\left(\dfrac{x}{2}\right)$; $y'' = -\left(\cos\left(\dfrac{x}{2}\right)\cdot\left(\dfrac{1}{2}\right)\right) = -\dfrac{1}{2}\cos\left(\dfrac{x}{2}\right)$

19. B $\int_2^3 \dfrac{x}{x^2+1}\,dx = \dfrac{1}{2}\int_2^3 \dfrac{2x\,dx}{x^2+1} = \dfrac{1}{2}\ln\left(x^2+1\right)\Big|_2^3 = \dfrac{1}{2}(\ln 10 - \ln 5) = \dfrac{1}{2}\ln 2$

20. C Consider the cases:

I. false if $f(x) = 1$

II. This is true by the Mean Value Theorem

III. false if the graph of f is a parabola with vertex at $x = \dfrac{a+b}{2}$.

Only II must be true.

21. C $x = x^2 - 3x + 3$ at $x = 1$ and at $x = 3$.

Area = $\int_1^3 \left(x - \left(x^2 - 3x + 3\right)\right)dx = \int_1^3 \left(-x^2 + 4x - 3\right)dx = \left(-\dfrac{1}{3}x^3 + 2x^2 - 3x\right)\Big|_1^3 = \dfrac{4}{3}$

22. C $2 = \ln x - \ln\dfrac{1}{x} = \ln x + \ln x \Rightarrow \ln x = 1 \Rightarrow x = e$

23. B By L'Hôpital's rule (which is no longer part of the AB Course Description),
$\lim_{x\to 0}\frac{f(x)}{g(x)}=\lim_{x\to 0}\frac{f'(x)}{g'(x)}=\frac{f'(0)}{g'(0)}=\frac{\cos 0}{1}=\frac{1}{1}=1$

Alternatively, $f'(x)=\cos x$ and $f(0)=0\Rightarrow f(x)=\sin x$. Also $g'(x)=1$ and $g(0)=0\Rightarrow g(x)=x$. Hence $\lim_{x\to 0}\frac{f(x)}{g(x)}=\lim_{x\to 0}\frac{\sin x}{x}=1$.

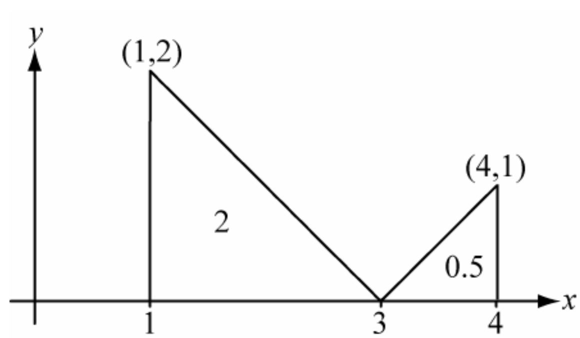
24. C Let $y=x^{\ln x}$ and take the ln of each side. $\ln y=\ln x^{\ln x}=\ln x\cdot\ln x$. Take the derivative of each side with respect to *x*. $\frac{y'}{y}=2\ln x\cdot\frac{1}{x}\Rightarrow y'=2\ln x\cdot\frac{1}{x}\cdot x^{\ln x}$

25. B Use the Fundamental Theorem of Calculus. $f'(x)=\frac{1}{x}$

26. E Use the technique of antiderivatives by parts: Let $u=x$ and $dv=\cos x\,dx$.
$\int_0^{\frac{\pi}{2}} x\cos x\,dx=\left(x\sin x-\int\sin x\,dx\right)\Big|_0^{\frac{\pi}{2}}=\left(x\sin x+\cos x\right)\Big|_0^{\frac{\pi}{2}}=\frac{\pi}{2}-1$

27. E The function is continuous at $x = 3$ since $\lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)=9=f(3)$. Also, the derivative as you approach $x=3$ from the left is 6 and the derivative as you approach $x=3$ from the right is also 6. These two facts imply that f is differentiable at $x = 3$. The function is clearly continuous and differentiable at all other values of *x*.

28. C The graph is a V with vertex at $x=3$. The integral gives the sum of the areas of the two triangles that the V forms with the horizontal axis for *x* from 1 to 4. These triangles have areas of 2 and 0.5 respectively.

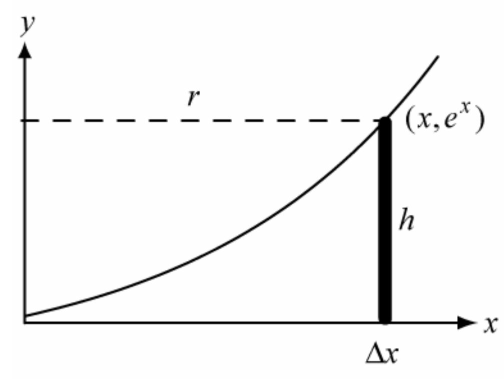


29. B This limit gives the derivative of the function $f(x)=\tan(3x)$. $f'(x)=3\sec^2(3x)$

30. A Shells (which is no longer part of the AB Course Description)

$\sum 2\pi rh\Delta x$, where $r = x, h = e^{2x}$

Volume = $2\pi\int_0^1 xe^{2x}dx$



31. C Let $y = f(x)$ and solve for x.

$y = \frac{x}{x+1}; xy + y = x;\ x(y-1) = -y;\ x = \frac{y}{1-y} \Rightarrow f^{-1}(x) = \frac{x}{1-x}$

32. A The period for $\sin\left(\frac{x}{2}\right)$ is $\frac{2\pi}{\frac{1}{2}} = 4\pi$.

33. A Check the critical points and the endpoints.

$f'(x) = 3x^2 - 6x = 3x(x-2)$ so the critical points are 0 and 2.

x	-2	0	2	4
$f(x)$	-8	12	8	28

Absolute maximum is at $x = 4$.

34. D The interval is $x = a$ to $x = c$. The height of a rectangular slice is the top curve, $f(x)$, minus the bottom curve, $g(x)$. The area of the rectangular slice is therefore $(f(x) - g(x))\Delta x$. Set up a Riemann sum and take the limit as Δx goes to 0 to get a definite integral.

35. B $4\cos\left(x + \frac{\pi}{3}\right) = 4\left(\cos x \cdot \cos\left(\frac{\pi}{3}\right) - \sin x \cdot \sin\left(\frac{\pi}{3}\right)\right)$

$$= 4\left(\cos x \cdot \frac{1}{2} - \sin x \cdot \frac{\sqrt{3}}{2}\right) = 2\cos x - 2\sqrt{3}\sin x$$

36. C $3x - x^2 = x(3-x) > 0$ for $0 < x < 3$

Average value = $\frac{1}{3}\int_0^3 \left(3x - x^2\right)dx = \frac{1}{3}\left(\frac{3}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^3 = \frac{3}{2}$

37. D Since $e^x > 0$ for all x, the zeros of $f(x)$ are the zeros of $\sin x$, so $x = 0, \pi, 2\pi$.

38. E $\int \left(\frac{1}{x}\int_1^x \frac{du}{u}\right)dx = \int \frac{1}{x}\ln x\,dx = \int \ln x\left(\frac{dx}{x}\right)$.This is $\int u\,du$ with $u = \ln x$, so the value is $\frac{(\ln x)^2}{2} + C$

39. E $\int_3^{10} f(x)\,dx = -\int_{10}^{3} f(x)\,dx$; $\int_1^3 f(x)\,dx = \int_1^{10} f(x)\,dx - \int_3^{10} f(x)\,dx = 4 - (-7) = 11$

40. B $x^2 + y^2 = z^2$, take the derivative of both sides with respect to t. $2x\cdot\frac{dx}{dt} + 2y\cdot\frac{dy}{dt} = 2z\cdot\frac{dz}{dt}$

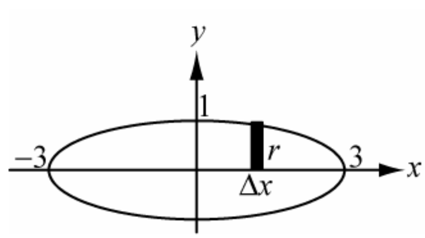
Divide by 2 and substitute: $4\cdot\frac{dx}{dt} + 3\cdot\frac{1}{3}\frac{dx}{dt} = 5\cdot 1 \Rightarrow \frac{dx}{dt} = 1$

41. A The statement makes no claim as to the behavior of f at $x = 3$, only the value of f for input arbitrarily close to $x = 3$. None of the statements are true.

42. C $\lim_{x\to\infty}\frac{x}{x+1} = \lim_{x\to\infty}\frac{\frac{x}{x}}{\frac{x}{x}+\frac{1}{x}} = \lim_{x\to\infty}\frac{1}{1+\frac{1}{x}} = 1$.

None of the other functions have a limit of 1 as $x \to \infty$

43. B The cross-sections are disks with radius $r = y$ where $y = \frac{1}{3}\sqrt{9 - x^2}$.



Volume $= \pi\int_{-3}^{3} y^2\,dx = 2\pi\int_0^3 \frac{1}{9}\left(9 - x^2\right)dx = \frac{2\pi}{9}\left(9x - \frac{1}{3}x^3\right)\Big|_0^3 = 4\pi$

44. C For I: $p(-x) = f\left(g(-x)\right) = f\left(-g(x)\right) = -f\left(g(x)\right) = -p(x) \Rightarrow p$ is odd.

For II: $r(-x) = f(-x) + g(-x) = -f(x) - g(x) = -\left(f(x) + g(x)\right) = -r(x) \Rightarrow r$ is odd.

For III: $s(-x) = f(-x)\cdot g(-x) = \left(-f(x)\right)\cdot\left(-g(x)\right) = f(x)\cdot g(x) = s(x) \Rightarrow s$ is not odd.

45. D Volume $= \pi r^2 h = 16\pi \Rightarrow h = 16r^{-2}$. $A = 2\pi rh + 2\pi r^2 = 2\pi\left(16r^{-1} + r^2\right)$

$\frac{dA}{dr} = 2\pi\left(-16r^{-2} + 2r\right) = 4\pi r^{-2}\left(-8 + r^3\right)$; $\frac{dA}{dr} < 0$ for $0 < r < 2$ and $\frac{dA}{dr} > 0$ for $r > 2$

The minimum surface area of the can is when $r = 2 \Rightarrow h = 4$.

1. A $\int_0^1 \left(x-x^2\right)dx = \frac{1}{2}x^2 - \frac{1}{3}x^3\Big|_0^1 = \frac{1}{2}-\frac{1}{3}=\frac{1}{6}$ only.

2. D $\int_0^1 x\left(x^2+2\right)^2 dx = \frac{1}{2}\int_0^1 \left(x^2+2\right)^2\left(2x\,dx\right) = \frac{1}{2}\cdot\frac{1}{3}\left(x^2+2\right)^3\Big|_0^1 = \frac{1}{6}\left(3^3-2^3\right)=\frac{19}{6}$

3. B $f(x)=\ln\sqrt{x}=\frac{1}{2}\ln x;\ f'(x)=\frac{1}{2}\cdot\frac{1}{x} \Rightarrow f''(x) = -\frac{1}{2x^2}$

4. E $\left(\frac{uv}{w}\right)' = \frac{(uv'+u'v)w-uvw'}{w^2} = \frac{uv'w+u'vw-uvw'}{w^2}$

5. C $\lim_{x\to a} f(x) = f(a)$ for all values of a except 2.
$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(x-2) = 0 \neq -1 = f(2)$

6. C $2y\cdot y' - 2x\cdot y' - 2y = 0 \Rightarrow y' = \frac{y}{y-x}$

7. A $\int_2^\infty \frac{dx}{x^2} = \lim_{L\to\infty}\int_2^L \frac{dx}{x^2} = \lim_{L\to\infty}\left(-\frac{1}{x}\right)\Big|_2^L = \lim_{L\to\infty}\left(\frac{1}{2}-\frac{1}{L}\right) = \frac{1}{2}$

8. A $f'(x)=e^x,\ f'(2)=e^2,\ \ln e^2 = 2$

9. D II does not work since the slope of f at $x=0$ is not equal to $f'(0)$. Both I and III could work. For example, $f(x)=e^x$ in I and $f(x)=\sin x$ in III.

