Der – Int – Calc – Thm - Gph Worksheet ________________

Derivatives

Basic derivatives: ($f'(x), \frac{dy}{dx}, y'$) Find y'.

1) $y = 4$
2) $y = 2x^3 + 3x^2 - 5x + 1$
3) $y = \sin x$

4) $y = \cos x$
5) $y = \tan x$
6) $y = \csc x$
7) $y = \sec x$

8) $y = \cot x$
9) $y = \ln x$
10) $y = e^x$

Basic derivatives with chain rule: $y' = operation(u)u'$

1) $y = (x^2 + 3x - 1)^3$
2) $y = \sin 4x$
3) $y = \cos x^2$

4) $y = \tan(3x + 1)$
5) $y = \cot 2x$
6) $y = \sec(3x - 4)$

7) $y = \csc(x^2 - 1)$
8) $y = \ln(2x + 3)$
9) $y = e^{3x^2 - 4}$

Product rule using basic derivatives: $y = f\,g \quad y' = f'g + f\,g'$

1) $y = \ln x \cdot x^2$

2) $y = \cos x \cdot e^x$

3) $y = \sin x \cdot \tan x$

4) $y = \cot x \cdot e^x$

5) $y = 2x^3\,e^x$

6) $y = \csc x \cdot \sec x$

Quotient rule using basic derivatives: $y = \dfrac{f}{g}, \quad y' = \dfrac{gf' - f\,g'}{g^2}$

1) $y = \dfrac{\cos x}{e^x}$

2) $y = \dfrac{3x^2 - 3x + 1}{\tan x}$

3) $y = \dfrac{\ln x}{\sin x}$

4) $y = \dfrac{\csc x}{x^4}$

5) $y = \dfrac{e^x}{\cot x}$

6) $y = \dfrac{\sec x}{\ln x}$

Special Derivatives

1) $\frac{d}{dx} f\left(3x^2\right)$ 2) $\frac{d}{dx}\left[f(x)+g(x)\right]$ 3) $\frac{d}{dx}\left[f(x)\cdot g(x)\right]$ 4) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$

Implicit Differentiation:

3) $3xy^2 - 5x + 12y + 2e^y = 5$

a) Find $\frac{dy}{dx}$ b) Find $\frac{dy}{dt}$

Logarithmic Differentiation:

4) $y = x^{\sin x}$

Fundamental Theorem of Calculus: (derivation of an integral)

5) $\frac{d}{dx}\left(\int_{2x}^{x^2} t^3 + \cos t \, dt\right)$ 6) $g(x) = 2x - 7 + \int_1^x f(t)\, dt$

Integrals

Basic Integrals (anti derivatives)

$$\int du = u + c \qquad \int k\, f(u)\, du = k\int f(u)\, du$$
$$\int \left(f(u) \pm g(u)\right) du = \int f(u)\, du \pm \int g(u)\, du$$

$\int u^n \ du = \dfrac{u^{n+1}}{n+1} + c$ Power rule good for any power except x^{-1} which is equal to $\dfrac{1}{x}$

examples: $\int x^3 \ dx = \dfrac{x^4}{4} + c, \quad \int x^{-3} \ dx = \dfrac{x^{-2}}{-2} + c, \quad \int x^{\frac{1}{2}} \ dx = \dfrac{2x^{\frac{3}{2}}}{3} + c$

1) $\int x^7 dx =$

2) $\int \sqrt[3]{x}\, dx =$

3) $\int \dfrac{dx}{x^{-3}} =$

U substitution and power rule:

4) $\int x\left(2x^2 - 5\right)^3 dx =$

5) $\int \dfrac{3x^2}{5x^3 - 7} dx =$

$\int u^{-1}\ du \quad same\ as \quad \int \frac{1}{u}\ du \quad same\ as \quad \int \frac{du}{u} = \ln|u| + c$

6) $\int \frac{1}{t} dt =$ 7) $\int \frac{x\,dx}{x^2-4} =$ 8) $\int \frac{1}{2y-3} dy =$

$\int e^u du = e^u + c$

9) $\int e^x\ dx =$ 10) $\int e^{2x}\ dx =$ 11) $\int 3xe^{x^2}\ dx =$

$\int \sin u\, du = -\cos u + c$

12) $\int \sin x\, dx =$ 13) $\int \sin 2x\, dx =$ 14) $\int x \sin x^2 dx =$

$\int \cos u\, du = \sin u + c$

15) $\int \cos x\, dx =$ 16) $\int -3\cos 4x\, dx =$ 17) $\int x^3 \cos 2x^4 dx =$

