AP Calc BC Convergence Practice

Name: Block: Seat:

1. Determine what values of p so that this is a converging alternate series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{p^n}$$

2. Consider

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

   (a) If you added up the first six terms, what is the difference between the partial sum and the actual sum? Since this is a decreasing series, isn't the maximum possible error related to the next term? What is the error bound ?

   (b) Since (approximation + error = actual value), is this an overestimate or an underestimate of the sum?

   (c) How many terms before the error is less than 0.01?

3. (1969 BC 45) What is the complete interval of convergence of

$$\sum_{k=1}^{\infty} \frac{(x+1)^k}{k^2}$$

From 1973 BC 19

4. Justify why the series converges or not

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

5. Justify why the series converges or not

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

6. Justify why the series converges or not

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

From 1988 BC 44

7. Justify why the series converges or not

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$$

8. Justify why the series converges or not

$$\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{3}{2}\right)^n$$

9. Justify why the series converges or not

$$\sum_{n=1}^{\infty} \frac{1}{n \ln n}$$

From 1988 BC 30

10.

$$\sum_{i=n}^{\infty} \left(\frac{1}{3}\right)^{i} =$$

From 1993 BC 16

11. Justify why the series converges or not

$$\sum_{k=3}^{\infty} \frac{2}{k^2+1}$$

12. Justify why the series converges or not

$$\sum_{k=1}^{\infty} \left(\frac{6}{7}\right)^{k}$$

13. Justify why the series converges or not

$$\sum_{k=2}^{\infty} \frac{(-1)^k}{k}$$

From 1997 BC 76

14. Justify why the series converges or not

$$\left\{ \frac{5n}{2n-1} \right\}$$

15. Justify why the series converges or not

$$\left\{ \frac{e^n}{n} \right\}$$

16. Justify why the series converges or not

$$\left\{ \frac{e^n}{1+e^n} \right\}$$

1. p > 1 by Alternating Series (Note them $p < 0$ this no longer alternates)
2. The Remainder $|S - S_n| \leq \frac{1}{49}$. The seventh term $a_7 = \frac{-1}{49}$ is negative, so the sum of the first six terms is an overestimate.
The tenth term is .01, so 11 terms needed.
3. Use ratio test to get $(-2, 0)$ then AST to test $\frac{(-1)^k}{k^2}$; p-series for $\frac{1}{k^2}$ so $[-2, 0]$
4. $p > 1$ converge, 5. $p = 1$ diverge, 6.AST: strictly alternates, decreases and $\lim a_n = 0$ converges
7. AST: strictly alternates, decreases and $\lim a_n = 0$ converges; 8. harmonic, $p = 1$ p-series, diverges
9. n^{th}-term test fails lim=0, ratio test fails lim=1, direct comparison with $\frac{1}{n}$ works, diverges
10. $\frac{3}{2}\left(\frac{1}{3}\right)^n$
11. let $b_n = \frac{2}{k^2}$ which is a convergent p-series, so by DCT a_n converges
12. geometric $r < 1$, converges; 13. convergent alt. harmonic harmonic