10. D This limit is the derivative of $\sin x$.

11. A The slope of the line is $-\frac{1}{7}$, so the slope of the tangent line at $x=1$ is $7 \Rightarrow f'(1)=7$.

12. B $v(t)=3t+C$ and $v(2)=10 \Rightarrow C=4$ and $v(t)=3t+4$.
Distance $= \int_0^2 (3t+4)\,dt = \frac{3}{2}x^2+4t\Big|_0^2 = 14$

13. B The Maclaurin series for $\sin t$ is $t - \frac{t^3}{3!} + \frac{t^5}{5!} - \cdots$. Let $t = 2x$.

$$\sin(2x) = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \cdots + \frac{(-1)^{n-1}(2x)^{2n-1}}{(2n-1)!} + \cdots$$

14 A Use the Fundamental Theorem of Calculus: $\sqrt{1+(x^2)^3} \cdot \frac{d(x^2)}{dx} = 2x\sqrt{1+x^6}$

15. E $x = t^2 + 1$, $\frac{dx}{dt} = 2t$, $\frac{d^2x}{dt^2} = 2$; $y = \ln(2t+3)$, $\frac{dy}{dt} = \frac{2}{2t+3}$; $\frac{d^2y}{dt^2} = -\frac{4}{(2t+3)^2}$

16. A Use the technique of antiderivatives by parts

$u = x \qquad dv = e^{2x}\,dx$

$du = dx \qquad v = \frac{1}{2}e^{2x}$

$$\frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}\,dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$$

17. D Use partial fractions:

$$\int_2^3 \frac{3}{(x-1)(x+1)}\,dx = \int_2^3 \left(\frac{1}{x-1} - \frac{1}{x+2}\right)dx = \left(\ln|x-1| - \ln|x+2|\right)\Big|_2^3 = \ln 2 - \ln 5 - \ln 1 + \ln 4 = \ln\frac{8}{5}$$

18. E $\Delta x = \frac{4-(-2)}{3} = 2$, $T = \frac{1}{2}(2)\left(\frac{e^4}{2} + 2\cdot\frac{e^2}{2} + 2\cdot\frac{e^0}{2} + \frac{e^{-2}}{2}\right) = \frac{1}{2}\left(e^4 + 2e^2 + 2e^0 + e^{-2}\right)$

19. B Make a sketch. $x < -2$ one zero, $-2 < x < 5$ no zeros, $x > 5$ one zero for a total of 2 zeros

20. E This is the definition of a limit.

21. D $\frac{1}{2}\int_1^3 \frac{1}{x}\,dx = \frac{1}{2}\ln x\Big|_1^3 = \frac{1}{2}(\ln 3 - \ln 1) = \frac{1}{2}\ln 3$

22. E Quick Solution: f' must have a factor of f which makes E the only option. Or,

$$\ln f(x) = x\ln(x^2+1) \Rightarrow \frac{f'(x)}{f(x)} = x\cdot\frac{2x}{x^2+1} + \ln(x^2+1) \Rightarrow f'(x) = f(x)\cdot\left(\frac{2x^2}{x^2+1} + \ln(x^2+1)\right)$$

23. E $r=0$ when $\cos 3\theta = 0 \Rightarrow \theta = \pm\frac{\pi}{6}$. The region is for the interval from $\theta = -\frac{\pi}{6}$ to $\theta = \frac{\pi}{6}$.

$$\text{Area} = \frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (4\cos 3\theta)^2 \, d\theta$$

24. D $f'(x) = 3x^2 - 4x$, $f(0) = 0$ and $f(2) = 0$. By the Mean Value Theorem,
$0 = \frac{f(2)-f(0)}{2-0} = f'(c) = 3c^2 - 4c$ for $c \in (0, 2)$. So, $c = 4/3$.

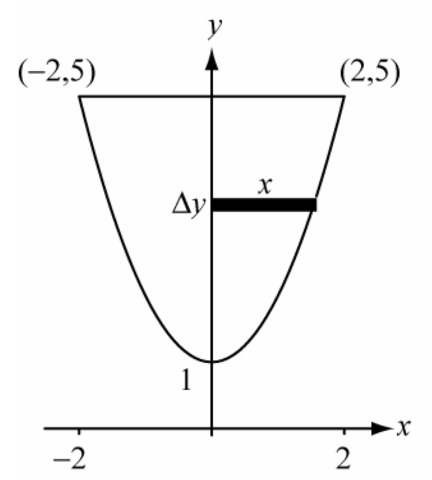
25. D Square cross-sections: $\sum y^2 \Delta x$ where $y = 4x^2$. Volume $= \int_0^1 16x^4\,dx = \frac{16}{5}x^4 \Big|_0^1 = \frac{16}{5}$.

26. C This is not true if f is not an even function.

27. B $y'(x) = 3x^2 + 2ax + b$, $y''(x) = 6x + 2a$, $y''(1) = 0 \Rightarrow a = -3$
$y(1) = -6$ so, $-6 = 1 + a + b - 4 \Rightarrow -6 = 1 - 3 + b - 4 \Rightarrow b = 0$

28. E
$$\frac{\frac{d}{dx}\left(\cos\left(\frac{\pi}{x}\right)\right)}{\cos\left(\frac{\pi}{x}\right)} = \frac{-\sin\left(\frac{\pi}{x}\right)\cdot\frac{d}{dx}\left(\frac{\pi}{x}\right)}{\cos\left(\frac{\pi}{x}\right)} = \frac{-\sin\left(\frac{\pi}{x}\right)\cdot\left(-\frac{\pi}{x^2}\right)}{\cos\left(\frac{\pi}{x}\right)} = \frac{\pi}{x^2}\tan\left(\frac{\pi}{x}\right)$$

29. B Disks: $\sum \pi x^2 \Delta y$ where $x^2 = y - 1$.
Volume $= \pi\int_1^5 (y-1)\,dy = \frac{\pi}{2}(y-1)^2\Big|_1^5 = 8\pi$



30. C This is an infinite geometric series with ratio $\frac{1}{3}$ and first term $\frac{1}{3^n}$.

$$\text{Sum} = \frac{\text{first}}{1-\text{ratio}} = \frac{\left(\frac{1}{3^n}\right)}{1-\frac{1}{3}} = \frac{3}{2}\cdot\left(\frac{1}{3^n}\right)$$

31. C This integral gives $\frac{1}{4}$ of the area of the circle with center at the origin and radius = 2.

$$\frac{1}{4}\left(\pi\cdot 2^2\right) = \pi$$

32. E No longer covered in the AP Course Description. The solution is of the form $y = y_h + y_p$ where y_h is the solution to $y' - y = 0$ and the form of y_p is $Ax^2 + Bx + K$. Hence $y_h = Ce^x$. Substitute y_p into the original differential equation to determine the values of A, B, and K.

Another technique is to substitute each of the options into the differential equation and pick the one that works. Only (A), (B), and (E) are viable options because of the form for y_h. Both (A) and (B) fail, so the solution is (E).

33. E $L = \int_0^2 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^2 \sqrt{1+\left(3x^2\right)^2}\,dx = \int_0^2 \sqrt{1+9x^4}\,dx$

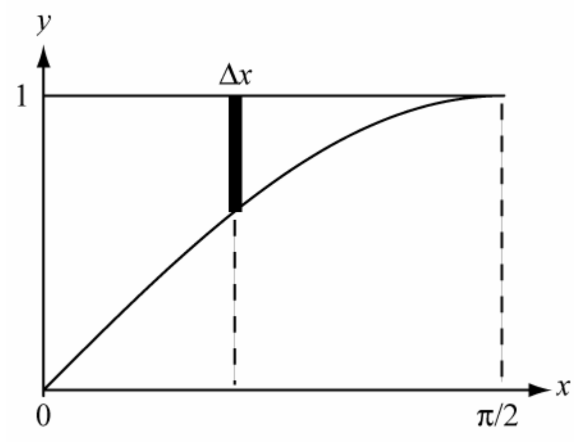
34. C At $t = 1$, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \left.\frac{4t^3+4t}{3t^2+1}\right|_{t=1} = \frac{8}{4} = 2$; the point at $t = 1$ is $(2,3)$. $y = 3 + 2(x-2) = 2x - 1$

35. A Quick solution: For large x the exponential function dominates any polynomial, so $\lim\limits_{x\to+\infty} \frac{x^k}{e^x} = 0$.

Or, repeated use of L'Hôpital's rule gives $\Rightarrow \lim\limits_{x\to+\infty}\frac{x^k}{e^x} = \lim\limits_{x\to+\infty}\frac{k!}{e^x} = 0$

36. E Disks: $\sum \pi(R^2 - r^2)\Delta x$ where $R = 1$, $r = \sin x$

Volume = $\pi\int_0^{\pi/2}(1-\sin^2 x)\,dx$



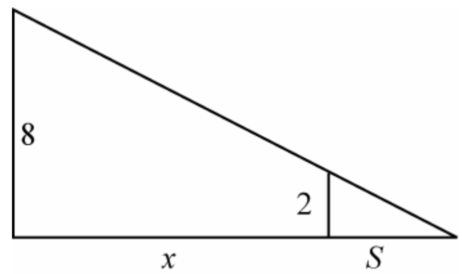
Note that the expression in (E) can also be written as

$$\pi\int_0^{\pi/2}\cos^2 x\,dx = -\pi\int_{\pi/2}^{0}\cos^2\left(\tfrac{\pi}{2}-x\right)dx$$
$$= \pi\int_0^{\pi/2}\sin^2 x\,dx$$

and therefore option (D) is also a correct answer.

37. D $\frac{x+S}{8} = \frac{S}{2} \Rightarrow x = 3S$

$\frac{dx}{dt} = 3\frac{dS}{dt} = 3\cdot\frac{4}{9} = \frac{4}{3}$



38. C Check $x = -1$, $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$ which is convergent by alternating series test

Check $x = 1$, $\sum_{n=1}^{\infty}\frac{1}{n}$ which is the harmonic series and known to diverge.