$\int \sec^2 u\,du \quad same\ as \quad \int (\sec u)^2\,du = \tan u + c$

18) $\int \sec^2 5x\,dx =$

19) $\int x^4 \left(\sec x^5\right)^2 dx =$

$\int \csc^2 u\,du \quad same\ as \quad \int (\csc u)^2\,du = -\cot u + c$

20) $\int \csc^2(-3x)\,dx =$

21) $\int -2x\csc^2\left(x^2\right)dx$

$\int \sec u \tan u\,du = \sec u + c$

22) $\int \sec 5x\,tan5x\,dx$

23) $\int x^5 \sec x^6 \tan x^6\,dx$

$\int \csc u \cot u\,du = -\csc u + c$

24) $\int \csc 3x \cot 3x\,dx$

Calculator

1) Find the zero's: (quadratic formula)

$$f(x) = 2x^2 - 5x - 8$$

2) Find the zero(s) (no quadratic, calculate zero in window)

$$f(x) = 2x^3 - 3x^2 + 2x - 5$$

3) Find the intersection:

Can be written 3 different ways: In 3 you only get the x coordinate (plug in the function to get the y coordinate)

a) $\begin{array}{l} y = 2x^3 - 4 \\ y = e^{-x} \end{array}$

b) $2x^3 - 4 = e^{-x}$

c) $0 = e^{-x} - 2x^3 + 4$

4) Limits:

$$\lim_{x \to 0} \frac{\sqrt{4+x}-2}{x}$$

$$\lim_{x \to -\infty} \left(\frac{x}{2} + \sqrt{\frac{x^2}{4} + x} \right)$$

5) On the interval $[0,12]$, find the area of the region bound by the curves

$$y = 12\sin\left(\frac{x}{4}\right) \quad \& \quad y = 7.$$

Find points of intersection, store the x values in A & B. Use Math 9, Vars, Y-Vars, etc. and A & B.

6) Given $v(t) = 12\sin\left(\frac{x}{4}\right),\ 0 \le t \le 20\ ,\ x(0) = -14$

a) Find $a(7)$. Use in window use Calc 6 and type in 7.

b) Find position $x(15)$

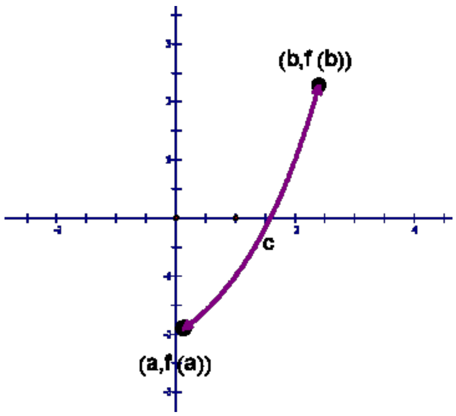
c) Find the distance traveled on the interval $[2,16]$

d) Find the displacement on the interval $[4,14]$

Theorems – Definitions – Graphs

Write a theorem or definition that each graph is trying to display.

1)



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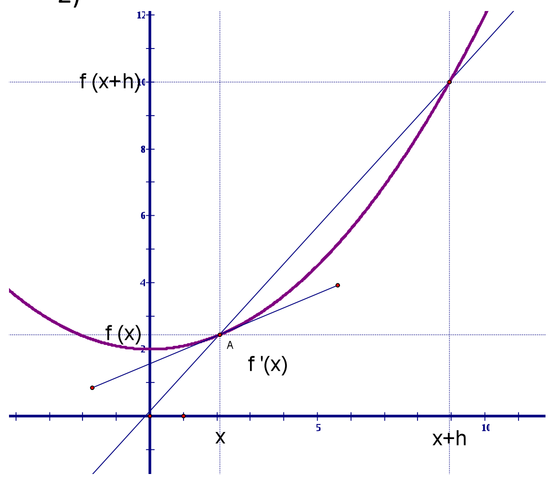

