Taylor Polynomials 2 Name: Block: Seat:

A polynomial function can be approximate most any function f near some value of $x = c$. If f has n derivitives at c, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \frac{f'''(c)}{3!}(x-c)^3 + \cdots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the n**th Taylor polynomial for** f **at** c
If $c = 0$ then

$$P_n(x) = f(0) + f'(0)(x) + \frac{f''(0)}{2!}(x)^2 + \frac{f'''(0)}{3!}(x)^3 + \cdots + \frac{f^{(n)}(0)}{n!}(x)^n$$

is called the n**th Maclaurin polynomial** for f.

1. Construct a fourth degree polynomial with the following behavior at $x = 0$.

$$\begin{aligned} P(0) &= 1 \\ P'(0) &= 2 \\ P''(0) &= 3 \\ P'''(0) &= 4 \\ P^{(4)}(0) &= 5 \end{aligned}$$

   Use a fourth degree polynomial $P(x) = ax^4 + bx^3 + cx^2 + dx + e$

2. Start with the fourth derivative and work backwards using differential equations to develop a formula for $P(x) = ax^4 + bx^3 + cx^2 + dx + e$

3. Construct a fourth degree polynomial that matches the behavior of $f(x) = \ln(1+x)$ at $x = 0$ by using successive derivitives of x

4. Use the method from the previous examples to find a fourth degree polynomial that approximates the behavior of $f(x) = \sqrt{1+2x}$ at $x = 0$

5. Create the fifth degree Taylor polynomial for $f(x) = e^{2x}$

6. Write the first nonzero terms and the general term for the Maclaurin series for $f(x) = \cos(2x)$

7. Suppose f has derivatives of all orders at $x = 1$. Suppose that the following values apply: $f(1) = 3$, $f'(1) = 4$, $f''(1) = -8$, $f'''(1) = -7$, an $f^{(4)}(1) = 7$.

   Write the fourth degree Taylor polynomial approximation for f centered at $x = 1$. Use this to approximate $f(1.1)$.

8. Write the fourth degree Taylor polynomial to approximate $f(x) = \cos(x)$ centered at $x = \frac{\pi}{4}$ Use this to approximate f(1.1).

Common MacLaurin Series

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots + x^n + \ldots = \sum_{n=0}^{\infty} x^n \ (for \ -1 < x < 1) \tag{1}$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \cdots + (-x)^{n-1} + \ldots = \sum (-1)^n x^n \ (for \ -1 < x < 1) \tag{2}$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + (-1)^{n-1}\frac{x^n}{n} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n} \ (for \ -1 < x < 1) \tag{3}$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{(-1)^n x^{2n+1}}{2n+1} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \ (for \ -1 \le x \le 1) \tag{4}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!} \ (for \ x \in \mathbb{R}) \tag{5}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{x^{2n+1}}{(2n+1)!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \ (for \ x \in \mathbb{R}) \tag{6}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{x^{2n}}{(2n)!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \ (for \ x \in \mathbb{R}) \tag{7}$$