39. C $\frac{dy}{y} = \sec^2 x\,dx \Rightarrow \ln|y| = \tan x + k \Rightarrow y = Ce^{\tan x}$. $y(0) = 5 \Rightarrow y = 5e^{\tan x}$

40. E Since f and g are inverses their derivatives at the inverse points are reciprocals. Thus,

$$g'(-2)\cdot f'(5) = 1 \Rightarrow g'(-2) = \frac{1}{-\frac{1}{2}} = -2$$

41. B Take the interval $[0,1]$ and divide it into n pieces of equal length and form the right Riemann Sum for the function $f(x) = \sqrt{x}$. The limit of this sum is what is given and its value is given by $\int_0^1 \sqrt{x}\,dx$

42. A Let $5-x=u$, $dx=-du$, substitute

$$\int_1^4 f(5-x)\,dx=\int_4^1 f(u)(-du)=\int_1^4 f(u)\,du=\int_1^4 f(x)\,dx=6$$

43. A This is an example of exponential growth, $B=B_0\cdot 2^{t/3}$. Find the value of t so $B=3B_0$.

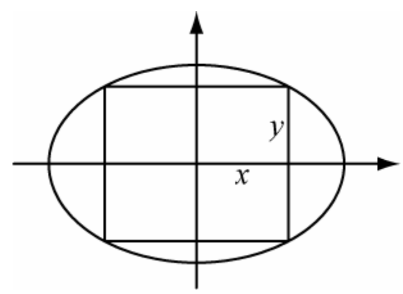
$$3B_0=B_0\cdot 2^{t/3}\Rightarrow 3=2^{t/3}\Rightarrow \ln 3=\frac{t}{3}\ln 2\Rightarrow t=\frac{3\ln 3}{\ln 2}$$

44. A I. Converges by Alternate Series Test

II Diverges by the nth term test: $\lim\limits_{n\to\infty}\frac{1}{n}\left(\frac{3}{2}\right)^n\neq 0$

III Diverges by Integral test: $\int_2^\infty \frac{1}{x\ln x}dx=\lim\limits_{L\to\infty}\ln\left(\ln x\right)\Big|_2^L=\infty$

45. B $A=(2x)(2y)=4xy$ and $y=\sqrt{4-\frac{4}{9}x^2}$.



So $A=8x\sqrt{1-\frac{1}{9}x^2}$.

$$A'=8\left(\left(1-\frac{1}{9}x^2\right)^{1/2}+\frac{1}{2}x\left(1-\frac{1}{9}x^2\right)^{-1/2}\left(-\frac{2}{9}x\right)\right)$$

$$=\frac{8}{9}\left(1-\frac{1}{9}x^2\right)^{-1/2}(9-2x^2)$$

$A'=0$ at $x=\pm 3,\frac{3}{\sqrt{2}}$. The maximum area occurs when $x=\frac{3}{\sqrt{2}}$ and $y=\sqrt{2}$. The value of the largest area is $A=4xy=4\cdot\frac{3}{\sqrt{2}}\cdot\sqrt{2}=12$

1. C $f'(x)=\frac{3}{2}x^{\frac{1}{2}}$; $f'(4)=\frac{3}{2}\cdot 4^{\frac{1}{2}}=\frac{3}{2}\cdot 2=3$

2. B Summing pieces of the form: (vertical)$\cdot$(small width), vertical $=\left(d-f(x)\right)$, width = Δx

   Area = $\int_a^b \left(d-f(x)\right)dx$

3. D Divide each term by n^3. $\lim\limits_{n\to\infty}\frac{3n^3-5n}{n^3-2n^2+1}=\lim\limits_{n\to\infty}\frac{3-\frac{5}{n^2}}{1-\frac{2}{n}+\frac{1}{n^3}}=3$

4. A $3x^2+3\left(y+x\cdot y'\right)+6y^2\cdot y'=0$; $y'(3x+6y^2)=-(3x^2+3y)$

   $y'=-\frac{3x^2+3y}{3x+6y^2}=-\frac{x^2+y}{x+2y^2}$

5. A $\lim\limits_{x\to -2}\frac{x^2-4}{x+2}=\lim\limits_{x\to -2}\frac{(x+2)(x-2)}{x+2}=\lim\limits_{x\to -2}(x-2)=-4$. For continuity $f(-2)$ must be –4.

6. D Area $=\int_3^4\frac{1}{x-1}dx=\left(\ln|x-1|\right)\Big|_3^4=\ln 3-\ln 2=\ln\frac{3}{2}$

7. B $y'=\frac{2\cdot(3x-2)-(2x+3)\cdot 3}{(3x-2)^2}$; $y'(1)=-13$. Tangent line: $y-5=-13(x-1)\Rightarrow 13x+y=18$

8. E $y'=\sec^2 x+\csc^2 x$

9. E $h(x)=f\left(|x|\right)=3|x|^2-1=3x^2-1$

10. D $f'(x)=2(x-1)\cdot\sin x+(x-1)^2\cos x$; $f'(0)=(-2)\cdot 0+1\cdot 1=1$

11. C $a(t)=6t-2$; $v(t)=3t^2-2t+C$ and $v(3)=25\Rightarrow 25=27-6+C$; $v(t)=3t^2-2t+4$

    $x(t)=t^3-t^2+4t+K$; Since $x(1)=10$, $K=6$; $x(t)=t^3-t^2+4t+6$.

12. B The only one that is true is II. The others can easily been seen as false by examples. For example, let $f(x) = 1$ and $g(x) = 1$ with $a = 0$ and $b = 2$. Then I gives $2 = 4$ and III gives $2 = \sqrt{2}$, both false statements.

13. A period $= \dfrac{2\pi}{B} = \dfrac{2\pi}{3}$

14. A Let $u = x^3 + 1$. Then $\displaystyle\int \frac{3x^2}{\sqrt{x^3+1}}\,dx = \int u^{-1/2}\,du = 2u^{1/2} + C = 2\sqrt{x^3+1} + C$

15. D $f'(x) = (x-3)^2 + 2(x-2)(x-3) = (x-3)(3x-7)$; $f'(x)$ changes from positive to negative at $x = \dfrac{7}{3}$.

16. B $y' = 2\dfrac{\sec x \tan x}{\sec x} = 2\tan x$; $y'(\pi/4) = 2\tan(\pi/4) = 2$. The slope of the normal line $-\dfrac{1}{y'(\pi/4)} = -\dfrac{1}{2}$

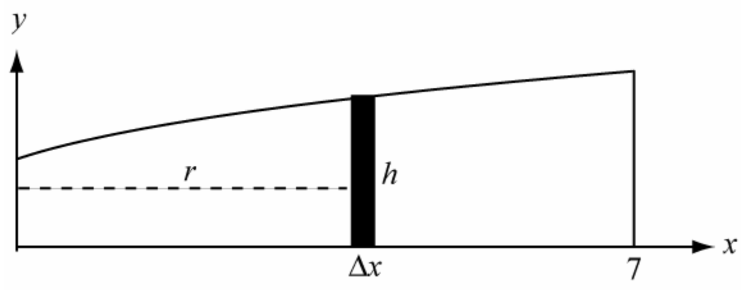
17. E Expand the integrand. $\displaystyle\int (x^2+1)^2\,dx = \int (x^4 + 2x^2 + 1)\,dx = \frac{1}{5}x^5 + \frac{2}{3}x^3 + x + C$

18. D Want c so that $f'(c) = \dfrac{f\left(\frac{3\pi}{2}\right) - f\left(\frac{\pi}{2}\right)}{\frac{3\pi}{2} - \frac{\pi}{2}} = \dfrac{\sin\left(\frac{3\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)}{\pi} = \dfrac{0}{\pi}$.

$f'(c) = \dfrac{1}{2}\cos\left(\dfrac{c}{2}\right) = 0 \Rightarrow c = \pi$

19. E The only one that is true is E. A consideration of the graph of $y = f(x)$, which is a standard cubic to the left of 0 and a line with slope 1 to the right of 0, shows the other options to be false.

20. B Use Cylindrical Shells which is no part of the AP Course Description. The volume of each shell is of the form $(2\pi rh)\Delta x$ with $r = x$ and $h = y$. Volume $= 2\pi\int_0^7 x(x+1)^{\frac{1}{3}}\,dx$.



21. C $y = x^{-2} - x^{-3}$; $y' = -2x^{-3} + 3x^{-4}$; $y'' = 6x^{-4} - 12x^{-5} = 6x^{-5}(x-2)$. The only domain value at which there is a sign change in y'' is $x = 2$. Inflection point at $x = 2$.

22. E $\int \frac{1}{x^2 - 2x + 2}\,dx = \int \frac{1}{(x^2 - 2x + 1) + 1}\,dx = \int \frac{1}{(x-1)^2 + 1}\,dx = \tan^{-1}(x-1) + C$

23. C A quick way to do this problem is to use the effect of the multiplicity of the zeros of f on the graph of $y = f(x)$. There is point of inflection and a horizontal tangent at $x = -2$. There is a horizontal tangent and turning point at $x = 3$. There is a horizontal tangent on the interval $(-2, 3)$. Thus, there must be 3 critical points. Also, $f'(x) = (x-3)^3(x+2)^4(9x-7)$.

24. A $f'(x) = \frac{2}{3}\left(x^2 - 2x - 1\right)^{-\frac{1}{3}}(2x-2)$, $f'(0) = \frac{2}{3}\cdot(-1)\cdot(-2) = \frac{4}{3}$

25. C $\frac{d}{dx}(2^x) = 2^x \cdot \ln 2$

26. D $v(t) = 4\sin t - t$; $a(t) = 4\cos t - 1 = 0$ at $t = \cos^{-1}(1/4) = 1.31812$; $v(1.31812) = 2.55487$

27. C $f'(x) = 3x^2 + 12 > 0$. Thus f is increasing for all x.

28. B $\int_1^{500}(13^x - 11^x)\,dx + \int_2^{500}(11^x - 13^x)\,dx = \int_1^{500}(13^x - 11^x)\,dx - \int_2^{500}(13^x - 11^x)\,dx$

$$= \int_1^2 (13^x - 11^x)\,dx = \left(\frac{13^x}{\ln 13} - \frac{11^x}{\ln 11}\right)\Bigg|_1^2 = \frac{13^2 - 13}{\ln 13} - \frac{11^2 - 11}{\ln 11} = 14.946$$

29. C Use L'Hôpital's Rule (which is no longer part of the AB Course Description).

$$\lim_{\theta\to 0}\frac{1-\cos\theta}{2\sin^2\theta}=\lim_{\theta\to 0}\frac{\sin\theta}{4\sin\theta\cos\theta}=\lim_{\theta\to 0}\frac{1}{4\cos\theta}=\frac{1}{4}$$

A way to do this without L'Hôpital's rule is the following

$$\lim_{\theta\to 0}\frac{1-\cos\theta}{2\sin^2\theta}=\lim_{\theta\to 0}\frac{1-\cos\theta}{2(1-\cos^2\theta)}=\lim_{\theta\to 0}\frac{1-\cos\theta}{2(1-\cos\theta)(1+\cos\theta)}=\lim_{\theta\to 0}\frac{1}{2(1+\cos\theta)}=\frac{1}{4}$$

30. C Each slice is a disk whose volume is given by $\pi r^2\Delta x$, where $r=\sqrt{x}$.

$$\text{Volume}=\pi\int_0^3(\sqrt{x})^2\,dx=\pi\int_0^3 x\,dx=\frac{\pi}{2}x^2\Big|_0^3=\frac{9}{2}\pi.$$

31. E $f(x)=e^{3\ln(x^2)}=e^{\ln(x^6)}=x^6;\ f'(x)=6x^5$

32. A $\int\frac{du}{\sqrt{a^2-u^2}}=\sin^{-1}\left(\frac{u}{a}\right)+C,\ a>0$

$$\int_0^{\sqrt{3}}\frac{dx}{\sqrt{4-x^2}}=\sin^{-1}\left(\frac{x}{2}\right)\Big|_0^{\sqrt{3}}=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)-\sin^{-1}(0)=\frac{\pi}{3}$$

33. B Separate the variables. $y^{-2}dy=2dx$; $-\frac{1}{y}=2x+C$; $y=\frac{-1}{2x+C}$. Substitute the point $(1,-1)$ to find the value of C. Then $-1=\frac{-1}{2+C}\Rightarrow C=-1$, so $y=\frac{1}{1-2x}$. When $x=2$, $y=-\frac{1}{3}$.

34. D Let x and y represent the horizontal and vertical sides of the triangle formed by the ladder, the wall, and the ground.
$x^2+y^2=25$; $2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$; $2(24)\frac{dx}{dt}+2(7)(-3)=0$; $\frac{dx}{dt}=\frac{7}{8}$.