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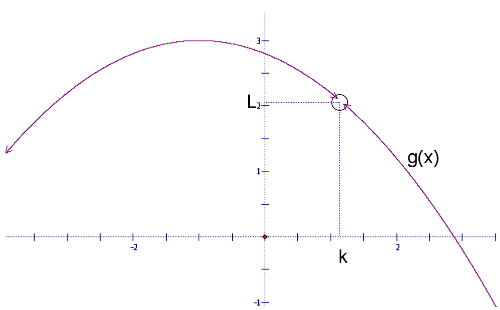

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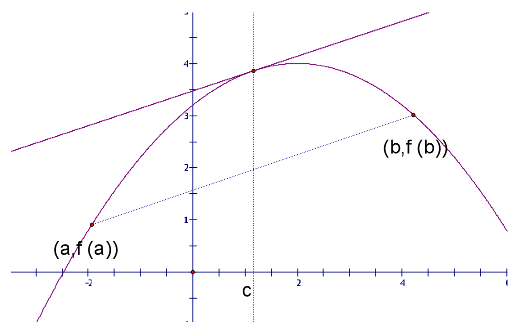

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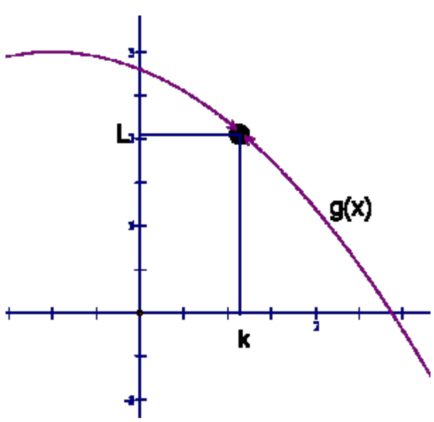

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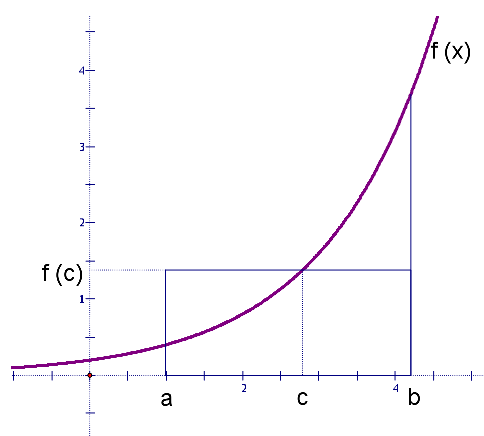

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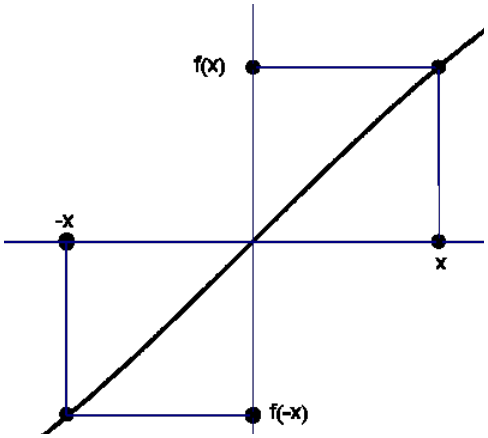