35. E For there to be a vertical asymptote at $x=-3$, the value of c must be 3. For $y=2$ to be a horizontal asymptote, the value of a must be 2. Thus $a+c=5$.

36. D Rectangle approximation = $e^0+e^1=1+e$
Trapezoid approximation. = $\left(1+2e+e^4\right)/2$.

Difference = $(e^4-1)/2=26.799$.

37. C I and II both give the derivative at a. In III the denominator is fixed. This is not the derivative of f at $x=a$. This gives the slope of the secant line from $(a, f(a))$ to $(a+h, f(a+h))$.

38. A $f'(x)=x^2-\sin x+C$, $f(x)=\frac{1}{3}x^3+\cos x+Cx+K$. Option A is the only one with this form.

39. D $A=\pi r^2$ and $C=2\pi r$; $\frac{dA}{dt}=2\pi r\frac{dr}{dt}$ and $\frac{dC}{dt}=2\pi\frac{dr}{dt}$. For $\frac{dA}{dt}=\frac{dC}{dt}$, $r=1$.

40. C The graph of $y=f(|x|)$ is symmetric to the y-axis. This leaves only options C and E. For $x>0$, x and $|x|$ are the same, so the graphs of $f(x)$ and $f(|x|)$ must be the same. This is option C.

41. D Answer follows from the Fundamental Theorem of Calculus.

42. B This is an example of exponential growth. We know from pre-calculus that $w=2\left(\frac{3.5}{2}\right)^{\frac{t}{2}}$ is an exponential function that meets the two given conditions. When $t=3$, $w=4.630$. Using calculus the student may translate the statement "increasing at a rate proportional to its weight" to mean exponential growth and write the equation $w=2e^{kt}$. Using the given conditions, $3.5=2e^{2k}$; $\ln(1.75)=2k$; $k=\frac{\ln(1.75)}{2}$; $w=2e^{t\cdot\frac{\ln(1.75)}{2}}$. When $t=3$, $w=4.630$.

43. B Use the technique of antiderivative by parts, which is no longer in the AB Course Description. The formula is $\int u\,dv=uv-\int v\,du$. Let $u=f(x)$ and $dv=x\,dx$. This leads to $\int x f(x)\,dx=\frac{1}{2}x^2 f(x)-\frac{1}{2}\int x^2 f'(x)\,dx$.

44. C $f'(x)=\ln x+x\cdot\frac{1}{x}$; $f'(x)$ changes sign from negative to positive only at $x=e^{-1}$.
$f(e^{-1})=-e^{-1}=-\frac{1}{e}$.

45. B Let $f(x) = x^3 + x - 1$. Then Newton's method (which is no longer part of the AP Course Description) gives

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n{}^3 + x_n - 1}{3x_n{}^2 + 1}$$

$$x_2 = 1 - \frac{1+1-1}{3+1} = \frac{3}{4}$$

$$x_3 = \frac{3}{4} - \frac{\left(\frac{3}{4}\right)^3 + \frac{3}{4} - 1}{3\left(\frac{3}{4}\right)^2 + 1} = \frac{59}{86} = 0.686$$

1. A $\int_0^1 (x - x^2)\,dx = \left(\frac{1}{2}x^2 - \frac{1}{3}x^3\right)\Big|_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

2. C $\lim_{x\to 0} \frac{2x^2 + 1 - 1}{x^2} = 2$

3. E $Q'(x) = p(x) \Rightarrow$ degree of Q is $n+1$

4. B If $x = 2$ then $y = 5$. $x\frac{dy}{dt} + y\frac{dx}{dt} = 0$; $2(3) + 5\cdot\frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt} = -\frac{6}{5}$

5. D $r = 2\sec\theta$; $r\cos\theta = 2 \Rightarrow x = 2$. This is a vertical line through the point $(2,0)$.

6. A $\frac{dx}{dt} = 2t,\ \frac{dy}{dt} = 3t^2$ thus $\frac{dy}{dx} = \frac{3t^2}{2t} = \frac{3}{2}t;\ \frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{\frac{d\left(\frac{dy}{dx}\right)}{dt}}{\frac{dx}{dt}} = \frac{\frac{3}{2}}{2t} = \frac{3}{4t}$

7. A $\int_0^1 x^3 e^{x^4}\,dx = \frac{1}{4}\int_0^1 e^{x^4}(4x^3 dx) = \frac{1}{4}e^{x^4}\Big|_0^1 = \frac{1}{4}(e-1)$

8. B $f(x) = \ln e^{2x} = 2x,\ f'(x) = 2$

9. D $f'(x) = \frac{2}{3}\cdot\frac{1}{x^{1/3}}$. This does not exist at $x = 0$. D is false, all others are true.

10. E I. $\ln x$ is continuous for $x > 0$
II. e^x is continuous for all x
III. $\ln(e^x - 1)$ is continuous for $x > 0$.

11. E $\int_4^\infty \frac{-2x}{\sqrt[3]{9 - x^2}}\,dx = \lim_{b\to\infty} \frac{3}{2}\left(9 - x^2\right)^{2/3}\Big|_4^b$. This limit diverges. Another way to see this without doing the integration is to observe that the denominator behaves like $x^{2/3}$ which has a smaller degree than the degree of the numerator. This would imply that the integral will diverge.

12. E $v(t) = 2\cos 2t + 3\sin 3t,\ a(t) = -4\sin 2t + 9\cos 3t,\ a(\pi) = -9$.

13. C $\frac{dy}{y} = x^2 dx,\ \ln|y| = \frac{1}{3}x^3 + C_1,\ y = Ce^{\frac{1}{3}x^3}$. Only C is of this form.

14. B The only place that $f'(x)$ changes sign from positive to negative is at $x = -3$.

15. D $f'(x) = e^{\tan^2 x} \cdot \frac{d\left(\tan^2 x\right)}{dx} = 2\tan x \cdot \sec^2 x \cdot e^{\tan^2 x}$

16. A I. Compare with p-series, $p = 2$

II. Geometric series with $r = \frac{6}{7}$

III. Alternating harmonic series

17. A Using implicit differentiation, $\frac{y + xy'}{xy} = 1$. When $x = 1$, $\frac{y + y'}{y} = 1 \Rightarrow y' = 0$.

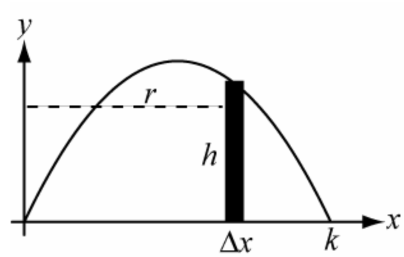
Alternatively, $xy = e^x,\ y = \frac{e^x}{x},\ y' = \frac{xe^x - e^x}{x^2} = \frac{e^x\left(x-1\right)}{x^2}.\ \ y'(1) = 0$

18. B $f'(x) \cdot e^{f(x)} = 2x \Rightarrow f'(x) = \frac{2x}{e^{f(x)}} = \frac{2x}{1+x^2}$

19. B Use cylindrical shells which is no longer part of the AP Course Description. Each shell is of the form $2\pi rh\Delta x$ where $r = x$ and $h = kx - x^2$. Solve the equation

$$10 = 2\pi\int_0^k x(kx - x^2)\,dx = 2\pi\left(\frac{kx^3}{3} - \frac{x^4}{4}\right)\Bigg|_0^k = 2\pi \cdot \frac{k^4}{12}.$$

$k = \sqrt[4]{\frac{60}{\pi}} \approx 2.0905$.



20. E $v(t) = -\frac{1}{2}e^{-2t} + 3$ and $x(t) = \frac{1}{4}e^{-2t} + 3t + 4$

21. A Use logarithms.
$\ln y = \frac{1}{3}\ln\left(x^2+8\right) - \frac{1}{4}\ln(2x+1)$; $\frac{y'}{y} = \frac{2x}{3\left(x^2+8\right)} - \frac{2}{4(2x+1)}$; at $(0,2)$, $y' = -1$.

22. B $f'(x) = x^2e^x + 2xe^x = xe^x(x+2)$; $f'(x) < 0$ for $-2 < x < 0$

23. D $L = \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^4 \sqrt{4t^2+1}\, dt$

24. C This is L'Hôpital's Rule.

25. D At $t = 3$, slope $= \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-te^{-t}+e^{-t}}{e^t} = \left.\frac{1-t}{e^{2t}}\right|_{t=3} = -\frac{2}{e^6} = -0.005$

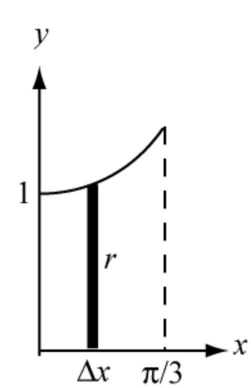
26. B $\frac{2e^{2x}}{1+(e^{2x})^2} = \frac{2e^{2x}}{1+e^{4x}}$

27. C This is a geometric series with $r = \frac{x-1}{3}$. Convergence for $-1 < r < 1$. Thus the series is convergent for $-2 < x < 4$.

28. A $v = \left(\frac{2t+2}{t^2+2t}, 4t\right)$, $v(2) = \left(\frac{6}{8}, 8\right) = \left(\frac{3}{4}, 8\right)$

29. E Use the technique of antiderivatives by parts: $u = x$ and $dv = \sec^2 x\, dx$
$\int x\sec^2 x\, dx = x\tan x - \int \tan x\, dx = x\tan x + \ln\left|\cos x\right| + C$

30. C Each slice is a disk with radius $r = \sec x$ and width Δx.
Volume $= \pi\int_0^{\pi/3} \sec^2 x\, dx = \pi \tan x\Big|_0^{\pi/3} = \pi\sqrt{3}$



31. A $s_n = \frac{1}{5}\left(\frac{5+n}{4+n}\right)^{100}$, $\lim_{n\to\infty} s_n = \frac{1}{5}\cdot 1 = \frac{1}{5}$

32. B Only II is true. To see that neither I nor III must be true, let $f(x)=1$ and let $g(x)=x^2-\frac{128}{15}$ on the interval [0, 5].

33. A The value of this integral is 2. Option A is also 2 and none of the others have a value of 2. Visualizing the graphs of $y=\sin x$ and $y=\cos x$ is a useful approach to the problem.

34. E Let $y=PR$ and x=RQ.
$x^2+y^2=40^2$, $2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$, $x\cdot\frac{3}{4}\left(-\frac{dy}{dt}\right)+y\frac{dy}{dt}=0 \Rightarrow y=\frac{3}{4}x$.
Substitute into $x^2+y^2=40^2$. $x^2+\frac{9}{16}x^2=40^2$, $\frac{25}{16}x^2=40^2$, $x=32$

35. A Apply the Mean Value Theorem to *F*. $F'(c)=\frac{F(b)-F(a)}{b-a}=\frac{0}{a}=0$. This means that there is number in the interval (a,b) for which F' is zero. However, $F'(x)=f(x)$. So, $f(x)=0$ for some number in the interval (a,b).

36. E $v=\pi r^2 h$ and $h+2\pi r=30 \Rightarrow v=2\pi\left(15r^2-\pi r^3\right)$ for $0<r<\frac{15}{\pi}$; $\frac{dv}{dr}=6\pi r\left(10-\pi r\right)$. The maximum volume is when $r=\frac{10}{\pi}$ because $\frac{dv}{dr}>0$ on $\left(0,\frac{10}{\pi}\right)$ and $\frac{dv}{dr}<0$ on $\left(\frac{10}{\pi},\frac{15}{\pi}\right)$.

37. B $\int_0^e f(x)\,dx=\int_0^1 x\,dx+\int_1^e \frac{1}{x}\,dx=\frac{1}{2}+\ln e=\frac{3}{2}$

38. C $\frac{dN}{dt}=kN \Rightarrow N=Ce^{kt}$. $N(0)=1000 \Rightarrow C=1000$. $N(7)=1200 \Rightarrow k=\frac{1}{7}\ln(1.2)$. Therefore $N(12)=1000e^{\frac{12}{7}\ln(1.2)}\approx 1367$.

39. C Want $\frac{y(4)-y(1)}{4-1}$ where $y(x)=\ln|x|+C$. This gives $\frac{\ln 4-\ln 1}{3}=\frac{1}{3}\ln 4=\frac{1}{3}\ln 2^2=\frac{2}{3}\ln 2$.

40. C The interval is $[0,2]$, $x_0=0,\ x_1=1,\ x_2=2$. $S=\frac{1}{3}\left(0+4\ln 2+0\right)=\frac{4}{3}\ln 2$. Note that Simpson's rule is no longer part of the BC Course Description.

41. C $f'(x) = (2x-3)e^{\left(x^2-3x\right)^2}$; $f' < 0$ for $x < \frac{3}{2}$ and $f' > 0$ for $x > \frac{3}{2}$.

Thus f has its absolute minimum at $x = \frac{3}{2}$.

42. E Suppose $\lim\limits_{x\to 0} \ln\left((1+2x)^{\csc x}\right) = A$. The answer to the given question is e^A.

Use L'Hôpital's Rule: $\lim\limits_{x\to 0} \ln\left((1+2x)^{\csc x}\right) = \lim\limits_{x\to 0} \frac{\ln(1+2x)}{\sin x} = \lim\limits_{x\to 0} \frac{2}{1+2x} \cdot \frac{1}{\cos x} = 2$.

43. A $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \Rightarrow \sin x^2 = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} - \cdots = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \cdots$

44. E By the Intermediate Value Theorem there is a c satisfying $a < c < b$ such that $f(c)$ is equal to the average value of f on the interval [a,b]. But the average value is also given by $\frac{1}{b-a}\int_a^b f(x)\,dx$. Equating the two gives option E.

Alternatively, let $F(t) = \int_a^t f(x)\,dx$. By the Mean Value Theorem, there is a c satisfying $a < c < b$ such that $\frac{F(b)-F(a)}{b-a} = F'(c)$. But $F(b)-F(a) = \int_a^b f(x)\,dx$, and $F'(c) = f(c)$ by the Fundamental Theorem of Calculus. This again gives option E as the answer. This result is called the Mean Value Theorem for Integrals.

45. D This is an infinite geometric series with a first term of $\sin^2 x$ and a ratio of $\sin^2 x$.

The series converges to $\frac{\sin^2 x}{1-\sin^2 x} = \tan^2 x$ for $x \neq (2k+1)\frac{\pi}{2}$, k an integer. The answer is therefore $\tan^2 1 = 2.426$.

1. C $\int_1^2 (4x^3 - 6x)\,dx = (x^4 - 3x^2)\Big|_1^2 = (16-12)-(1-3) = 6$

2. A $f(x) = x(2x-3)^{\frac{1}{2}};\quad f'(x) = (2x-3)^{\frac{1}{2}} + x(2x-3)^{-\frac{1}{2}} = (2x-3)^{-\frac{1}{2}}(3x-3) = \dfrac{(3x-3)}{\sqrt{2x-3}}$

3. C $\int_a^b (f(x)+5)\,dx = \int_a^b f(x)dx + 5\int_a^b 1\,dx = a + 2b + 5(b-a) = 7b - 4a$

4. D $f(x) = -x^3 + x + \dfrac{1}{x};\quad f'(x) = -3x^2 + 1 - \dfrac{1}{x^2};\quad f'(-1) = -3(-1)^2 + 1 - \dfrac{1}{(-1)^2} = -3+1-1 = -3$

5. E $y = 3x^4 - 16x^3 + 24x^2 + 48;\quad y' = 12x^3 - 48x^2 + 48x;\quad y'' = 36x^2 - 96x + 48 = 12(3x-2)(x-2)$
$y'' < 0$ for $\dfrac{2}{3} < x < 2$, therefore the graph is concave down for $\dfrac{2}{3} < x < 2$

6. C $\dfrac{1}{2}\int e^{\frac{t}{2}}\,dt = e^{\frac{t}{2}} + C$

7. D $\dfrac{d}{dx}\cos^2(x^3) = 2\cos(x^3)\left(\dfrac{d}{dx}(\cos(x^3)\right) = 2\cos(x^3)(-\sin(x^3)\left(\dfrac{d}{dx}(x^3)\right)$
$= 2\cos(x^3)(-\sin(x^3)(3x^2)$

8. C The bug change direction when v changes sign. This happens at $t = 6$.

9. B Let A_1 be the area between the graph and t-axis for $0 \le t \le 6$, and let A_2 be the area between the graph and the t-axis for $6 \le t \le 8$ Then $A_1 = 12$ and $A_2 = 1$. The total distance is $A_1 + A_2 = 13$.

10. E $y = \cos(2x);\quad y' = -2\sin(2x);\quad y'\left(\dfrac{\pi}{4}\right) = -2$ and $y\left(\dfrac{\pi}{4}\right) = 0;\quad y = -2\left(x - \dfrac{\pi}{4}\right)$

11. E Since f' is positive for $-2 < x < 2$ and negative for $x < -2$ and for $x > 2$, we are looking for a graph that is increasing for $-2 < x < 2$ and decreasing otherwise. Only option E.

12. B $y = \dfrac{1}{2}x^2;\quad y' = x;$ We want $y' = \dfrac{1}{2} \Rightarrow x = \dfrac{1}{2}$. So the point is $\left(\dfrac{1}{2}, \dfrac{1}{8}\right)$.

13. A $f'(x) = \dfrac{|4-x^2|}{x-2}$; f is decreasing when $f' < 0$. Since the numerator is non-negative, this is only when the denominator is negative. Only when $x < 2$.

14. C $f(x) \approx L(x) = 2 + 5(x-3)$; $L(x) = 0$ if $0 = 5x - 13 \Rightarrow x = 2.6$

15. B Statement B is true because $\lim\limits_{x\to a^-} f(x) = 2 = \lim\limits_{x\to a^+} f(x)$. Also, $\lim\limits_{x\to b} f(x)$ does not exist because the left- and right-sided limits are not equal, so neither (A), (C), nor (D) are true.

16. D The area of the region is given by $\int_{-2}^{2}(5-(x^2+1))dx = 2(4x - \frac{1}{3}x^3)\Big|_0^2 = 2\left(8 - \frac{8}{3}\right) = \frac{32}{3}$

17. A $x^2 + y^2 = 25;\ 2x + 2y\cdot y' = 0;\ x + y\cdot y' = 0;\ y'(4,3) = -\frac{4}{3};$

$x + y\cdot y' = 0 \Rightarrow 1 + y\cdot y'' + y'\cdot y' = 0;\ 1 + (3)y'' + \left(-\frac{4}{3}\right)\cdot\left(-\frac{4}{3}\right) = 0;\ y'' = -\frac{25}{27}$

18. C $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}dx$ is of the form $\int e^u du$ where $u = \tan x$..

$\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}dx = e^{\tan x}\Big|_0^{\frac{\pi}{4}} = e^1 - e^0 = e - 1$

19. D $f(x) = \ln|x^2 - 1|$; $f'(x) = \frac{1}{x^2-1}\cdot\frac{d}{dx}(x^2-1) = \frac{2x}{x^2-1}$

20. E $\frac{1}{8}\int_{-3}^{5}\cos x\,dx = \frac{1}{8}(\sin 5 - \sin(-3)) = \frac{1}{8}(\sin 5 + \sin 3)$; Note: Since the sine is an odd function, $\sin(-3) = -\sin(3)$.

21. E $\lim\limits_{x\to 1}\frac{x}{\ln x}$ is nonexistent since $\lim\limits_{x\to 1}\ln x = 0$ and $\lim\limits_{x\to 1} x \neq 0$.

22. D $f(x) = (x^2-3)e^{-x};\ f'(x) = e^{-x}(-x^2+2x+3) = -e^{-x}(x-3)(x+1);\ f'(x) > 0$ for $-1 < x < 3$

23. A Disks where $r = x$. $V = \pi\int_0^2 x^2 dy = \pi\int_0^2 y^4 dy = \frac{\pi}{5}y^5\Big|_0^2 = \frac{32\pi}{5}$

24. B Let $[0,1]$ be divided into 50 subintervals. $\Delta x = \frac{1}{50}$; $x_1 = \frac{1}{50}, x_2 = \frac{2}{50}, x_3 = \frac{3}{50}, \cdots, x_{50} = 1$

Using $f(x) = \sqrt{x}$, the right Riemann sum $\sum_{i=1}^{50} f(x_i)\Delta x$ is an approximation for $\int_0^1 \sqrt{x}\, dx$.

25. A Use the technique of antiderivatives by parts, which was removed from the AB Course Description in 1998.

$$u = x \qquad dv = \sin 2x\, dx$$
$$du = dx \qquad v = -\frac{1}{2}\cos 2x$$

$$\int x\sin(2x)\, dx = -\frac{1}{2}x\cos(2x) + \int \frac{1}{2}\cos(2x)\, dx = -\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C$$

1997 Calculus AB Solutions: Part B

76. E $\quad f(x)=\dfrac{e^{2x}}{2x};\ f'(x)=\dfrac{2e^{2x}\cdot 2x-2e^{2x}}{4x^2}=\dfrac{e^{2x}(2x-1)}{2x^2}$

77. D $\quad y=x^3+6x^2+7x-2\cos x$. Look at the graph of $y''=6x+12+2\cos x$ in the window [–3,–1] since that domain contains all the option values. y'' changes sign at $x=-1.89$.

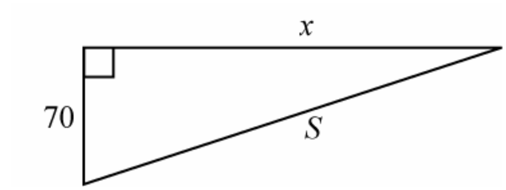
78. D $\quad F(3)-F(0)=\int_0^3 f(x)dx=\int_0^1 f(x)dx+\int_1^3 f(x)dx=2+2.3=4.3$

(Count squares for $\int_0^1 f(x)dx$)

79. C $\quad$ The stem of the questions means $f'(2)=5$. Thus f is differentiable at $x = 2$ and therefore continuous at $x = 2$. We know nothing of the continuity of f'. I and II only.

80. A $\quad f(x)=2e^{4x^2};\ f'(x)=16xe^{4x^2}$; We want $16xe^{4x^2}=3$. Graph the derivative function and the function $y = 3$, then find the intersection to get $x=0.168$.

81. A $\quad$ Let x be the distance of the train from the crossing. Then $\dfrac{dx}{dt}=60$.



$S^2=x^2+70^2 \Rightarrow 2S\dfrac{dS}{dt}=2x\dfrac{dx}{dt}\Rightarrow\dfrac{dS}{dt}=\dfrac{x}{S}\dfrac{dx}{dt}$.

After 4 seconds, $x=240$ and so $S=250$.