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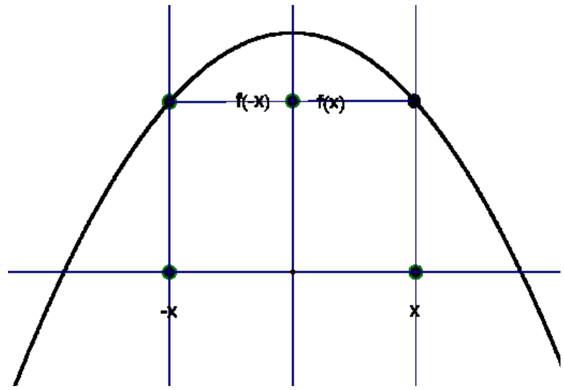
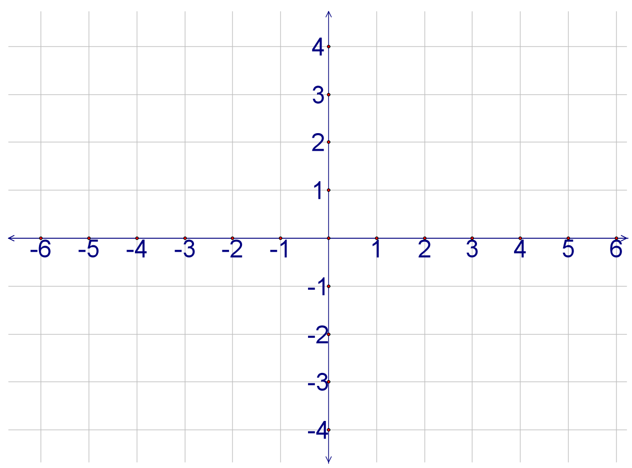
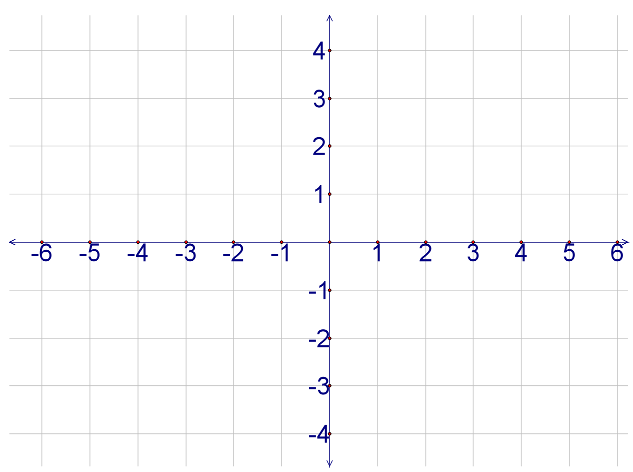


Sketch the function that satisfies the given information.

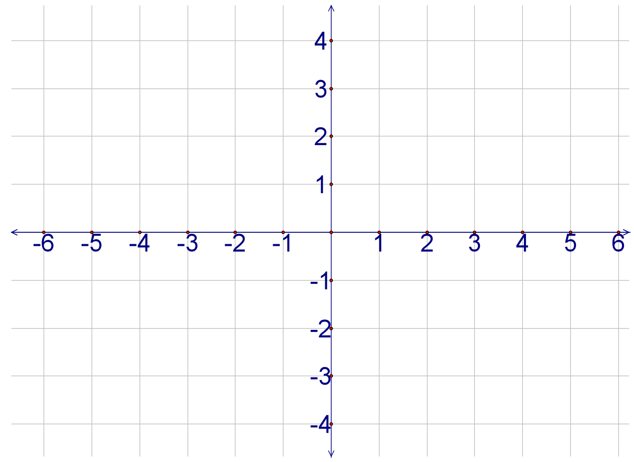
9) Domain $[-6,5]$ $\lim_{x\to -3^{+}} f(x) = -1$, $\lim_{x\to -3^{-}} f(x) = 4$, $f(-3) = 2$, $\lim_{x\to 2} f(x) = -1$, $f(2) = 0$



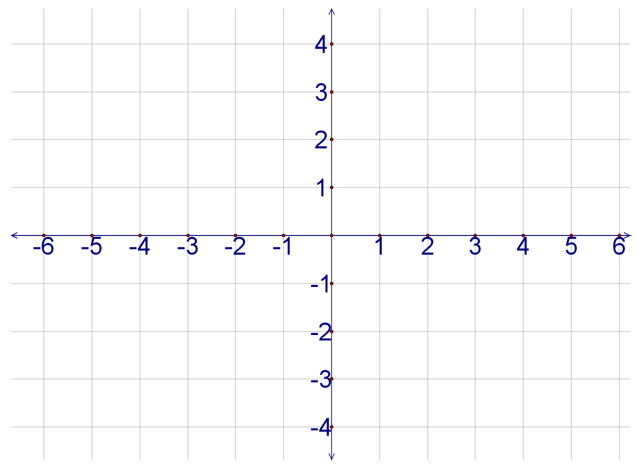
10) Domain $(-\infty,\infty)$, $f(x)$ is continuous at $x = -1$, but not differentiable and $f(-1) = 3$. The x-intercept is -5 and the y-intercept is 2