Therefore $\dfrac{dS}{dt}=\dfrac{240}{250}(60)=57.6$

82. B $\quad P(x)=2x^2-8x;\ P'(x)=4x-8;\ P'$ changes from negative to positive at $x = 2$. $P(2)=-8$

83. C $\quad \cos x=x$ at $x=0.739085$. Store this in A. $\int_0^A(\cos x-x)\,dx=0.400$

84. C $\quad$ Cross sections are squares with sides of length y.

Volume $=\int_1^e y^2dx=\int_1^e \ln x\,dx=(x\ln x-x)\Big|_1^e=(e\ln e-e)-(0-1)=1$

85. C $\quad$ Look at the graph of f' and locate where the y changes from positive to negative. $x=0.91$

86. A $\quad f(x)=\sqrt{x};\ f'(x)=\dfrac{1}{2\sqrt{x}};\ \ \dfrac{1}{2\sqrt{c}}=2\cdot\dfrac{1}{2\sqrt{1}}\ \Rightarrow\ c=\dfrac{1}{4}$

87. B $a(t) = t + \sin t$ and $v(0) = -2 \Rightarrow v(t) = \frac{1}{2}t^2 - \cos t - 1$; $v(t) = 0$ at $t = 1.48$

88. E $f(x) = \int_a^x h(x)dx \Rightarrow f(a) = 0$, therefore only (A) or (E) are possible. But $f'(x) = h(x)$ and therefore f is differentiable at $x = b$. This is true for the graph in option (E) but not in option (A) where there appears to be a corner in the graph at $x = b$. Also, Since h is increasing at first, the graph of f must start out concave up. This is also true in (E) but not (A).

89. B $T = \frac{1}{2} \cdot \frac{1}{2}(3 + 2 \cdot 3 + 2 \cdot 5 + 2 \cdot 8 + 13) = 12$

90. D

$F(x) = \frac{1}{2}\sin^2 x$	$F'(x) = \sin x \cos x$	Yes
$F(x) = \frac{1}{2}\cos^2 x$	$F'(x) = -\cos x \sin x$	No
$F(x) = -\frac{1}{4}\cos(2x)$	$F'(x) = \frac{1}{2}\sin(2x) = \sin x \cos x$	Yes

1997 Calculus BC Solutions: Part A

1. C $\int_0^1 \sqrt{x}(x+1)\,dx = \int_0^1 x^{\frac{3}{2}} + x^{\frac{1}{2}}\,dx = \frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}}\Big|_0^1 = \frac{16}{15}$

2. E $x = e^{2t}$, $y = \sin(2t)$; $\frac{dy}{dx} = \frac{2\cos(2t)}{2e^{2t}} = \frac{\cos(2t)}{e^{2t}}$

3. A $f(x) = 3x^5 - 4x^3 - 3x$; $f'(x) = 15x^4 - 12x^2 - 3 = 3(5x^2+1)(x^2-1) = 3(5x^2+1)(x+1)(x-1)$;
f' changes from positive to negative only at $x = -1$.

4. C $e^{\ln x^2} = x^2$; so $xe^{\ln x^2} = x^3$ and $\frac{d}{dx}(x^3) = 3x^2$

5. C $f(x) = (x-1)^{\frac{3}{2}} + \frac{1}{2}e^{x-2}$; $f'(x) = \frac{3}{2}(x-1)^{\frac{1}{2}} + \frac{1}{2}e^{x-2}$; $f'(2) = \frac{3}{2} + \frac{1}{2} = 2$

6. A $y = (16-x)^{\frac{1}{2}}$; $y' = -\frac{1}{2}(16-x)^{-\frac{1}{2}}$; $y'(0) = -\frac{1}{8}$; The slope of the normal line is 8.

7. C The slope at $x = 3$ is 2. The equation of the tangent line is $y - 5 = 2(x-3)$.

8. E Points of inflection occur where f' changes from increasing to decreasing, or from decreasing to increasing. There are six such points.

9. A f increases for $0 \le x \le 6$ and decreases for $6 \le x \le 8$. By comparing areas it is clear that f increases more than it decreases, so the absolute minimum must occur at the left endpoint, $x = 0$.

10. B $y = xy + x^2 + 1$; $y' = xy' + y + 2x$; at $x = -1, y = 1$; $y' = -y' + 1 - 2 \Rightarrow y' = -\frac{1}{2}$

11. C $\int_1^\infty x(1+x^2)^{-2}\,dx = \lim_{L\to\infty} -\frac{1}{2}(1+x^2)^{-1}\Big|_1^L = \lim_{L\to\infty} \frac{1}{4} - \frac{1}{2(1+L^2)} = \frac{1}{4}$

12. A f' changes from positive to negative once and from negative to positive twice. Thus one relative maximum and two relative minimums.

13. B $a(t) = 2t - 7$ and $v(0) = 6$; so $v(t) = t^2 - 7t + 6 = (t-1)(t-6)$. Movement is right then left with the particle changing direction at $t = 1$, , therefore it will be farthest to the right at $t = 1$.

14. C Geometric Series. $r = \frac{3}{8} < 1 \Rightarrow$ convergence. $a = \frac{3}{2}$ so the sum will be $S = \dfrac{\frac{3}{2}}{1-\frac{3}{8}} = 2.4$

15. D $x = \cos^3 t,\ y = \sin^3 t$ for $0 \le t \le \frac{\pi}{2}$. $L = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$$L = \int_0^{\pi/2} \sqrt{(-3\cos^2 t \sin t)^2 + (3\sin^2 t \cos t)^2}\, dt = \int_0^{\pi/2} \sqrt{9\cos^4 t \sin^2 t + 9\sin^4 t \cos^2 t}\, dt$$

16. B $\lim_{h\to 0} \frac{e^h - 1}{2h} = \frac{1}{2} \lim_{h\to 0} \frac{e^h - e^0}{h} = \frac{1}{2} f'(0)$, where $f(x) = e^x$ and $f'(0) = 1$. $\lim_{h\to 0} \frac{e^h - 1}{2h} = \frac{1}{2}$

17. B $f(x) = \ln(3-x);\ f'(x) = \frac{1}{x-3},\ f''(x) = -\frac{1}{(x-3)^2},\ f'''(x) = \frac{2}{(x-3)^3};$

$f(2) = 0,\ f'(2) = -1,\ f''(2) = -1,\ f'''(2) = -2;\ \ a_0 = 0,\ a_1 = -1,\ a_2 = -\frac{1}{2},\ a_3 = -\frac{1}{3}$

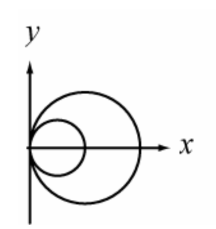
$f(x) \approx -(x-2) - \frac{(x-2)^2}{2} - \frac{(x-2)^3}{3}$

18. C $x = t^3 - t^2 - 1,\ y = t^4 + 2t^2 - 8t;\ \ \frac{dy}{dx} = \frac{4t^3 + 4t - 8}{3t^2 - 2t} = \frac{4t^3 + 4t - 8}{t(3t-2)}$. Vertical tangents at $t = 0, \frac{2}{3}$

19. D $\int_{-4}^{4} f(x)dx - 2\int_{-1}^{4} f(x)dx = (A_1 - A_2) - 2(-A_2) = A_1 + A_2$

20. E $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n \cdot 3^n}$. The endpoints of the interval of convergence are when $(x-2) = \pm 3;\ \ x = -1, 5$.
Check endpoints: $x = -1$ gives the alternating harmonic series which converges. $x = 5$ gives the harmonic series which diverges. Therefore the interval is $-1 \le x < 5$.

21. A Area $= 2 \cdot \frac{1}{2} \int_0^{\pi/2} ((2\cos\theta)^2 - \cos^2\theta)\, d\theta = \int_0^{\pi/2} 3\cos^2\theta\ d\theta$



22. C $g'(x) = f(x)$. The only critical value of g on (a, d) is at $x = c$. Since g' changes from positive to negative at $x = c$, the absolute maximum for g occurs at this relative maximum.

23. E $x = 5\sin\theta$; $\dfrac{dx}{dt} = 5\cos\theta \cdot \dfrac{d\theta}{dt}$; When $x = 3, \cos\theta = \dfrac{4}{5}$; $\dfrac{dx}{dt} = 5\left(\dfrac{4}{5}\right)(3) = 12$

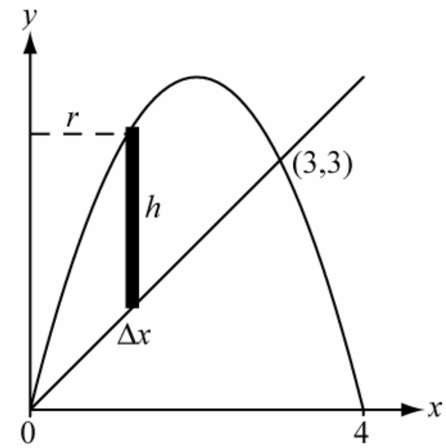
24. D $f'(x) = \sin(x^2) = x^2 - \dfrac{(x^2)^3}{3!} + \cdots = x^2 - \dfrac{1}{6}x^6 + \cdots \Rightarrow f(x) = \dfrac{1}{3}x^3 - \dfrac{1}{42}x^7 + \cdots$ The coefficieint of x^7 is $-\dfrac{1}{42}$.

25. A This is the limit of a right Riemann sum of the function $f(x) = \sqrt{x}$ on the interval $[a,b]$, so

$$\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{x_i}\,\Delta x = \int_a^b \sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\Bigg|_a^b = \frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$$

76. D Sequence I $\to \frac{5}{2}$; sequence II $\to \infty$; sequence III $\to 1$. Therefore I and III only.

77. E Use shells (which is no longer part of the AP Course Description.)



$\sum 2\pi r h \Delta x$ where $r = x$ and

$h = 4x - x^2 - x$

Volume =

$2\pi \int_0^3 x(4x - x^2 - x)\, dx = 2\pi \int_0^3 (3x^2 - x^3)\, dx$

78. A $\lim\limits_{h \to 0} \frac{\ln(e+h) - 1}{h} = \lim\limits_{h \to 0} \frac{\ln(e+h) - \ln e}{h} = f'(e)$ where $f(x) = \ln x$

79. D Count the number of places where the graph of $y(t)$ has a horizontal tangent line. Six places.

80 B Find the first turning point on the graph of $y = f'(x)$. Occurs at $x = 0.93$.

81. D f assumes every value between -1 and 3 on the interval $(-3, 6)$. Thus $f(c) = 1$ at least once.

82. B $\int_0^x (t^2 - 2t)\, dt \ge \int_2^x t\, dt$; $\frac{1}{3}x^3 - x^2 \ge \frac{1}{2}x^2 - 2$. Using the calculator, the greatest x value on the interval $[0, 4]$ that satisfies this inequality is found to occur at $x = 1.3887$.

83. E $\frac{dy}{y} = (1 + \ln x)\, dx$; $\ln|y| = x + x\ln x - x + k = x \ln x + k$; $|y| = e^k e^{x \ln x} \Rightarrow y = Ce^{x \ln x}$. Since $y = 1$ when $x = 1$, $C = 1$. Hence $y = e^{x \ln x}$.

84. C $\int x^2 \sin x\, dx$; Use the technique of antiderivatives by parts with $u = x^2$ and $dv = \sin x\, dx$. It will take 2 iterations with a different choice of u and dv for the second iteration.

$$\begin{aligned}\int x^2 \sin x\, dx &= -x^2 \cos x + \int 2x \cos x\, dx \\ &= -x^2 \cos x + \left(2x \sin x - \int 2 \sin x\, dx\right) \\ &= -x^2 \cos x + 2x \sin x + 2 \cos x + C\end{aligned}$$

85. D I. Average rate of change of f is $\frac{f(3) - f(1)}{3-1} = \frac{5}{2}$. True

II. Not enough information to determine the average value of f. False

III. Average value of f' is the average rate of change of f. True

86. A Use partial fractions. $\frac{1}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}$; $1 = A(x+3) + B(x-1)$

Choose $x = 1 \Rightarrow A = \frac{1}{4}$ and choose $x = -3 \Rightarrow B = -\frac{1}{4}$.

$$\int \frac{1}{(x-1)(x+3)} dx = \frac{1}{4}\left[\int \frac{1}{x-1} dx - \int \frac{1}{x+3} dx\right] = \frac{1}{4} \ln\left|\frac{x-1}{x+3}\right| + C$$

87. B Squares with sides of length x. Volume = $\int_0^2 x^2 dy = \int_0^2 (2-y)\, dy$

88. C $f(x) = \int_0^{x^2} \sin t\, dt$; $f'(x) = 2x \sin(x^2)$; For the average rate of change of f we need to determine $f(0)$ and $f(\sqrt{\pi})$. $f(0) = 0$ and $f\left(\sqrt{\pi}\right) = \int_0^{\pi} \sin t\, dt = 2$. The average rate of change of f on the interval is $\frac{2}{\sqrt{\pi}}$. See how many points of intersection there are for the graphs of $y = 2x \sin(x^2)$ and $y = \frac{2}{\sqrt{\pi}}$ on the interval $\left[0, \sqrt{\pi}\right]$. There are two.

89. D $f(x)=\int_1^x \frac{t^2}{1+t^5}\,dt;\ f(4)=\int_1^4 \frac{t^2}{1+t^5}\,dt=0.376$

Or, $f(4)=f(1)+\int_1^4 \frac{x^2}{1+x^5}\,dx=0.376$

Both statements follow from the Fundamental Theorem of Calculus.

90. B $F(x)=kx;\ 10=4k\ \Rightarrow\ k=\frac{5}{2};$ Work $=\int_0^6 F(x)dx=\int_0^6 \frac{5}{2}x\,dx=\frac{5}{4}x^2\Big|_0^6=45$ inch-lbs

1998 Calculus AB Solutions: Part A

1. D $y' = x^2 + 10x$; $y'' = 2x + 10$; y'' changes sign at $x = -5$

2. B $\int_{-1}^{4} f(x)dx = \int_{-1}^{2} f(x)dx + \int_{2}^{4} f(x)dx$

   = Area of trapezoid(1) – Area of trapezoid(2) = $4 - 1.5 = 2.5$

3. C $\int_{1}^{2} \frac{1}{x^2} dx = \int_{1}^{2} x^{-2} dx = -x^{-1}\Big|_1^2 = \frac{1}{2}$

4. B This would be false if f was a linear function with non-zero slope.

5. E $\int_{0}^{x} \sin t\, dt = -\cos t\Big|_0^x = -\cos x - (-\cos 0) = -\cos x + 1 = 1 - \cos x$

6. A Substitute $x = 2$ into the equation to find $y = 3$. Taking the derivative implicitly gives $\frac{d}{dx}\left(x^2 + xy\right) = 2x + xy' + y = 0$. Substitute for x and y and solve for y'.

   $4 + 2y' + 3 = 0;\; y' = -\frac{7}{2}$

7. E $\int_{1}^{e} \frac{x^2 - 1}{x} dx = \int_{1}^{e} x - \frac{1}{x} dx = \left(\frac{1}{2}x^2 - \ln x\right)\Bigg|_1^e = \left(\frac{1}{2}e^2 - 1\right) - \left(\frac{1}{2} - 0\right) = \frac{1}{2}e^2 - \frac{3}{2}$

8. E $h(x) = f(x)g(x)$ so, $h'(x) = f'(x)g(x) + f(x)g'(x)$. It is given that $h'(x) = f(x)g'(x)$. Thus, $f'(x)g(x) = 0$. Since $g(x) > 0$ for all x, $f'(x) = 0$. This means that f is constant. It is given that $f(0) = 1$, therefore $f(x) = 1$.

9. D Let $r(t)$ be the rate of oil flow as given by the graph, where t is measured in hours. The total number of barrels is given by $\int_{0}^{24} r(t)dt$. This can be approximated by counting the squares below the curve and above the horizontal axis. There are approximately five squares with area 600 barrels. Thus the total is about $3{,}000$ barrels.

10. D $f'(x) = \frac{(x-1)(2x) - (x^2 - 2)(1)}{(x-1)^2}$; $f'(2) = \frac{(2-1)(4) - (4-2)(1)}{(2-1)^2} = 2$

11. A Since f is linear, its second derivative is zero. The integral gives the area of a rectangle with zero height and width $(b - a)$. This area is zero.

12. E $\lim_{x \to 2^-} f(x) = \ln 2 \neq 4\ln 2 = \lim_{x \to 2^+} f(x)$. Therefore the limit does not exist.

13. B At $x = 0$ and $x = 2$ only. The graph has a non-vertical tangent line at every other point in the interval and so has a derivative at each of these other x's.

14. C $v(t) = 2t - 6$; $v(t) = 0$ for $t = 3$

15 D By the Fundamental Theorem of Calculus, $F'(x) = \sqrt{x^3 + 1}$, thus $F'(2) = \sqrt{2^3 + 1} = \sqrt{9} = 3$.

16. E $f'(x) = \cos(e^{-x}) \cdot \frac{d}{dx}(e^{-x}) = \cos(e^{-x})\left(e^{-x} \cdot \frac{d}{dx}(-x)\right) = -e^{-x}\cos(e^{-x})$

17. D From the graph $f(1) = 0$. Since $f'(1)$ represents the slope of the graph at $x = 1$, $f'(1) > 0$. Also, since $f''(1)$ represents the concavity of the graph at $x = 1$, $f''(1) < 0$.

18. B $y' = 1 - \sin x$ so $y'(0) = 1$ and the line with slope 1 containing the point (0,1) is $y = x + 1$.

19. C Points of inflection occur where f'' changes sign. This is only at $x = 0$ and $x = -1$. There is no sign change at $x = 2$.

20. A $\int_{-3}^{k} x^2\,dx = \frac{1}{3}x^3\Big|_{-3}^{k} = \frac{1}{3}\left(k^3 - (-3)^3\right) = \frac{1}{3}\left(k^3 + 27\right) = 0$ only when $k = -3$.

21. B The solution to this differential equation is known to be of the form $y = y(0) \cdot e^{kt}$. Option (B) is the only one of this form. If you do not remember the form of the solution, then separate the variables and antidifferentiate.
$\frac{dy}{y} = k\,dt$; $\ln|y| = kt + c_1$; $|y| = e^{kt + c_1} = e^{kt}e^{c_1}$; $y = ce^{kt}$.

22. C f is increasing on any interval where $f'(x) > 0$. $f'(x) = 4x^3 + 2x = 2x(2x^2 + 1) > 0$.
Since $(x^2 + 1) > 0$ for all x, $f'(x) > 0$ whenever $x > 0$.

23. A The graph shows that f is increasing on an interval (a, c) and decreasing on the interval (c, b), where $a < c < b$. This means the graph of the derivative of f is positive on the interval (a, c) and negative on the interval (c, b), so the answer is (A) or (E). The derivative is not (E), however, since then the graph of f would be concave down for the entire interval.

24. D The maximum acceleration will occur when its derivative changes from positive to negative or at an endpoint of the interval. $a(t) = v'(t) = 3t^2 - 6t + 12 = 3(t^2 - 2t + 4)$ which is always positive. Thus the acceleration is always increasing. The maximum must occur at $t = 3$ where $a(3) = 21$

25. D The area is given by $\int_0^2 x^2 - (-x)\,dx = \left(\frac{1}{3}x^3 + \frac{1}{2}x^2\right)\Big|_0^2 = \frac{8}{3} + 2 = \frac{14}{3}$.

26. A Any value of k less than 1/2 will require the function to assume the value of 1/2 at least twice because of the Intermediate Value Theorem on the intervals [0,1] and [1,2]. Hence $k = 0$ is the only option.

27. A $\frac{1}{2}\int_0^2 x^2\sqrt{x^3+1}\,dx = \frac{1}{2}\int_0^2 (x^3+1)^{\frac{1}{2}}\left(\frac{1}{3}\cdot 3x^2\right)dx = \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{2}{3}(x^3+1)^{\frac{3}{2}}\Big|_0^2 = \frac{1}{9}\left(9^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) = \frac{26}{9}$

28. E $f'(x) = \sec^2(2x)\cdot\frac{d}{dx}(2x) = 2\sec^2(2x)$; $f'\left(\frac{\pi}{6}\right) = 2\sec^2\left(\frac{\pi}{3}\right) = 2(4) = 8$

76. A From the graph it is clear that f is not continuous at $x = a$. All others are true.

77. C Parallel tangents will occur when the slopes of f and g are equal. $f'(x) = 6e^{2x}$ and $g'(x) = 18x^2$. The graphs of these derivatives reveal that they are equal only at $x = -0.391$.

78. B $A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$. However, $C = 2\pi r$ and $\frac{dr}{dt} = -0.1$. Thus $\frac{dA}{dt} = -0.1C$.

79. A The graph of the derivative would have to change from positive to negative. This is only true for the graph of f'.

80. B Look at the graph of $f'(x)$ on the interval (0,10) and count the number of x-intercepts in the interval.

81. D Only II is false since the graph of the absolute value function has a sharp corner at $x = 0$.

82. E Since F is an antiderivative of f, $\int_1^3 f(2x)\,dx = \frac{1}{2}F(2x)\Big|_1^3 = \frac{1}{2}\big(F(6) - F(2)\big)$

83. B $\lim\limits_{x\to a} \frac{x^2 - a^2}{x^4 - a^4} = \lim\limits_{x\to a} \frac{x^2 - a^2}{(x^2 - a^2)(x^2 + a^2)} = \lim\limits_{x\to a} \frac{1}{(x^2 + a^2)} = \frac{1}{2a^2}$

84. A A known solution to this differential equation is $y(t) = y(0)e^{kt}$. Use the fact that the population is $2y(0)$ when $t = 10$. Then

$2y(0) = y(0)e^{k(10)} \Rightarrow e^{10k} = 2 \Rightarrow k = (0.1)\ln 2 = 0.069$

85. C There are 3 trapezoids. $\frac{1}{2}\cdot 3\big(f(2) + f(5)\big) + \frac{1}{2}\cdot 2\big(f(5) + f(7)\big) + \frac{1}{2}\cdot 1\big(f(7) + f(8)\big)$

86. C Each cross section is a semicircle with a diameter of y. The volume would be given by

$$\int_0^8 \frac{1}{2}\pi\left(\frac{y}{2}\right)^2 dx = \frac{\pi}{8}\int_0^8 \left(\frac{8-x}{2}\right)^2 dx = 16.755$$

87. D Find the x for which $f'(x) = 1$. $f'(x) = 4x^3 + 4x = 1$ only for $x = 0.237$. Then $f(0.237) = 0.115$. So the equation is $y - 0.115 = x - 0.237$. This is equivalent to option (D).

88. C $F(9)-F(1)=\int_1^9 \frac{(\ln t)^3}{t}\,dt = 5.827$ using a calculator. Since $F(1)=0$, $F(9)=5.827$.

Or solve the differential equation with an initial condition by finding an antiderivative for $\frac{(\ln x)^3}{x}$. This is of the form $u^3 du$ where $u=\ln x$. Hence $F(x)=\frac{1}{4}(\ln x)^4+C$ and since $F(1)=0$, $C=0$. Therefore $F(9)=\frac{1}{4}(\ln 9)^4=5.827$

89. B The graph of $y=x^2-4$ is a parabola that changes from positive to negative at $x=-2$ and from negative to positive at $x=2$. Since g is always negative, f' changes sign opposite to the way $y=x^2-4$ does. Thus f has a relative minimum at $x=-2$ and a relative maximum at $x=2$.