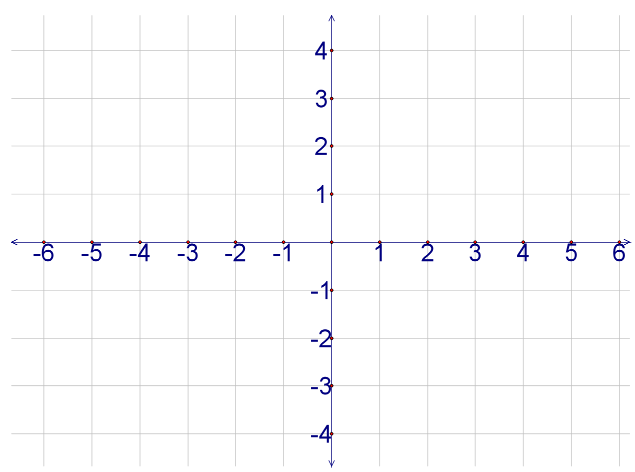
11) $f(-4)=3$, $f'(-2)=0$, $f'(3)=0$, $f'(x)<0$ in the interval $(-6,-2)$, $f'(x)>0$ in the intervals $(-2,3)$ and $(3,5)$, f is continuous on the interval $[-6,5]$



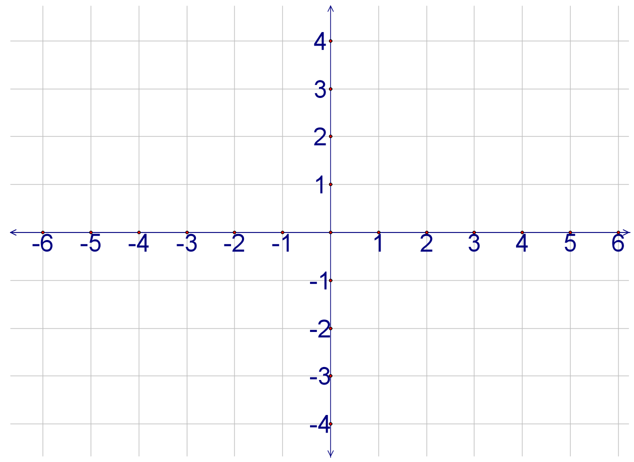
12) f is continuous on the interval $(-3,2)$, $\lim_{x\to -3^+} f(x) \to -\infty$, $\lim_{x\to 2^-} f(x) \to \infty$, $f''(x)<0$ in the interval $(-3,-1)$ and $f''(x)>0$ in the interval $(-1,2)$.



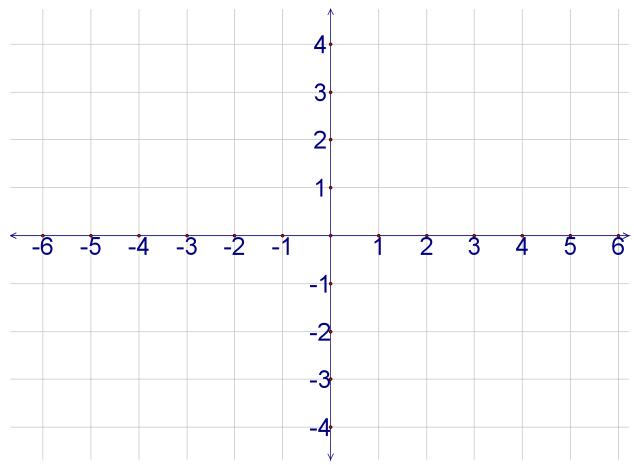
13) The domain of f is $[-4,3)$, $f'(x)>0$ for all x, $f'(2)=0$, there is an absolute minimum of -3, but no absolute maximum.



14) The domain of f is $\left(-2,5\right)$, there is a relative (local) minimum at $x=0$ and a relative maximum at $x=3$ and a point of inflection at $x=2$, but no absolute maximum or absolute minimum



15) The domain of f is $\left[-4,5\right]$, $\lim_{x\to -1^-} f(x)=2$, $\lim_{x\to -1^+} f(x)=4$, $f(-1)=2$, on the interval $\left(-4,-1\right)$ both $f'\left(x\right)\ and\ f''\left(x\right)>0$, on the interval $\left(-1,5\right)$ both $f'\left(x\right)\ and\ f''\left(x\right)<0$.



16) The domain of f is $\left[-4,5\right]$, there are 3 critical points at $x=-2, 1, 3$, absolute (global) maximum at $x=-4$, absolute (global) minimum at $x=1$, relative maximum at $x=3$, but no relative extreme at $x=-2$.