90. D The area of a triangle is given by $A=\frac{1}{2}bh$. Taking the derivative with respect to t of both sides of the equation yields $\frac{dA}{dt}=\frac{1}{2}\left(\frac{db}{dt}\cdot h+b\cdot\frac{dh}{dt}\right)$. Substitute the given rates to get $\frac{dA}{dt}=\frac{1}{2}(3h-3b)=\frac{3}{2}(h-b)$. The area will be decreasing whenever $\frac{dA}{dt}<0$. This is true whenever $b>h$.

91. E I. True. Apply the Intermediate Value Theorem to each of the intervals $[2,5]$ and $[5,9]$.

II. True. Apply the Mean Value Theorem to the interval $[2,9]$.

III. True. Apply the Intermediate Value Theorem to the interval $[2,5]$.

92. D $\int_k^{\frac{\pi}{2}} \cos x\,dx = 0.1 \Rightarrow \sin\left(\frac{\pi}{2}\right)-\sin k = 0.1 \Rightarrow \sin k = 0.9$. Therefore $k=\sin^{-1}(0.9)=1.120$.

1. C f will be increasing when its derivative is positive.
$f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3)$ $f'(x) = 3(x+3)(x-1) > 0$ for $x < -3$ or $x > 1$.

2. A $\frac{dx}{dt} = 5$ and $\frac{dy}{dt} = 3 \Rightarrow \frac{dy}{dx} = \frac{3}{5}$

3. D Find the derivative implicitly and substitute. $2y \cdot y' + 3(xy+1)^2(x \cdot y' + y) = 0$;
$2(-1) \cdot y' + 3((2)(-1)+1)^2((2) \cdot y' + (-1)) = 0$; $-2y' + 6 \cdot y' - 3 = 0$; $y' = \frac{3}{4}$

4. A Use partial fractions. $\frac{1}{x^2 - 6x + 8} = \frac{1}{(x-4)(x-2)} = \frac{1}{2}\left(\frac{1}{x-4} - \frac{1}{x-2}\right)$

$$\int \frac{1}{x^2 - 6x + 8}\,dx = \frac{1}{2}\left(\ln|x-4| - \ln|x-2|\right) + C = \frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C$$

5. A $h'(x) = f'\left(g(x)\right) \cdot g'(x)$; $h''(x) = f''\left(g(x)\right) \cdot g'(x) \cdot g'(x) + f'\left(g(x)\right) \cdot g''(x)$
$h''(x) = f''\left(g(x)\right) \cdot \left(g'(x)\right)^2 + f'\left(g(x)\right) \cdot g''(x)$

6. E The graph of h has 2 turning points and one point of inflection. The graph of h' will have 2 x-intercepts and one turning point. Only (C) and (E) are possible answers. Since the first turning point on the graph of h is a relative maximum, the first zero of h' must be a place where the sign changes from positive to negative. This is option (E).

7. E $\int_1^e \frac{x^2 - 1}{x}\,dx = \int_1^e x - \frac{1}{x}\,dx = \left(\frac{1}{2}x^2 - \ln x\right)\Big|_1^e = \left(\frac{1}{2}e^2 - 1\right) - \left(\frac{1}{2} - 0\right) = \frac{1}{2}e^2 - \frac{3}{2}$

8. B $y(x) = -\frac{1}{3}\left(\cos x\right)^3 + C$; Let $x = \frac{\pi}{2}$, $0 = -\frac{1}{3}\left(\cos\frac{\pi}{2}\right)^3 + C \Rightarrow C = 0$. $y(0) = -\frac{1}{3}\left(\cos 0\right)^3 = -\frac{1}{3}$

9. D Let $r(t)$ be the rate of oil flow as given by the graph, where t is measured in hours. The total number of barrels is given by $\int_0^{24} r(t)dt$. This can be approximated by counting the squares below the curve and above the horizontal axis. There are approximately five squares with area 600 barrels. Thus the total is about 3,000 barrels.

10. E $v(t) = \left(3t^2 - 1, 6(2t-1)^2\right)$ and $a(t) = \left(6t, 24(2t-1)\right) \Rightarrow a(1) = (6, 24)$

11. A Since f is linear, its second derivative is zero and the integral gives the area of a rectangle with zero height and width $(b-a)$. This area is zero.

12. E $\lim_{x\to 2^-} f(x) = \ln 2 \neq 4\ln 2 = \lim_{x\to 2^+} f(x)$. Therefore the limit does not exist.

13. B At $x = 0$ and $x = 2$ only. The graph has a non-vertical tangent line at every other point in the interval and so has a derivative at each of these other x's.

14. E $\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}$; $\sin 1 \approx 1 - \frac{1^3}{3!} + \frac{1^5}{5!} = 1 - \frac{1}{6} + \frac{1}{120}$

15. B Use the technique of antiderivatives by parts. Let $u = x$ and $dv = \cos x\,dx$.

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx = x\sin x + \cos x + C$$

16. C Inflection point will occur when f'' changes sign. $f'(x) = 15x^4 - 20x^3$.
$f''(x) = 60x^3 - 60x^2 = 60x^2(x-1)$. The only sign change is at $x = 1$.

17. D From the graph $f(1) = 0$. Since $f'(1)$ represents the slope of the graph at $x = 1$, $f'(1) > 0$. Also, since $f''(1)$ represents the concavity of the graph at $x = 1$, $f''(1) < 0$.

18. B
I. Divergent. The limit of the nth term is not zero.
II. Convergent. This is the same as the alternating harmonic series.
III. Divergent. This is the harmonic series.

19. D Find the points of intersection of the two curves to determine the limits of integration.
$4\sin\theta = 2$ when $\sin\theta = 0.5$; this is at $\theta = \frac{\pi}{6}$ and $\frac{5\pi}{6}$. Area $= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left((4\sin\theta)^2 - (2)^2\right) d\theta$

20. E $\left.\frac{d\left(\sqrt[3]{x}\right)}{dt}\right|_{x=8} = \left.\frac{1}{3}x^{-\frac{2}{3}}\cdot\frac{dx}{dt}\right|_{x=8} = \frac{1}{3}(8)^{-\frac{2}{3}}\cdot\frac{dx}{dt} = \frac{1}{3}\cdot\frac{1}{4}\cdot\frac{dx}{dt} = \frac{1}{12}\cdot\frac{dx}{dt} \Rightarrow k = 12$

21 C The length of this parametric curve is given by $\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^1 \sqrt{(t^2)^2 + t^2}\,dt$.

22. A This is the integral test applied to the series in (A). Thus the series in (A) converges. None of the others must be true.

23. E I. False. The relative maximum could be at a cusp.
II. True. There is a critical point at $x = c$ where $f'(c)$ exists
III. True. If $f''(c) > 0$, then there would be a relative minimum, not maximum

24. C All slopes along the diagonal $y = -x$ appear to be 0. This is consistent only with option (C). For each of the others you can see why they do not work. Option (A) does not work because all slopes at points with the same x coordinate would have to be equal. Option (B) does not work because all slopes would have to be positive. Option (D) does not work because all slopes in the third quadrant would have to be positive. Option (E) does not work because there would only be slopes for $y > 0$.

25. C $\int_0^\infty x^2 e^{-x^3}\,dx = \lim_{b\to\infty}\int_0^b x^2 e^{-x^3}\,dx = \lim_{b\to\infty} -\frac{1}{3}e^{-x^3}\Big|_0^b = \frac{1}{3}$.

26. E As $\lim_{t\to\infty}\frac{dP}{dt} = 0$ for a population satisfying a logistic differential equation, this means that $P \to 10{,}000$.

27. D If $f(x) = \sum_{n=0}^{\infty} a_n x^n$, then $f'(x) = \sum_{n=0}^{\infty} n a_n x^{n-1} = \sum_{n=1}^{\infty} n a_n x^{n-1}$.

$$f'(1) = \sum_{n=1}^{\infty} n a_n 1^{n-1} = \sum_{n=1}^{\infty} n a_n$$

28. C Apply L'Hôpital's rule. $\lim_{x\to 1}\frac{\int_1^x e^{t^2}\,dt}{x^2-1} = \lim_{x\to 1}\frac{e^{x^2}}{2x} = \frac{e}{2}$

76. D The first series is either the harmonic series or the alternating harmonic series depending on whether k is odd or even. It will converge if k is odd. The second series is geometric and will converge if $k < 4$.

77. E $f'(t) = \left(-e^{-t}, -\sin t\right)$; $f''(t) = \left(e^{-t}, -\cos t\right)$.

78. B $A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$. However, $C = 2\pi r$ and $\frac{dr}{dt} = -0.1$. Thus $\frac{dA}{dt} = -0.1C$.

79. A None. For every positive value of a the denominator will be zero for some value of x.

80. B The area is given by $\int_{-\frac{2}{3}}^{\frac{2}{3}} \left(1 + \ln(\cos^4 x)\right) dx = 0.919$

81. B $\frac{dy}{dx} = \sqrt{1 - y^2}$; $\frac{d^2 y}{dx^2} = \frac{d}{dx}\left((1 - y^2)^{\frac{1}{2}}\right) = \frac{1}{2}\left(1 - y^2\right)^{-\frac{1}{2}} \cdot (-2y) \cdot \frac{dy}{dx} = -y$

82. B $\int_3^5 \left[f(x) + g(x)\right] dx = \int_3^5 \left[2g(x) + 7\right] dx = 2\int_3^5 g(x)\, dx + (7)(2) = 2\int_3^5 g(x)\, dx + 14$

83. C Use a calculator. The maximum for $\left| \ln x - \left(\frac{(x-1)}{1} - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} \right) \right|$ on the interval $0.3 \le x \le 1.7$ occurs at $x = 0.3$.

84. B You may use the ratio test. However, the series will converge if the numerator is $(-1)^n$ and diverge if the numerator is 1^n. Any value of x for which $|x + 2| > 1$ in the numerator will make the series diverge. Hence the interval is $-3 \le x < -1$.

85. C There are 3 trapezoids. $\frac{1}{2} \cdot 3\left(f(2) + f(5)\right) + \frac{1}{2} \cdot 2\left(f(5) + f(7)\right) + \frac{1}{2} \cdot 1\left(f(7) + f(8)\right)$

86. C Each cross section is a semicircle with a diameter of y. The volume would be given by

$$\int_0^8 \frac{1}{2}\pi\left(\frac{y}{2}\right)^2 dx = \frac{\pi}{8}\int_0^8 \left(\frac{8-x}{2}\right)^2 dx = 16.755$$

87. D Find the x for which $f'(x)=1$. $f'(x)=4x^3+4x=1$ only for $x=0.237$. Then $f(0.237)=0.115$. So the equation is $y-0.115=x-0.237$. This is equivalent to option (D).

88. C From the given information, f is the derivative of g. We want a graph for f that represents the slopes of the graph g. The slope of g is zero at a and b. Also the slope of g changes from positive to negative at one point between a and b. This is true only for figure (C).

89. A The series is the Maclaurin expansion of e^{-x}. Use the calculator to solve $e^{-x}=x^3$.

90. A Constant acceleration means linear velocity which in turn leads to quadratic position. Only the graph in (A) is quadratic with initial $s=2$.

91. E $v(t)=11+\int_0^t a(x)\,dx \approx 11+\left[2\cdot 5+2\cdot 2+2\cdot 8\right]=41$ ft/sec.

92. D $f'(x)=2x-2,\ f'(2)=2,$ and $f(2)=3$, so an equation for the tangent line is $y=2x-1$. The difference between the function and the tangent line is represented by $(x-2)^2$. Solve $(x-2)^2<0.5$. This inequality is satisfied for all x such that $2-\sqrt{0.5}<x<2+\sqrt{0.5}$. This is the same as $1.293<x<2.707$. Thus the largest value in the list that satisfies the inequality is 2.7.